Buenos días!

Curso: Dinámica de fenómenos Críticos

Semestre 2 La CoNGA

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Continuidad espacio de fases

$$\frac{\partial P}{\partial t} = -\sum_i \frac{\partial J_i}{\partial q_i}$$

$$\begin{array}{cc} q_1, & q_2 \\ \downarrow & \downarrow \\ p & x \end{array}$$

$$\frac{\partial P}{\partial t} = 0 \implies \frac{\partial J_i}{\partial q_i} = 0$$

y

$$\frac{dH}{dt} = \sum_i \frac{\partial H}{\partial q_i}\frac{\partial q_i}{\partial t} = \sum_i \underbrace{\frac{\partial H}{\partial q_i} V_i}_{\text{ortogonalidad}} = 0$$

$$\frac{\partial H}{\partial q_1} V_1 + \frac{\partial H}{\partial q_2} V_2 = \dot{q}_2 \dot{q}_1 - \dot{q}_1 \dot{q}_2 = 0$$

Flujo Hamiltoniano en el espacio de fase

$$\bar{\nabla}_{\bar{x}} \cdot \bar{v} = 0$$

Flujo incompresible

$$\sum_i \frac{\partial v_i}{\partial x_i} = 0$$

$$\Rightarrow \sum_i \frac{\partial J_i}{\partial \xi_i} = 0$$

$$P \propto e^{-H/T}$$

distribución canónica

$$J_i = v_i \underbrace{P}_{Ce^{-H/T}} - D_i \frac{\partial P}{\partial \xi_i} - \frac{\Gamma_i}{T}\frac{\partial H}{\partial \xi_i} P$$

$$\frac{\partial J_i}{\partial \xi_i} = \sum_i \left(\cancel{\frac{\partial v_i}{\partial \xi_i}}^{\,0} e^{-H/T} + \cancel{v_i \left(\frac{-1}{T}\right) \frac{\partial H}{\partial \xi_i}}^{\,0} e^{-H/T} \right.$$

$$\left. - D_i \frac{\partial^2 e^{-H/T}}{\partial \xi_i} - \frac{\Gamma_i}{T} \frac{\partial}{\partial \xi_i}\left(\frac{\partial H}{\partial \xi_i} e^{-H/T} \right) \right)$$

$$= \sum_i \frac{D_i}{T} \left[\frac{\partial^2 H}{\partial \xi_i^2} e^{-H/T} - \frac{1}{T}\left(\frac{\partial H}{\partial \xi_i}\right)^2 e^{-H/T} \right]$$

$$- \Gamma_i/T \left[\qquad \text{''} \qquad \right] = 0$$

$D_i = \Gamma_i$ relación de Einstein

η $D = \frac{k_B T}{e}\mu \sim \tau$ de colisión

Término de Fricción

$P \propto e^{-H/T}$ distribución canónica

(Hw) $\frac{\partial q_i}{\partial t} = V_i - \frac{\Gamma_i}{T}\frac{\partial H}{\partial q_i} + \xi_i(t)$ ←

resolve osc.

$\frac{\partial q_2}{\partial t} = \frac{q_1}{m} = V_2$ ←

$2D_i \delta_{ij}\delta(t-t') = \langle \xi_i(t)\xi_j(t')\rangle$

τ_+ , τ_- función de

$$\frac{1}{\tau_1} = \frac{\Gamma_1}{T m} \; ; \quad \frac{1}{\tau_2} = \left(\frac{K}{m}\right)^{1/2}$$

Si suponemos $\frac{\Gamma_1}{T m} >> \left(\frac{K}{m}\right)^{1/2}$ ||

$$\boxed{\frac{1}{\tau_+} \simeq \frac{\Gamma_1 / T}{m}} \qquad \boxed{\frac{1}{\tau_-} \simeq \frac{K T}{\Gamma_1}}$$

τ_+ modo rápido

τ_- " lento

$$\frac{1}{\tau_+} > \frac{1}{\tau_-} \Leftrightarrow \tau_+ < \tau_-$$

Eliminación de modos rápidos

- Análogo a la eliminación de escalas espaciales pequeñas y quedarnos con escalas mayores

$$k = \frac{2\pi}{\lambda} \qquad \lambda_< \qquad \lambda_>$$

- En el espacio nos ocupamos de la renormalización de H
- En el tiempo nos ocupamos de la renormalización de la ecuación de mov

$$\frac{\partial q_1}{\partial t} = V_1 - \frac{\Gamma_1}{T}\frac{\partial H}{\partial q_1} + S_1 \qquad \forall t$$

$$t > t_a$$

$$\frac{q_1}{m} = \frac{\partial q_2}{\partial t} = V_2 - \frac{\Gamma_2}{T}\frac{\partial H}{\partial q_2} + S_2$$

poso inicial $\Gamma_2 = 0 \quad S_2 = 0$

$$\frac{\partial q_1/m}{\partial t} + \underbrace{\frac{\Gamma_1}{mT}}_{\gamma}\frac{q_1}{m} = \underbrace{-\frac{K}{m} q_2 + S}_{\text{inhomogeneidad}}$$

$$\frac{q_1}{m} = -\frac{K}{m}\int_{-\infty}^{t} dt'\, e^{-\tilde{\gamma}(t-t')}\, q_2(t') + \int_{-\infty}^{t} dt'\, e^{-\gamma(t-t')}\, \xi_1(t')/m$$

$\xi'(t)$

$\gamma = \frac{1}{\tau_+}$ $\quad$ γ grande debido a que τ_+ es el tiempo más corto

queremos la física $\quad$ $\tau_+ \ll t < \tau_-$ $\quad$ renormalización temporal

$$\frac{\partial q_2}{\partial t} = -\frac{K_1}{m}\int_{-\infty}^{t} dt'\, e^{-\gamma(t-t')}\, q_2 + \xi'(t)$$

q varia poco

$$= -\boxed{\frac{K_1}{m}}\, q_2(t)\int_{-\infty}^{t} dt\, e^{-\gamma(t-t')} + \xi'(t)$$

$$H_{q_2} = \frac{1}{2}K_1 q_2^2$$

$$\frac{\partial H}{\partial q_2} = K_1 q_2$$

$$= -\frac{\Gamma'}{T}\frac{\partial H}{\partial q_2} + \xi'(t)$$

q_2 modo lento

$$\Gamma' = \frac{T}{m} \left. \frac{e^{-\gamma(t-t')}}{\gamma} \right|_{-\infty}^{t} = \frac{T}{m\gamma}$$

$$\Gamma' = \frac{T}{\cancel{m}} \cdot \frac{1}{\frac{\Gamma_1}{\cancel{m} T}} = \frac{T^2}{\Gamma_1}$$

$$\frac{\Gamma'}{T} = \frac{T}{\Gamma_1} = \frac{1}{K\tau_-} \qquad \frac{\Gamma'}{T} \propto \frac{\tau_-}{K}$$

$$\tau_- = \frac{\Gamma_1}{KT} \quad \frac{1}{K\tau_-} = \frac{T}{\Gamma_1}$$

$$\boxed{\frac{\Gamma'}{T} K = \frac{1}{\tau_-}} \Leftarrow$$

$$\langle \xi_1(t)\, \xi_1(t') \rangle = 2D_1\, \delta(t-t')$$

HW

$$\langle \xi'(t)\, \xi'(t') \rangle = \underbrace{2D_1 \left(\frac{T}{T_1}\right)^2}\frac{\gamma}{2}\, \underbrace{e^{-\gamma|t-t'|}}$$

$$\langle \xi'(t)\, \xi'(t') \rangle \sim 2D'\, \delta(t-t')$$

$q_1\,,\; q_2$

$\downarrow$

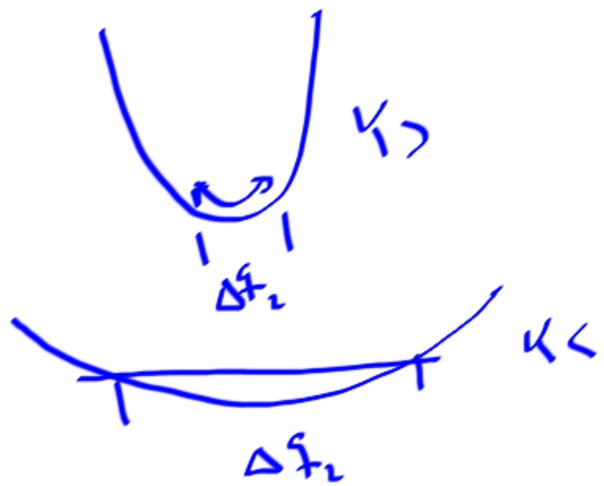
$\tau_\pm$

y modos

$$q_\pm = \tau_\mp\, q_1 + q_2$$

$$\frac{\partial \langle q_- \rangle}{\partial t} = -\frac{\langle q_- \rangle}{\tau_-}\;;\quad \frac{\partial \langle q_+ \rangle}{\partial t} = -\frac{\langle q_+ \rangle}{\tau_+}$$

$$\tau_- = \frac{T}{\Gamma' K} \qquad K \text{ costante del resorte}$$



$$P(q_2) \sim e^{-\frac{1}{2}K\frac{q_2^2}{T}} \sim e^{-\frac{q_2^2}{2T/K}} \quad //$$

$$\langle q_2^2 \rangle = \sigma^2 = T/K$$

evoluzione diffusiva

$$\langle q_2^2 \rangle = D' \tau_- = D' \frac{T}{\Gamma' K}$$

Teoría de Van-Hove $\equiv$ Teoría de Campo medio de la ecuación cinética

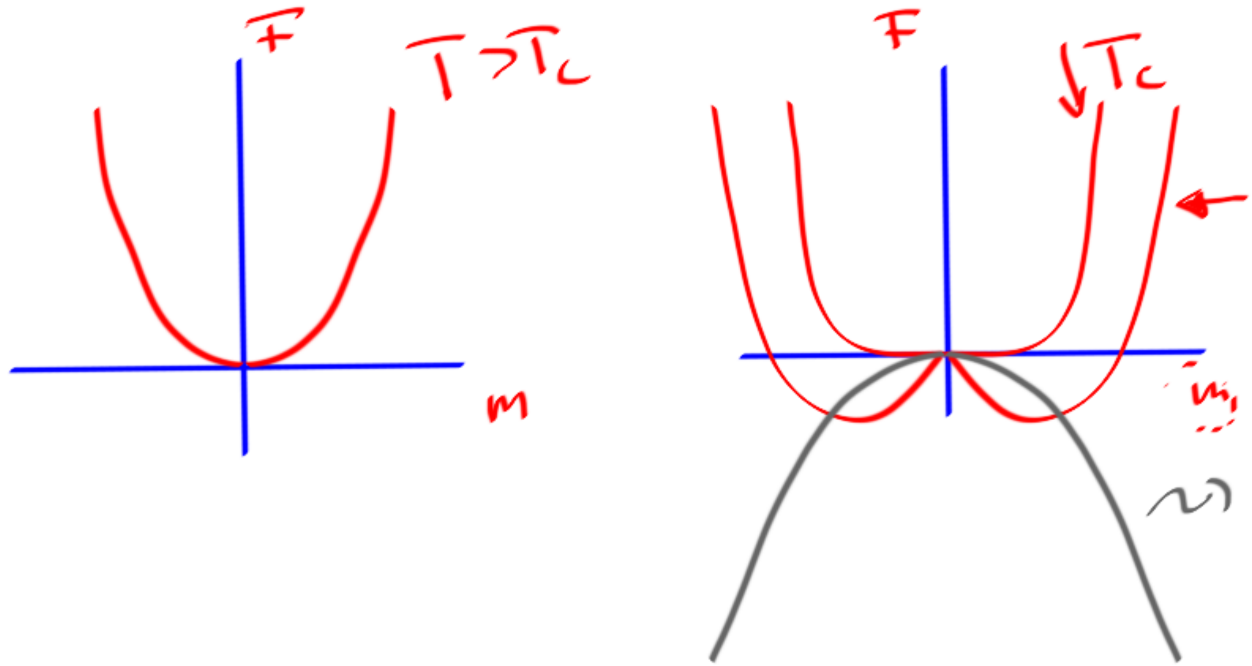
Mode coupling Theory

- incorporar ∞ modos
- No hay solución general para la ec estocástica

$T > T_c$

Modelo Gaussiano

$$\beta H = \int d^3 r \left[a_2 m^2 + c (\nabla m)^2 \right]$$



Modello Gaussiano $T < T_c$

Modi σ_k componente di Fourier di $m(\bar{r})$

$$m = \frac{1}{\sqrt{L^d}} \sum_k e^{i\bar{k}\cdot\bar{r}} \sigma_{\bar{k}}$$

$$\frac{1}{T}\ \beta H = \sum_{k<\Lambda} (a_2 + ck^2)|\sigma_k|^2 \qquad k = \frac{2\pi}{\lambda}$$

$$=$$

$k_> = \frac{2\pi}{a}$

$k < \Lambda$

$2\pi/\lambda < \Lambda$

$\frac{2\pi}{\Lambda} < \lambda$

$$\frac{\partial \sigma_k}{\partial t} = -\frac{\Gamma_k}{\cancel{T}}\, 2\cancel{T}(a_2 + ck^2)\sigma_k \quad \sim \frac{\partial H}{\partial \sigma_k}$$

$$+ \zeta_k$$

$$\langle \zeta_k(t)\,\zeta_{k'}(t')\rangle = 2\Gamma_k\,\delta_{k,-k'}\,\delta(t-t')$$

Supongamos que $\Gamma_k = \Gamma$

T. Fourier inversa

espacio directo

$$\begin{cases} \dfrac{\partial m}{\partial t} = -2\Gamma\left(a_2 m - c\nabla^2 m\right) + \zeta(t) \\ \langle \zeta(x',t')\,\zeta(x,t)\rangle = 2\Gamma\,\delta(x-x')\,\delta(t-t') \end{cases}$$

$$\frac{1}{\tau_k} = \overbrace{2\Gamma\left(a_2 + ck^2\right)}$$

infinitos tiempos característicos

en el límite de $k \to 0$ $\lambda \to \infty$

$$\tau_k \simeq \frac{1}{2a_2}\frac{1}{\Gamma}$$

$a_2'(T-T_c)$ → cercanía a la transición

$$\tau_k = [2a_2'(T-T_c)]^{-1}\,\Gamma^{-1}$$

$$T \to T_c^+$$

τ_k diverge alentamiento crítico!