Buenos días!

Curso: Dinámica F. Críticos

Semestre 2 La CoNGA

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Teoria di Van-Hove

Teoria Gaussiana

parametro di ordine

$$\beta \mathcal{H} = \int d^3r \; a_2 m^2 + c(\nabla m)^2 + \cancel{m^4}$$

F.T.

$\sim \frac{2\pi}{\lambda}$ $\qquad \lambda \sim a$ $\sim$ parametro di reticolo

$$m = \frac{1}{\sqrt{V}} \sum_k e^{i\vec{k}\cdot\vec{r}} \sigma_k$$

$k_B = 1$

$$\beta \mathcal{H} = \sum_{k<\Lambda} (a_2 + ck^2) |\sigma_k|^2$$

$$\frac{\partial \sigma_k}{\partial t} \overset{\sim \text{ modos}}{=} -\frac{\Gamma_k}{\cancel{T}}\, 2\cancel{T}\, \underbrace{(a_2 + ck^2)\sigma_k}_{\frac{\partial \mathcal{H}}{\partial \sigma_k}} + \zeta_k$$

$$\langle \zeta_k(t)\, \zeta(t') \rangle = 2\Gamma_k\, \delta_{-k,k'}\, \delta(t-t')$$

$$\delta(k+k')$$

Modos desacoplados

$$\frac{1}{\tau_k} = 2\Gamma_k\,(a_2 + ck^2)$$

si $k \to 0$ $\lambda \to \infty$ y Γ constante

$$a_2 = a_2'(T - T_c)$$

revisar teoría de Landau (Kardar)

$$\tau_k = \frac{1}{2a_2}\frac{1}{\Gamma}$$

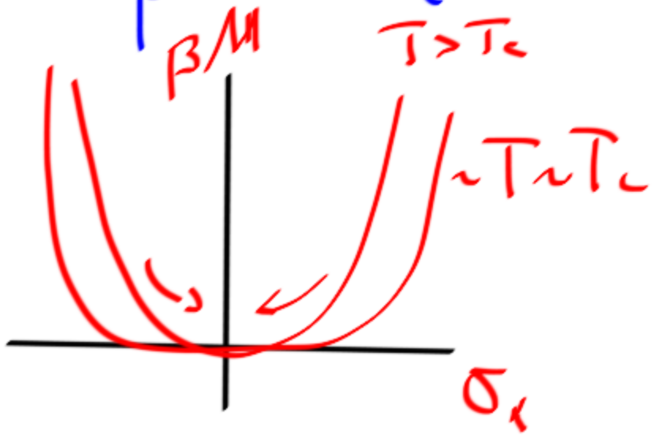
$$\tau_k = \frac{1}{2\Gamma a_2'}(T - T_c)^{-1}$$

$T \to T_c^+$ τ_k diverge

alentamiento critico

critical slowing down

$$\beta \mathcal{H} = (a_2 + ck^2)\,|\sigma_k|^2$$



F. Críticos estáticos

T. C. Medio

$$\xi = \left(\frac{a_2}{c}\right)^{-1/2} = \left(\frac{a_2'}{c}\right)^{-1/2} \underbrace{(T-T_c)}_{t}^{-1/2}$$

$$\tau_k = \left[2\Gamma(a_2 + ck^2)\right]^{-1} \sim \xi^2 F(k\xi)$$

$$\tau_k = \left[\frac{2\Gamma c}{\xi^2}\left(1 + \xi^2 k^2\right)\right]^{-1} = \xi^2 \left[2\Gamma c\right]^{-1} \left(1+\xi^2 k^2\right)^{-1}$$

$\tau_k = \xi^z f(k\xi)$ hipótese de escalamento dinâmico

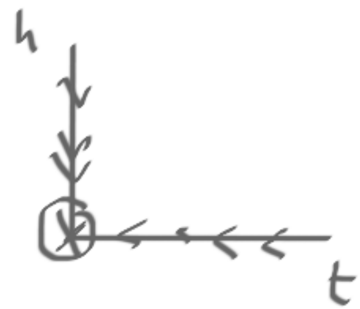
Gaussiano $\longleftrightarrow$ T. Campo Médio

$z = 2$ expoente dinâmico

Repasso

$t = T - T_c$ $\quad h$ campo magnético

$$f(t,h) = |t|^2 \, g\left(h/|t|^{\Delta} \right)$$



$h = 0$

$g(0) \sim \text{const}$

$f(t,h) \sim t^2 \, \text{const}$

Aproximación por h a $t=0$

$$f(t,h) = |t|^{2} g(\infty)$$

$$g(\infty) = \left(\frac{h}{|t|^{\Delta}}\right)^{\alpha}$$

$$= |t|^{2}\left(\frac{h}{|t|^{\Delta}}\right)^{2/\Delta}$$

$$2 = \Delta\alpha$$

$$\alpha = \frac{2}{\Delta}$$

$$f(t,h) = h^{2/\Delta}$$

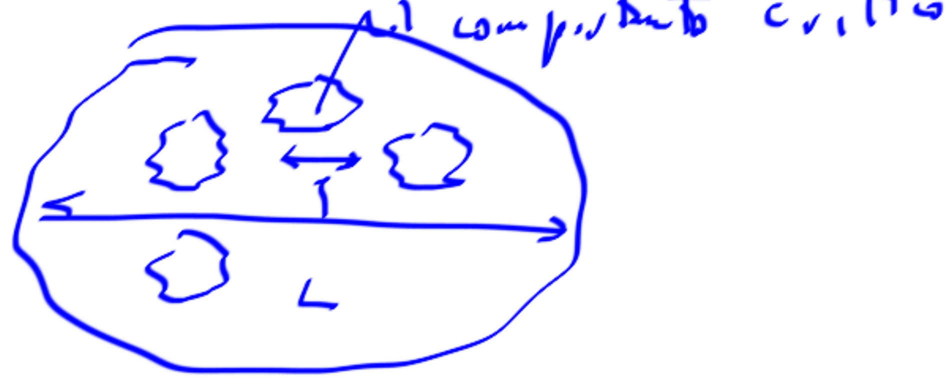
$$\tau_h = \xi^z f(h\xi) \qquad h=0 \quad \lambda \to \infty$$

$$\tau_h \simeq \xi^z = |t|^{-\nu z} \qquad \text{critical slowing down}$$

$$\tau_h \simeq \xi^z (h\xi)^\alpha \qquad 1/\xi = 0 \quad \xi \to \infty$$

$$\alpha = -z$$

$$\tau_h \simeq c\, h^{-z} \qquad \lambda < \xi$$



$$\tau_k = c\,\xi^2 \frac{1}{1+\xi^2 k^2} \quad \begin{cases} \to c\,\xi^2 & k=0 \\ \to c\,k^{-2} & \xi\to\infty \end{cases}$$

Comentarios:

A) $\Gamma_k = \Gamma$ dinámica no-conservada

B) $\Gamma_k = \gamma k^2$ dinámica conservada $\quad \boxed{\Gamma_k = \Gamma + \gamma k^2 + \mathcal{O}(k^4)} \ \ldots$

Hohenberg + Halperin

$$\tau_k = \left[2\left(a_2 + c k^2\right)\right]^{-1} \Gamma_k^{-1}$$

$$= \left(2 c k^2\right)^{-1} \frac{1}{\gamma k^2} \sim \frac{1}{2c\gamma} k^{-4}$$

y nueva clase de univ. $z=4$

– Formulación de la dinámica de un campo

1) Sintonización de un parámetro para acercarme a criticalidad

2) Invariancia de escala general (leyes de escala sin sintonizar parámetros)

$$\beta \mathcal{H} = \int d x^d \left[\frac{r}{2} m^2 + u m^4 + \frac{K}{2} (\nabla m)^2 + \dots \right]$$

$$F_i(\bar{x}) = -\frac{\delta \mathcal{H}}{\delta m_i(\bar{x})} = -r\, m_i - 4u\, m_i |\bar{m}|^2 + k\nabla^2 m_i$$

- Greiner Cuantización de campos (Teoría clásica de campos)

F_i

Modelo estocástico

$$m\ddot{\vec{x}} = -\frac{\dot{\vec{x}}}{\mu} - \frac{\partial V}{\partial \bar{x}} + f_{\text{aleatoria}}$$

desprecio efectos inerciales

$$\frac{\dot{\vec{x}}}{\mu} = -\frac{\partial V}{\partial \bar{x}} + f_{\text{aleatoria}}$$

grano grueso

$$\dot{\vec{x}} = -\mu \frac{\partial V}{\partial \bar{x}} + f_{\text{aleat.}}$$

$$\frac{\partial \vec{m}}{\partial t} = -\mu r \vec{m} - 4\mu \underbrace{\underline{\underline{u}}\, m^2 \vec{m}}_{m^4 = \vec{m}\cdot\vec{m}\ \vec{m}\cdot\vec{m} = m_i m_i m_j m_j} + \mu K \nabla^2 \vec{m} + \vec{\eta}(x,t)$$

$\vec{m}\cdot\vec{m}$

H.W $\longrightarrow$ derivación de ecuación

$u = 0$

Gaussiano

$$m(\vec{q},t) = \int dx^d\, e^{i\vec{q}\cdot\vec{x}}\, m(\vec{x},t)$$

$$\frac{\partial m_i(q,t)}{\partial t} = -\mu \left(r + K q^2 \right) m_i(q,t) + \eta(q,t)$$

$\frac{1}{\tau_q} \rightarrow \underline{\underline{z = 2}}$

$$\langle \eta_i(\xi,t)\,\eta_j(\xi',t')\rangle = \int d^d_x\, d^d_{x'}\; e^{i\bar{\xi}\cdot\bar{x}}\, e^{i\xi'\cdot x'}$$

$$\langle \eta_i(\bar{x},t)\,\eta_j(\bar{x}',t')\rangle$$

$$2D\,\delta_{ij}\,\delta^d(\bar{x}-\bar{v}')\,\delta(t-t')$$

$$= 2D\,\delta_{ij}\;\delta(t-t')\,\delta(\xi+\xi')$$

$$\delta_{\xi,-\xi'}$$

Modello Gaussiano

H.W. $\beta \mathcal{H} = \int \frac{d^d q}{(2\pi)^d} \frac{(r + K q^2)}{2} |m(q)|^2$

$e^{-\beta \mathcal{H}}$ $\int dx \, e^{-\frac{x^2}{2\sigma^2}}$

$k_B = 1$ $\langle m(q) m(q) \rangle = \frac{T}{(r + K q^2)}$

$\langle m_i(\bar{q}) m_j(\bar{q}') \rangle = (2\pi)^d \delta(q + q') \delta_{ij} \frac{D/n}{r + K q^2}$

$k_B T \mu = D$ Relación de Einstein

Fokker - Planck

$$\frac{\partial P}{\partial t} = - \int d^d x \, \frac{\delta}{\delta m_i(\vec{x})} \overbrace{\left[-\mu \frac{\delta \mathcal{H}}{\delta m_i(\vec{x})} P - D \frac{\delta P}{\delta m_i(\vec{x})} \right]}^{J}$$

Si $\frac{\partial P}{\partial t} = 0 \Rightarrow P_{eq} \propto e^{-\beta \mathcal{H}}$

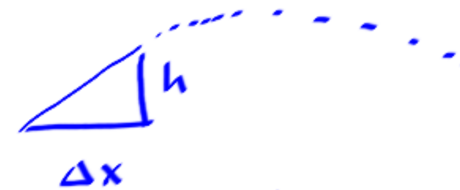
Autosintonización (Invariancia de escala genérica)

— Pilas de arena Per Bak
criticalidad autoorganizada

Ondas capilares:

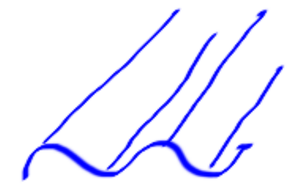
Fenómenos de escalas

Tensión superficial



$$\beta \mathcal{H} = \sigma \Delta A \quad \sim \text{incremento de longitud}$$

$$= \sigma \left(\sqrt{\Delta x^2 + h^2} - \Delta x \right)$$



$$\beta \mathcal{H} = \int dx^d \left(\sqrt{1 + (\nabla h)^2} - 1 \right)$$

$$\nabla h \to 0 \qquad 1 + \frac{1}{2} (\nabla h)^2 - 1$$

$$\beta \mathcal{H} = \frac{\sigma}{2} \int dx^d \, (\nabla h)^2 \quad \text{elastico}$$

gravity $= \int dx^d \int_0^h \rho g\, h(x)\, dh = \int dx^d \rho g \frac{h^2(x)}{2}$

$$\beta \mathcal{H} = \int dx^d \left[\frac{\sigma}{2} (\nabla h)^2 + \frac{\rho g}{2} h^2 \right]$$

incompressible

$$\frac{\delta \beta \mathcal{H}}{\delta h(y)} = -\sigma \nabla^2 h + \rho g h$$

$$\frac{\partial h(x,t)}{\partial t} = -\frac{\mu}{T} \frac{\delta \beta \mathcal{H}}{\delta h(x)} + \eta$$

$$\frac{1}{\tau_q} = \left(\rho g + \mu\sigma q^2\right) \qquad z = 2$$

$$t_{min} = \frac{a^2}{\mu\sigma} \qquad t_{max} = \frac{L^2}{\mu\sigma}$$

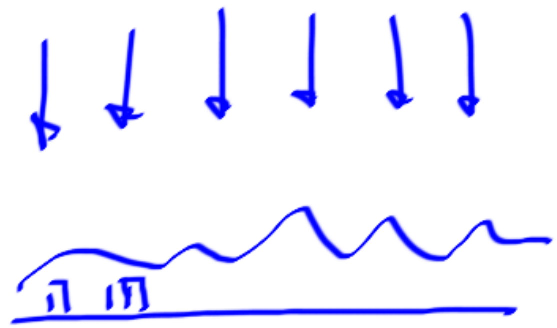
$$\lambda_{capilar} = \sqrt{\frac{\sigma}{\rho g}}$$

Ver discusión de Kardar. HW 9.53 – 9.60

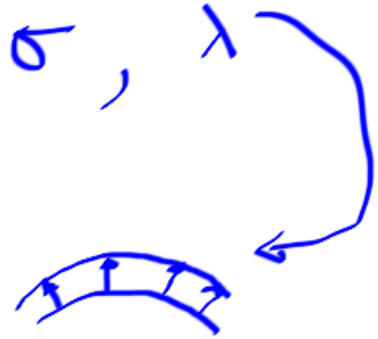
$$w^2 = \langle [h(x,t) - h(x',t')] \rangle = |x-x'|^{2\chi} g\left(\frac{|t-t'|}{|x-x'|^z}\right)$$

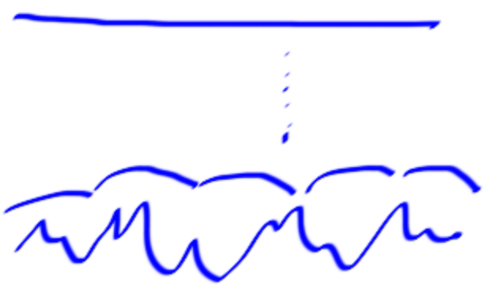
$$w \sim t^\beta \qquad \beta = \chi/z$$

KPZ: No-lin



ν, σ, λ





Ec. de Burgers

$$\frac{\partial h}{\partial t} = u + \nu \nabla^2 h + \frac{\lambda}{2} (\nabla h)^2 + \eta$$

difusión superficial

crecimiento lateral

análisis dimensional

z

ν, λ como escalan $t, \bar{x}$

Tauber, Zia, Schlottmann

Henkel