Buenas tardes!

Proyectos MC + DM

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Clase 19

Probabilidad conjunta

Ec. Liouville ecuacion de movimiento de un sistema de N particulas $\Rightarrow$ densidad de probabilidad

Jerarquia BBGKY $\qquad f_n(x_1, x_2, x_3 \ldots x_n)$ $\qquad$ funcion de dist. de n part.

distribuciones conjuntas

Dados $X_1, X_2, X_3 \ldots X_n$ variables aleatorias cada una tomada de una dist. podemos construir un vector

$$\underline{X} = (X_1, X_2, X_3 \ldots X_n)$$

proceso estocástico

$$P(X \in \beta) = \int_{\beta} f(x_1, \dots, x_n)\, dx_1 \dots dx_n$$

densidad

densidad de probabilidad marginal

densidad reducidas

reducida

$$f_X(x) = \int f_{XY}(x,y)\, dy$$

densidad de prob de x independ de lo que hace y regla de Bayes

$$f_{Y|X}(y|x) = \frac{f_{YX}(x,y)}{f_X(x)}$$

$$E[Y \mid X=x] = \int y \, f_{Y|X}(y|x) \, dx$$

si variables aleatorias son indep.

$$\Rightarrow f(x_1, \dots x_n) = f(x_1) f(x_2) \dots f(x_n)$$

if $X_1, X_2 \ldots X_n$ are indp

$$E[a + b_1X_1 + b_2X_2 + \ldots b_nX_n]$$

$$= a + b_1\mu_1 + b_2\mu_2 + \ldots + b_n\mu_n$$

$$E[X_i] = \mu_i$$

$$E[X_1X_2 \ldots X_n] = \mu_1\mu_2\mu_3 \ldots \mu_n$$

Covarianza

$$\mathrm{Cov}(X,Y) = E[(X-\overset{E[X]}{\mu_x})(Y-\overset{E[Y]}{\mu_y})]$$

$$\mathrm{Var}(X) = \mathrm{Cov}(X,X)$$

$$\rho(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sigma_x \, \sigma_y}$$ Cov. escalada

$$\sigma_x = \sqrt{\mathrm{Var}(X)}$$

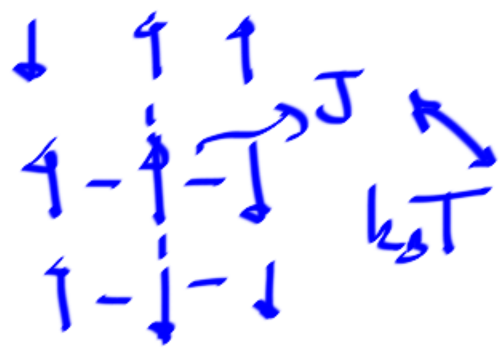
$$Cov(X,Y) = E[XY] - E[X]E[Y]$$

si X, Y son indep.

$$= 0$$

$\boxed{\text{Ej } 1}$

[illegible]



Funciones de variables aleatorias:

$X \longrightarrow$ se extrae de una distribución determinada

cual es la distribución de una función de X

$f_X(x)$ $\qquad z = g(x)$

$f_Z(z)$?

Ex

sea $Z = aX + b$

con $a \neq 0$

$$F_Z(z) = P(Z \le z)$$

$$= P\left(X \le \frac{z-b}{a}\right)$$

$$= F_X\left(\frac{z-b}{a}\right)$$

ni integrando

$$f_Z(z) = \frac{d}{dz} P(Z \le z) = \frac{d}{dz} P\left(X \le \frac{z-b}{a}\right) = \frac{d}{dx} P\left(X \le \frac{z-b}{a}\right) \frac{dx}{dz}$$

$$x = \frac{z-b}{a} \qquad \frac{dx}{dz} = \frac{1}{a}$$

$$f_z(z) = f_x\left(\frac{z-b}{a}\right) \cdot \frac{1}{a}$$ pero $a > 0$

s. $a < 0$ $\qquad z = -|a| X + b$

$$f_z(z) = f_x\left(\frac{z-b}{a}\right) \frac{1}{|a|}$$

$\boxed{\text{Ej } 2}$

probar

Ex from General

invertible

$$z = g(x) \Rightarrow x = g^{-1}(z)$$

$$F_z(z) = P(z \leq z) = P(x \leq g^{-1}(z))$$

$$= F_x(g^{-1}(z))$$

$$f_z(z) = \frac{d\,P(z \leq z)}{dz} = \frac{d}{dx} P(x \leq g^{-1}(z)) \cdot \frac{dx}{dz}$$

$$f_z(z) = f_X(g^{-1}(t)) \frac{d g^{-1}(t)}{dt} \quad //$$

<u>Ex</u>

$$z = x^2 \rightsquigarrow g^{-1}(z)$$

$$x = \pm\sqrt{z}$$

$$\boxed{Ej.}$$

$$f_z(z) = f_x(\sqrt{z}) \frac{1}{\sqrt{z}}$$

Ex

$\sigma'^2 = \sigma^2 a^2$

$\langle x \rangle = 0$

$\langle z \rangle = b$

$f_x =$ Gaussiana

$= \frac{1}{\sqrt{2\pi}} e^{-x^2/2\sigma^2}$

$\langle z \rangle = a \cancel{\langle x \rangle}^{0} + b$

$z = a x + b$

y

$f_z(z) = \frac{1}{|a|} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\left(\frac{z-b}{a}\right)^2 \frac{1}{2\sigma^2}}$

Forma canonica

$= \frac{1}{\sqrt{2\pi\sigma'^2}} e^{-\frac{(y-\langle y \rangle)^2}{2\sigma'^2}}$

$$f_x = \text{Gaussian} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-x^2/2\sigma^2}$$

y

Eg.

$$z = a e^{x}$$

$$x = \ln \frac{z}{a} = g^{-1}(z)$$

$$f_z(z) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}\left(\ln \frac{z}{a}\right)^2} \cdot \frac{a}{z}$$

log normal

Procesos estocásticos (multivariables)

Transformaciones lineales

Sea $\vec{x} = (x_1 \ldots x_n)^T = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$

y sea A una matriz $m \times n$

la transformación de $\vec{x} \Rightarrow \vec{z}$

$$\vec{z} = A\vec{x}$$

Como se comporta $\vec{\mu}_z$, $\overleftrightarrow{\Sigma}_z$

1) $\vec{\mu}_z = A\vec{\mu}_x$

2) $\overleftrightarrow{\Sigma}_z = A\,\Sigma_x A^T$

1) $\vec{\mu}_z = E[\vec{z}\,] = E[A\vec{x}\,] = A\,E[\vec{x}\,]$

$\vec{\mu}_z = A\vec{\mu}_x$ //

$$\vec{\Sigma}_z = E\left[(\bar{z}-\bar{\mu}_z)(z-\mu_z)^T\right]$$

$$= E\left[A(\bar{x}-\bar{\mu}_x)\left(A(\bar{x}-\bar{\mu}_x)\right)^T\right]$$

$$= E\left[A(\bar{x}-\bar{\mu}_x)(\bar{x}-\bar{\mu}_x)^T A^T\right]$$

$$= A\; E\left[(\bar{x}-\bar{\mu}_x)(x-\mu_x)^T\right] A^T$$

$$\vec{\Sigma}_z = A\,\vec{\Sigma}_x\,A^T$$

$$f_{\vec{Z}}(\bar{z}) = \frac{f_{\vec{X}}(A^{-1}z)}{|A|} \quad \mathbb{E}_{j}$$

$|A| \rightarrow$ determinante de A.

que ocurre si $\vec{Z} = g(\vec{X})$

$$f_{\bar{Z}}(\bar{z}) = f_{\vec{X}}\left(g^{-1}(\vec{z})\right) \left| J_z(g^{-1}) \right| \ \text{det}$$

$$\rightsquigarrow \quad J_x(g) = \begin{vmatrix} \partial g_1^{-1}/\partial x_1 & \cdots & \partial g_1^{-1}/\partial x_n \\ \partial g_n^{-1}/\partial x_1 & \cdots & \partial g_n^{-1}/\partial x_n \end{vmatrix}$$

Gaussian multidimensional

Una variable $f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ $\boxed{Ej}$

Varianza unidad

$\rightsquigarrow$ probar normal

Si hacemos la transform

$$z = \mu + \sigma X \Rightarrow X = \frac{z-\mu}{\sigma}$$

$$f_z(z) = \frac{1}{\sqrt{2\pi}\,|\sigma|} e^{-\frac{1}{2}\left(\frac{z-\mu}{\sigma}\right)^2}$$

en n dimensiones

$\vec{X} = (X_1, \dots X_n)$

X_i independientes

$$f_{\vec{X}}(\vec{X}) = \frac{1}{(2\pi)^{n/2}} e^{-\frac{1}{2}(X_1, \dots X_n)\begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix}}$$

$$e^{-\frac{1}{2}(X_1, \dots X_n)(X_1, \dots X_n)^T}$$

$$= \frac{1}{(2\pi)^{n/2}} e^{-\frac{1}{2} X X^T}$$

$$\vec{Z} = \vec{\mu} + B\vec{X}$$

$$\Sigma_X = BB^T$$

$$E[\vec{Z}] = \vec{\mu}$$

$$\vec{Y} = \bar{z} - \bar{\mu} = B\vec{X}$$

$(x_1, \dots, x_n)$

$$f_{\bar{Y}}(\bar{y}) = \frac{f_{\bar{X}}(B^{-1}\bar{y})}{|B|} \qquad \mu$$

$$= \frac{1}{|B|(2\pi)^{n/2}} e^{-\frac{1}{2}\left((B^{-1}y)^T(B^{-1}y)\right)}$$

$$= \frac{1}{|B|(2\pi)^{n/2}} e^{-\frac{1}{2} y^T (B^{-1})^T B^{-1} y}$$

$$(B^{-1})^T B^{-1} = (B^T)^{-1} B^{-1} = (B^T B)^{-1}$$
$$= \Sigma_x^{-1}$$

$$B^T B = \Sigma$$

$$|B^T B| = |\Sigma|$$

$$|B^T||B| = |B|^2 = |\Sigma|$$

$$|B| = \sqrt{|\Sigma|}$$

$$f_{\bar{y}}(\bar{y}) = \frac{1}{\sqrt{|\Sigma|}(2\pi)^{n/2}} e^{-\frac{1}{2} y^T (\Sigma^{-1}) y}$$

$$= \frac{1}{\sqrt{(2\pi)^n |\Sigma|}} e^{-\frac{1}{2} (t-\mu)^T \Sigma^{-1} (t-\mu)}$$

Teorema límite próxima clase

$(B^T)^{-1} = (B^{-1})^T$ Ej

- $h(x) = \int_{\operatorname{tg} x}^{x^2} f(t)\,dt$

- $\dfrac{d\,h(x)}{dx} = -f(\operatorname{tg} x)\sec^2 x + 2 f(x^2)\, x$

demostrar //.

Proxima clase

Viernes 14 horas

Clase 20