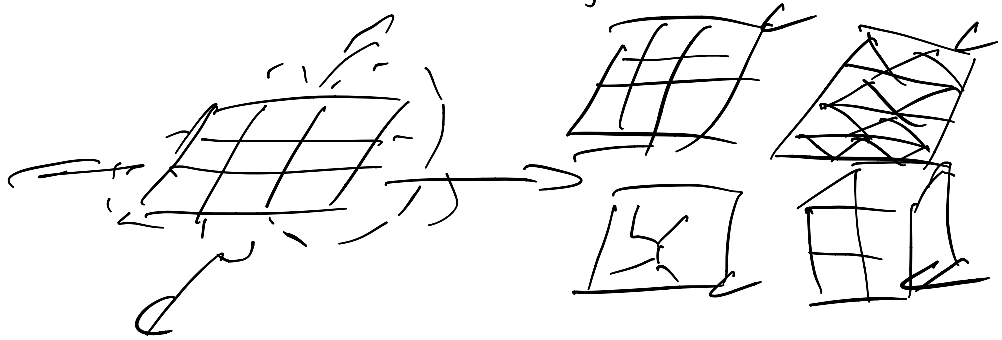
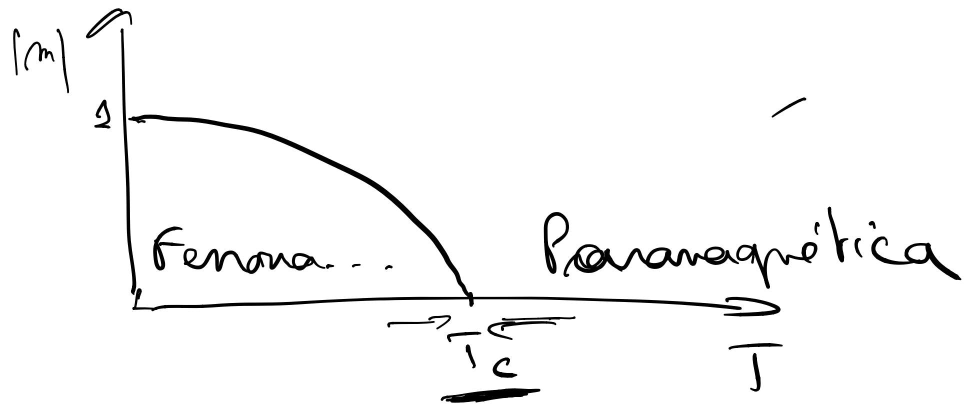
Condiciones de borde periódicas.





exponentes críticos

$T \lesssim T_c$ $\quad |m| \sim (T_c - T)^{\boxed{\beta}}$

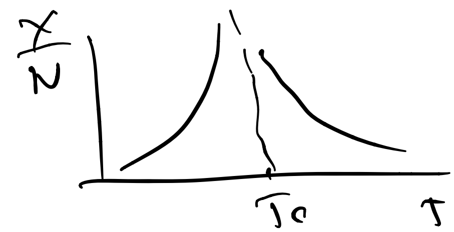
$$\langle \sigma_i \sigma_j \rangle - \langle \sigma_i \rangle \langle \sigma_j \rangle \underset{\substack{T \sim T_c \\ |i-j| \to \infty}}{\sim} \frac{e^{-\frac{|i-j|}{\xi}}}{|i-j|^{d-2+\boxed{\eta}}}$$

$$\xi \xrightarrow[T \to T_c]{} \infty \qquad \xi \underset{T \to T_c}{\sim} |T - T_c|^{-\boxed{\nu}} \text{ en principio}$$

podría haber un ν_+ y ν_- en general $\nu_+ = \nu_-$

$$\chi = \beta \sum_{i,j}{}' \left[\langle\sigma_i\sigma_j\rangle - \langle\sigma_i\rangle\langle\sigma_j\rangle\right]$$

$$\frac{\chi}{N}\Big|_{h=0} \sim |T-T_c|^{-\boxed{\gamma}}$$



$$\frac{C}{N} \sim |T-T_c|^{-\boxed{\alpha}}$$

→ Universalidad!

$$T - T_c = t \qquad \xi \gg a$$

$$\chi \sim \beta N \sum_{j}{}' \left[\langle\sigma_0\sigma_j\rangle - \langle\sigma_0\rangle\langle\sigma_j\rangle\right]$$

$$\frac{\chi}{N} \sim \beta \int \left[\langle\sigma(0)\,\sigma(\vec{r})\rangle - \langle\,\rangle^2\right] d^d\vec{r}$$

$$\sim \frac{1}{r^{d-2+\eta}}\, e^{-r/\xi} \sim \xi^{2-\eta}$$

para $\xi \sim t^{-\nu}$

$$\frac{\chi}{N} \sim t^{-\nu(2-\eta)} \sim t^{-\gamma}$$

$$\boxed{\frac{\gamma}{\nu} = 2-\eta}$$

$E \sim k_B T_c$

$[f] = \left[\frac{F}{N}\right] = \frac{E}{L^d}$ $\quad L \Leftrightarrow \xi$

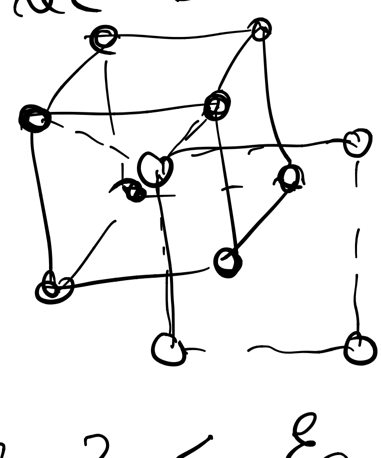
$$f \sim \frac{k_B T_c}{\xi^d} \sim t^{\nu d}$$

$$\frac{C}{N} \sim \frac{\partial^2}{\partial T^2} f \sim t^{\nu d - 2} \sim t^{-\alpha}$$

$$\Rightarrow \boxed{2-\alpha = \nu d} = 2\beta + \gamma$$

3) Aplicación a las mezclas binarias (3-D)

en una red BCC



Cu y Zn

$$\varepsilon_{Cu-Zn} < \varepsilon_{Cu-Cu}, \varepsilon_{Zn-Zn}$$

N átomos, $\frac{N}{2}$ Cu, $\frac{N}{2}$ Zn

$$E = N_{Cu-Cu}\,\varepsilon_{Cu-Cu} + N_{Zn-Zn}\,\varepsilon_{Zn-Zn} + N_{Zn-Cu}\,\varepsilon_{Zn-Cu}$$

$\frac{N}{2}$ átomos de Cu participan a $8 \times \frac{N}{2} = \cancel{4} \, 4N$ ligaduras. Cu-Cu o Cu-Zn

$$\Rightarrow 2N_{Cu-Cu} + N_{Cu-Zn} = 4N$$

$$2N_{Zn-Zn} + N_{Cu-Zn} = 4N$$

$$\Rightarrow N_{Cu-Cu} = N_{Zn-Zn} = 2N - \frac{N_{Cu-Zn}}{2}$$

$$\Rightarrow E = \boxed{\underline{2N}\left(\varepsilon_{Cu-Cu} + \varepsilon_{Zn-Zn}\right)} = \text{cte.}$$

$$+ \underline{N_{Cu-Zn}\left(\varepsilon_{Cu-Zn} - \frac{\varepsilon_{Cu-Cu} + \varepsilon_{Zn-Zn}}{2}\right)}$$

definimos

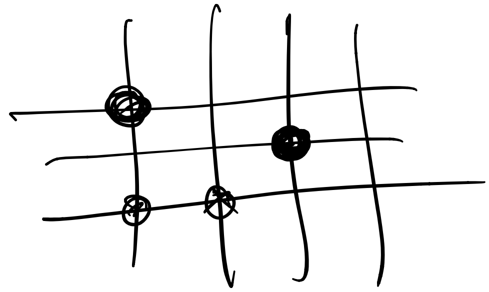
en el subláttice A: $\sigma_i = \begin{cases} +1 & \text{si Cu} \\ -1 & \text{si Zn} \end{cases}$

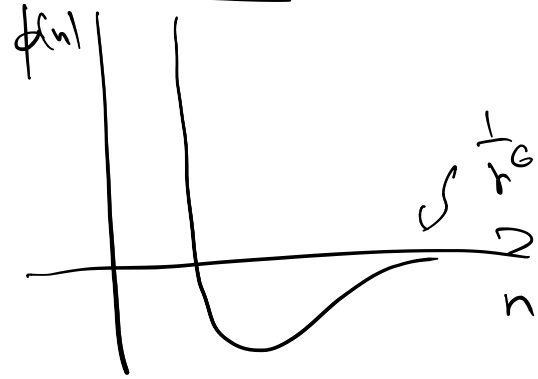
" " B: $\sigma_j = \begin{cases} -1 & \text{si Cu} \\ +1 & \text{si Zn} \end{cases}$

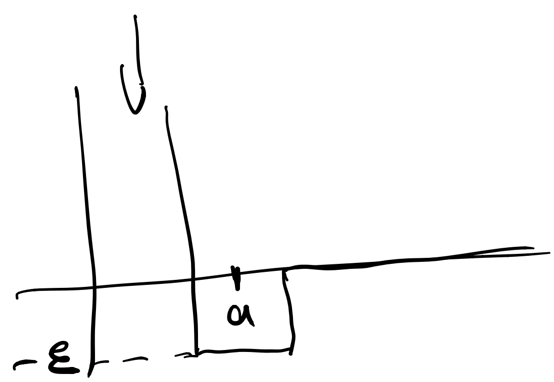
$$E \neq H = -J \sum_{\langle i,j \rangle}' \sigma_i \sigma_j + cte$$

si $J = \dfrac{\varepsilon_{cu-cu} + \varepsilon_{zn-zn}}{4} - \dfrac{\varepsilon_{cu-zn}}{2} > 0$

4) Gas en la red (Lattice gas)







$$E = \frac{1}{2} \sum_{i,j}' \phi(|i-j|)$$

$n_i = \dfrac{1+\sigma_i}{2} \qquad \sigma_i = \pm 1$

número de partículas en "i"

$$N_p = \sum_i n_i = \frac{N}{2} + \frac{1}{2}\sum_i \sigma_i$$

N = número de sítios

$$\underline{N_p = \frac{N}{2} + \frac{1}{2} M}$$

$$Z = \sum_{\{n_i\}} e^{-\beta\left[E - \mu \sum_i n_i\right]} \longleftrightarrow Z_{Ising}^{(h)}$$

$$E = \sum_{\forall i,j} \phi(|i-j|)\, n_i n_j$$

$$E - \mu \sum_i n_i \rightarrow -\underbrace{\frac{\varepsilon}{4}}_{J} \sum_{\langle ij \rangle} \sigma_i \sigma_j - \underbrace{\left(\frac{2\mu + z\varepsilon}{4}\right)}_{h} \sum_i \sigma_i$$

z = número de vizinhos de cada sítio.

Fine-tuning

$$\rho \sim m \qquad \frac{1}{\rho^2}\frac{\partial \rho}{\partial \mu} = k \frac{\chi}{N}$$

$$\rho^2 k \sim \frac{\chi}{N}$$

5) Modelo de Heisenberg


sitio "i"

se define $\vec{S}_i$ tq $\vec{S}_i \cdot \vec{S}_i = 1$

$$\boxed{H(\vec{h}) = -\sum_{i,j} J_{i,j}\, \vec{S}_i \cdot \vec{S}_j - \vec{h} \cdot \sum_i \vec{S}_i}$$

$\vec{h} = \vec{0}$

$$\vec{S}_i = \begin{pmatrix} S_i^x \\ S_i^y \\ S_i^z \end{pmatrix}$$

$R^T R = Id$

$\det R = \pm 1$

$\boxed{O(3)}$

subgrupo $SO(3) \longrightarrow \det = 1$.

$$\begin{pmatrix} S^x \\ S^y \\ S^z \end{pmatrix}$$