$$Z = \sum_{\{\sigma\}} e^{-\beta H\{\sigma\}} \qquad H = -\sum_{ij} \frac{J_{ij}}{2}\sigma_i\sigma_j$$

simetría $\mathbb{Z}_2$ $\forall_i\, \sigma_i \to -\sigma_i$

Transf. de H. S.

$$Z = \text{cte} \int \mathcal{D}\{\phi\}\, e^{-S\{\phi\}}$$

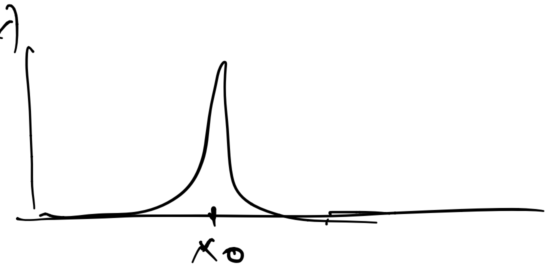
$$\mathcal{D}\{\phi\} = \lim_{N\to\infty} \prod_i d\phi_i$$

$$S(\phi) = \int d^D\vec{r} \left[\frac{c}{2}(\vec{\nabla}\phi)^2 + \frac{a_2}{2}\phi^2 + \frac{a_4}{4}\phi^4 + \dots \right]$$

$$\boxed{c = \frac{\ell^2}{\beta \hat{J}(0)}} \; ; \; a_2 = \frac{k_B T}{\hat{J}(0)} - \frac{1}{a^d}$$

acción de Ginzburg-Landau

$$Z = \int_{-\infty}^{\infty} dx\, e^{-f(x)}$$



$$\approx$$

$$Z \sim e^{-f(x_0)} \qquad -\ln Z \sim f(x_0)$$

* Configuraciones que minimizan $S(\phi)$.
* obs la simetría de $S(\phi)$

$$\phi(\vec{r}) \to -\phi(\vec{r}) \quad \text{simetría } \mathbb{Z}_2$$

$$\phi \sim m(\vec{r})$$

$$Z\{h_i\} = \sum_{\{\sigma\}} e^{\frac{\beta}{2} \sum_{ij} \sigma_i J_{ij} \sigma_j + \sum_i h_i \sigma_i}$$

$$m_i = \langle \sigma_i \rangle = \frac{1}{Z} \frac{\partial Z}{\partial h_i}$$

$$\sum_i \phi_i \sigma_i \to \sum_i (\phi_i + h_i) \sigma_i$$

$$\frac{1}{Z} \frac{\partial Z}{\partial h_i}\Big|_{h_i=0} \sim \frac{1}{Z} \frac{\partial Z}{\partial \phi_i} \sim \langle \phi_i \rangle$$

* config. qui min. $S(\phi)$

$$\to \frac{c}{2} (\vec{\nabla}\phi)^2, \quad c>0$$

$$\vec{\nabla}\phi \cdot \vec{\nabla}\phi = \|\vec{\nabla}\phi\|^2 \geq 0$$

$$\boxed{\phi = \text{cte} = m}$$

$$S(\phi) = \int d^d\vec{r} \left[\frac{c}{2}(\vec{\nabla}\phi)^2 + g(\phi) \right]$$

$$g(\phi) = \frac{a_2}{2}\phi^2 + \frac{a_4}{4}\phi^4$$

si T grande $a_2 > 0$

T pequeña $a_2 < 0$

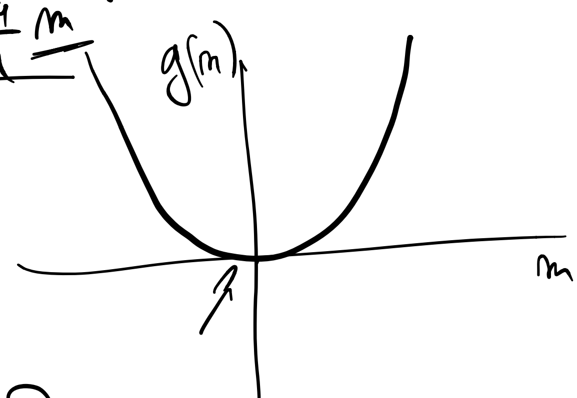
T_c el punto donde $a_2 = 0$

$t = T - T_c$ $\quad a_2 = at$

ponemos $\phi(\vec{r}) = m$

$$g(m) = \frac{a_2}{2}m^2 + \frac{a_4}{4}m^4$$

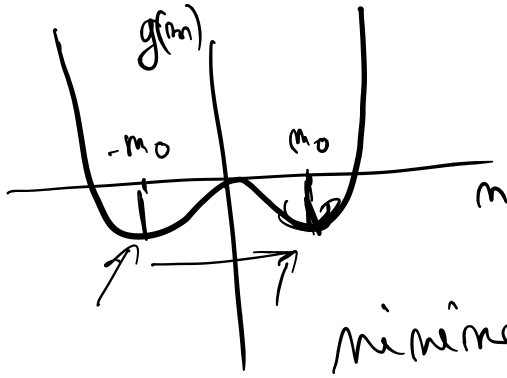
si $t > 0$ $(T > T_c)$



mínimo es en $m = 0$

$\langle \phi \rangle = 0 \rightarrow$ fase Para

si $T < T_c$ $\quad t < 0$, $a_2 < 0$



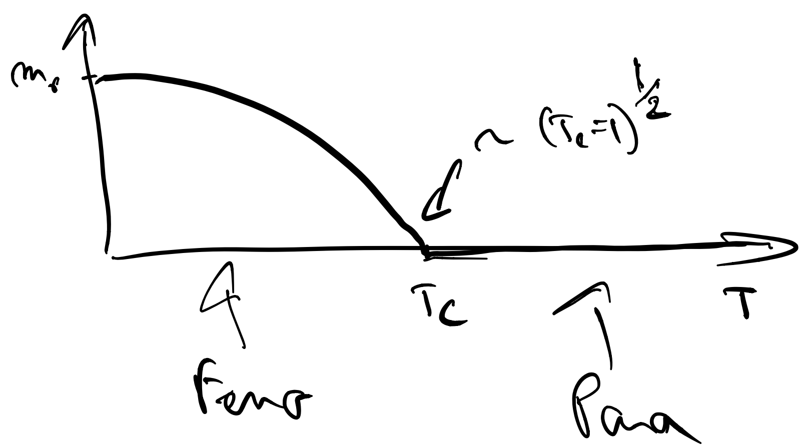
mínimo para $m = \pm m_0$ Fero

m_0 es solución de

$$g'(m) = 0 \qquad m_0\left(a_2 + a_4 m_0^2\right) = 0$$

$$m_0 = \pm\sqrt{\frac{-a_2}{a_4}} = \pm\sqrt{\frac{-at}{a_4}}$$

$$\Rightarrow \boxed{m_0 = \pm\sqrt{\frac{a(T_c-T)}{a_4}} \qquad \boxed{\beta = \frac{1}{2}}}$$



Para el caso para $T > T_c$

$m_0 = 0$

$$\phi(\vec{r}) = m_0 + \delta\phi(\vec{r})$$
$$\Downarrow$$
$$0$$

$$g(m_0 + \delta\phi) = g(m_0) + \frac{1}{2} g''(m_0)\, \delta\phi^2 + \dots$$

$$S(\delta\phi) \simeq \int d^d\vec{r} \left[\frac{c}{2} \left(\vec{\nabla}\delta\phi(\vec{r})\right)^2 + \frac{1}{2} g''(m_0)\, \delta\phi^2(\vec{r}) + \dots \right]$$

aproximación gaussiana.

$$g''(m_0 = 0) = a_2 > 0$$

$$S(\delta\phi) \simeq \int d^d\vec{r} \left[\frac{c}{2} \left(\vec{\nabla}\delta\phi\right)^2 + \frac{a_2}{2} \delta\phi^2 + \dots \right]$$

$$[a_2] \sim L^{-2} \qquad a_2 = \frac{1}{\xi^2}$$

$$a_2 = at \quad \Rightarrow \xi \sim \sqrt{\frac{1}{at}} \sim t^{-1/2}$$

$$\Rightarrow \boxed{\nu = \frac{1}{2}}$$

el caso Ferro

mínimo de S es para

$$\phi = \pm m_0 = \pm \sqrt{\frac{-a_2}{a_4}}$$

$$\phi = m_0 + \delta\phi$$

$$S(\phi) = \int d^d\vec{r}\left[\frac{c}{2}(\vec{\nabla}\delta\phi)^2 + \frac{1}{2}g''(m_0)\,\delta\phi^2\right]$$

$$g''(m_0) = a_2 + 3a_4 m_0^2 = -2a_2 > 0$$

$$S(\phi) = \int d^d\vec{r}\left[\underbrace{\frac{c}{2}(\vec{\nabla}\delta\phi)^2 - a_2\,\delta\phi^2}\right]$$

positivo.

$$\frac{1}{\xi^2}$$

$$\xi \sim (-a_2)^{-1/2}$$

$$\sim (-t)^{-1/2} \qquad \boxed{\nu = \frac{1}{2}}$$

$$\boxed{\text{Si } a_4 < 0}$$

$$S(\phi) = \int d^d\vec{r}\left[\frac{c}{2}(\vec{\nabla}\phi)^2 + \underbrace{\frac{a_2}{2}\phi^2 + \frac{a_4}{4}\phi^4 + \frac{a_6}{6}\phi^6}_{g(\phi)}\right]$$

$$Z \sim \underline{\int \mathcal{D}\phi\, e^{-S}} \qquad \underline{\int dx\, e^{+x^4} \to \infty}$$

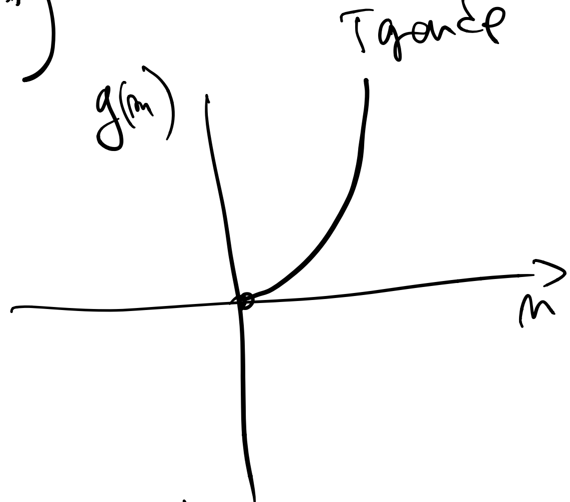
$$a_6 > 0$$

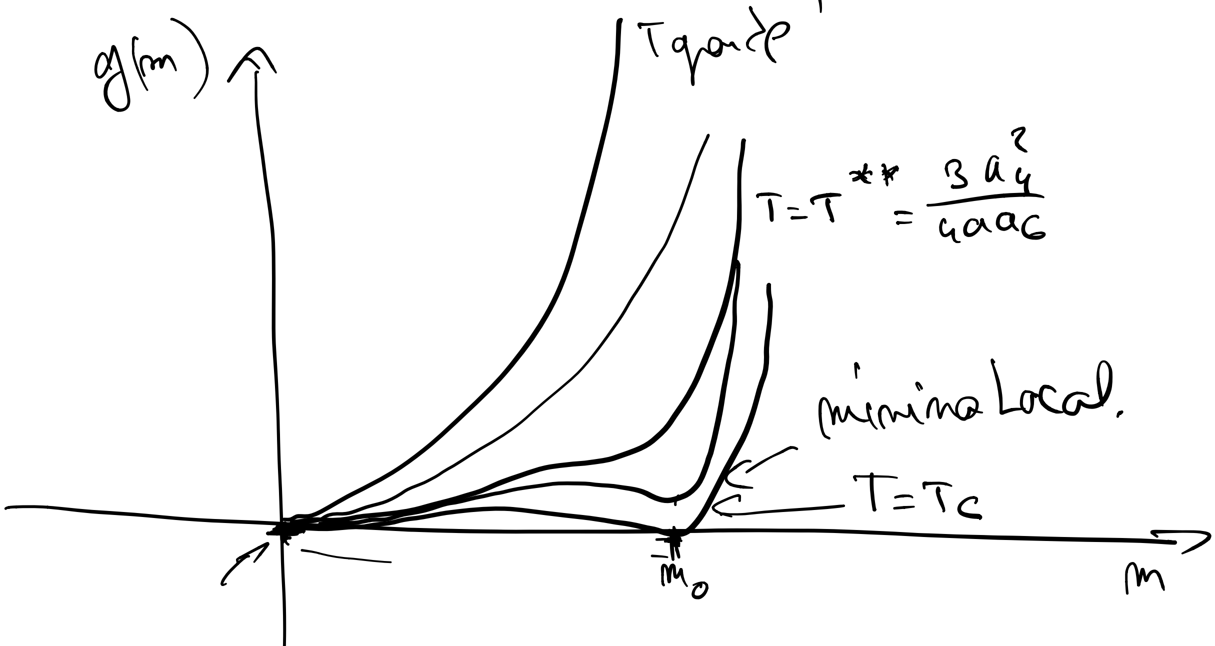
mínimo $\phi(\vec{r}) = m$ homogéneo.

$$g(m) = \frac{a_2}{2}\phi^2 + \frac{a_4}{4}\phi^4 + \frac{a_6}{6}\phi^6$$

$$a_2 = a(T - T^*)$$

T grande





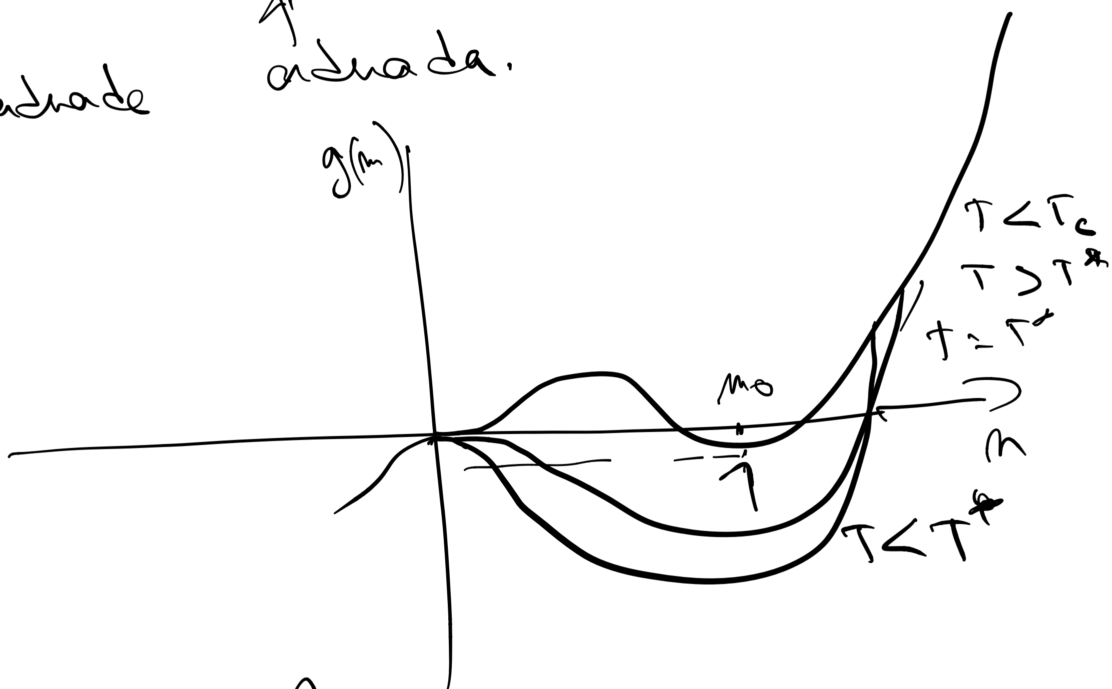
Soluciones de $g'(m) = 0$

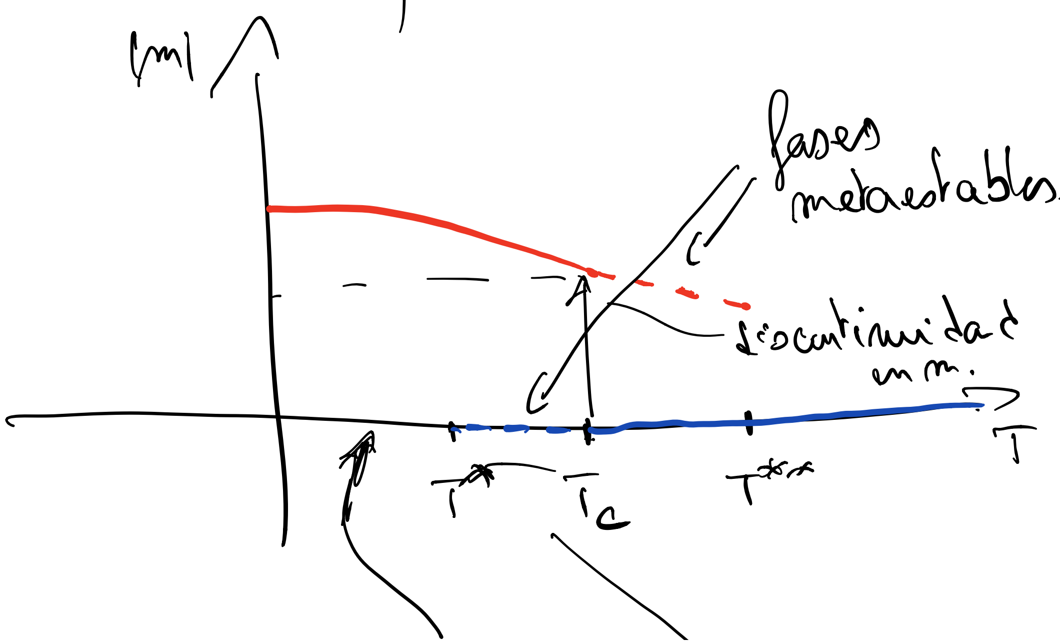
$$\longrightarrow m_0 = \pm \left[\frac{2|a_4|}{a_6} + \frac{2|a_4|}{a_6} \sqrt{1 - \frac{4a(T-T^*)a_6}{a_4^2}} \right]$$

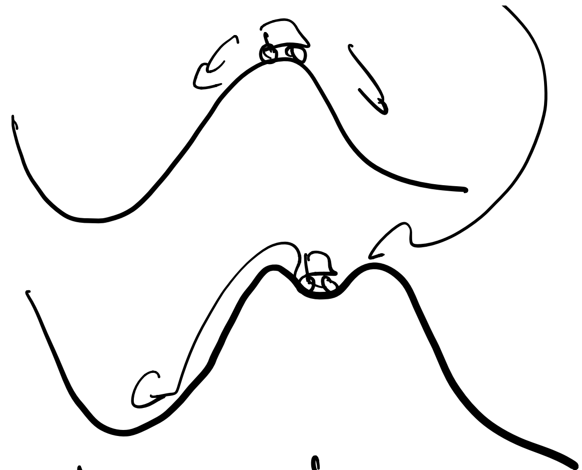
Para $T = T_c$ co-existen las fases diferentes

$m=0$ y $m = \pm m_0$

desordenada   ordenada.



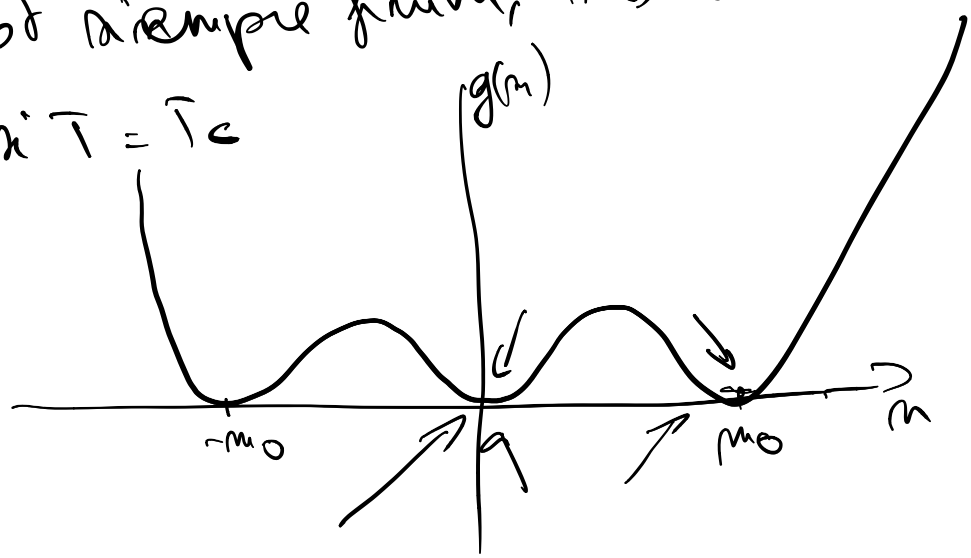




→ Transición de 1er orden.

ξ, la longitud de correlación
es siempre finita, incluso en $T=T_c$

si $T = T_c$



$$\phi = 0 + \delta\phi$$

$$S[\phi] = \int d^d\vec{r}\,\left[\frac{c}{2}\left(\vec{\nabla}\delta\phi\right)^2 + \frac{\mu_0}{2}\delta\phi^2\right]$$

$$\mu_0 = g''(m)\Big|_{m=0,\,T=T_c} \begin{array}{l} >0 \\ \neq 0 \end{array}$$

$\xi_0 \sim \mu_0^{-1/2}$ finito ya que $\mu_0 > 0$

$\phi = m_0 + \delta\phi$

$$\Rightarrow \boxed{S[\phi] = \int d^dr \left[\frac{c}{2}(\vec{\nabla}\delta\phi)^2 + \frac{\mu_m}{2}\delta\phi^2 \right]}$$

$$\mu_m = g''(m)\Big|_{m_0} > 0$$

$\xi_m \sim \mu_m^{-1/2}$ finito.

Para $T = T_c$

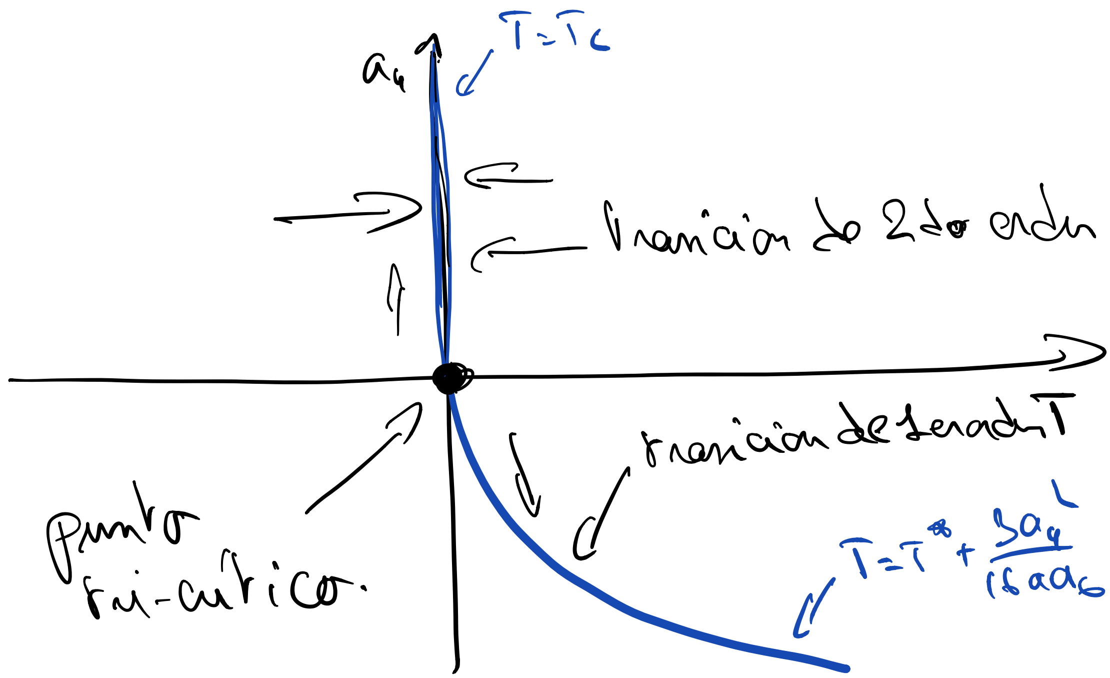
co-existe 2 fases, cada una

con un ξ finito

T_c temperatura

de transición de fase

$$T_c \sim T^* + \frac{3a_4^2}{16 a a_6}$$



Punto tri-crítico: $a_4 = 0$

$$S(\phi) = \int d^d x \left[\frac{c}{2}(\vec{\nabla}\phi)^2 + \frac{a_2}{2}\phi^2 + \frac{a_6}{6}\phi^6 \right]$$

mínimo de S suponiendo $\phi = m$

$$g(m) = \frac{a_2}{2} m^2 + \frac{a_6}{6} m^6$$

si $a_2 > 0 \to$ mínimo $m = 0$

$a_2 < 0 \to$ mínimo $at + a_6 m^4 = 0$

$$\Rightarrow m = \pm \left(\frac{-at}{a_6} \right)^{1/4} \to \boxed{\beta = \frac{1}{4}}$$

Modelo de Blume-Capel (1966)
en red cuadrada.

$S_i = +1, 0, -1$ como S^z de un espín 1

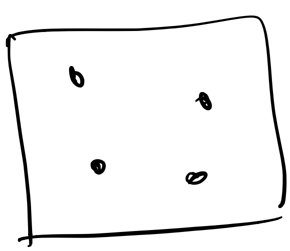
$\to H = -J \sum_{\langle ij\rangle}' S_i S_j - D \sum_i' S_i^2$

$D > 0 \to$ más vale poner $S_i = \pm 1$ que $S_i = 0$

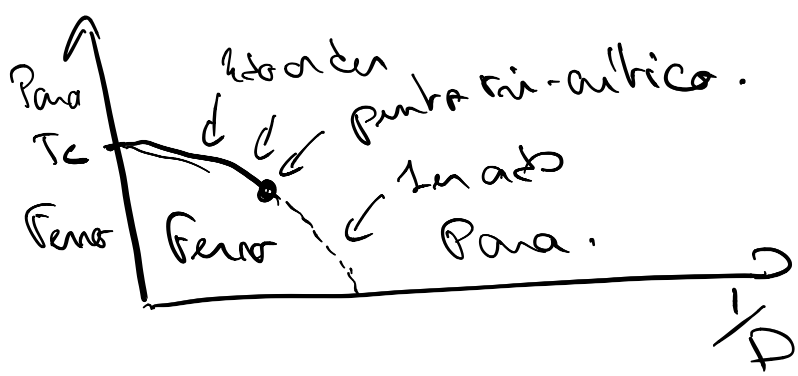
$\to$ simetría $\mathbb{Z}_2$ si $S_i \to -S_i$

$D \to \infty \to$ Ising.

$\to S_i = 0 \to$ impurezas no magnéticas.

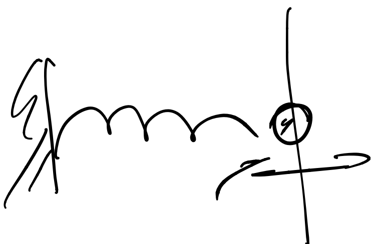


annealed impurities.



$$\hat{J}(q) \sim \hat{J}(0) + q^2 \hat{J}''(0) + \dots$$

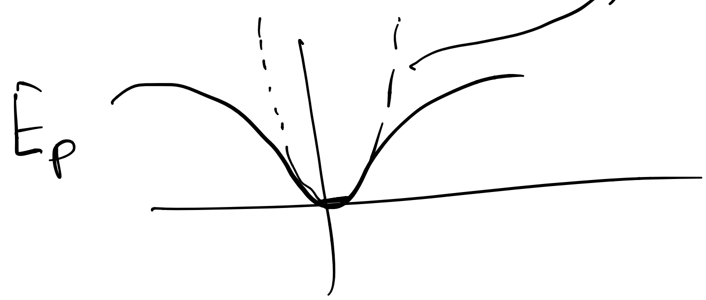
$$\operatorname{ch}\phi \sim \phi^2 \cdot \phi^4 \cdot\cdot\, \phi^6 + \dots$$



$x = 0$

$$\vec{F} = -kx\, e_x$$

$$E_p = \frac{1}{2} k x^2$$



III El Modelo Gaussiano

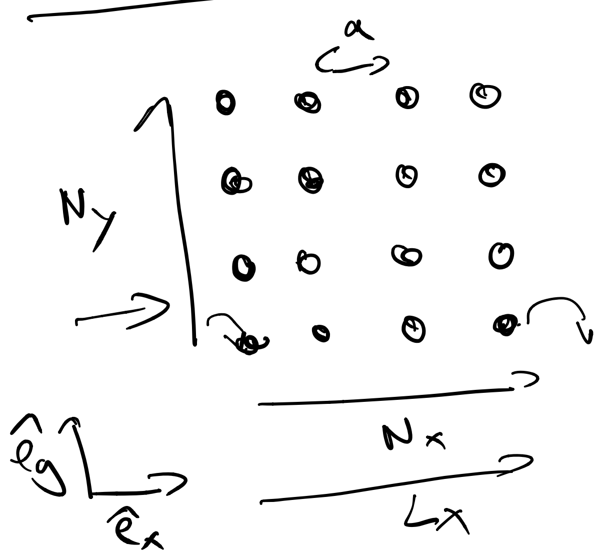
$$Z = \int \mathcal{D}\phi\, e^{-S}$$

$$Z = \int_{-\infty}^{\infty} dx\, e^{-f(x)} \sim e^{-f(x_0)}$$

$$f(x) = \alpha x^2$$

$$\int_{-\infty}^{\infty} e^{-\alpha x^2}\,dx = \sqrt{\frac{\pi}{\alpha}}$$

1) Sistema discreto y finito



2 dimensiones

$N = N_x N_y \dots$

$L_x = a N_x$

$L_y = a N_y$

etc...

$$\vec r = m_x a \hat e_x + m_y a \hat e_y + \dots$$

$$m_x = 0, 1, 2, \dots, N_x - 1$$

$$m_y = 0, 1, \dots, N_y - 1$$

Cond. de Borde periódicas. CBP.

$$\vec r \equiv \vec r + N_x a \hat e_x \equiv \vec r + N_y a \hat e_y \text{ etc...}$$

en cada sitio se define una variable real $\phi_{\vec r} \equiv \phi_{\vec r + N_x a \hat e_x}$ etc..

$-\infty < \phi_{\vec{n}} < \infty$

Modos de Fourier

entero.

$$e^{i\vec{k}\cdot\vec{n}} \quad , \quad \vec{k} = \frac{2\pi n_x}{L_x}\hat{e}_x + \frac{2\pi n_y}{L_y}\hat{e}_y + \cdots$$

Con $\underbrace{-\frac{N_x}{2} < n_x \leq \frac{N_x}{2}}$ o $0 \leq n_x < N_x$

zona de Brillouin.

ex (comenzar por 1-D)

usando $\displaystyle\sum_{n=0}^{N-1} \alpha^n = \frac{1-\alpha^N}{1-\alpha}$ $\quad \alpha \neq 1$

$= N$ si $\alpha = 1$

demuestra

$$\sum_{\forall \vec{n}} \frac{e^{i(\vec{k}_1-\vec{k}_2)\cdot\vec{n}}}{N} = \delta_{\vec{k}_1,\vec{k}_2}$$

$$= \delta_{n_x^1,n_x^2}\,\delta_{n_y^1,n_y^2}$$

$$y \quad \sum_{\forall \vec{k} \in 1.B}' \frac{e^{i\vec{k}\cdot(\vec{r}_1 - \vec{r}_2)}}{N} = \delta_{\vec{r}_1, \vec{r}_2}$$

$$= \delta_{m_x^1, m_x^2} \cdots$$

$$\hat{\phi}_{\vec{k}} = \sum_{\forall \vec{n}}' e^{i\vec{k}\cdot\vec{n}} \phi_{\vec{n}}$$

i) $\phi_{\vec{n}} = \sum_{\vec{k} \in 1.B}' \frac{e^{-i\vec{k}\cdot\vec{n}}}{N} \hat{\phi}_{\vec{k}}$

ii) $\phi_{\vec{n}}$ real $\rightarrow \hat{\phi}_{\vec{k}} = \hat{\phi}^*_{-\vec{k}}$

iii) $\sum_{\vec{n}}' \phi_{\vec{n}}^2 = \sum_{\vec{k}}' \frac{1}{N} |\hat{\phi}_{\vec{k}}|^2$

iv) $\sum_{\vec{n}}' \phi_{\vec{n}} \phi_{\vec{n}+d\hat{e}_x} = \frac{1}{2} \sum_{\vec{n}}' [\phi_{\vec{n}-d\hat{e}_x} \phi_{\vec{n}} + \phi_{\vec{n}} \phi_{\vec{n}+d\hat{e}_x}]$

$d = na$

$$= \frac{1}{N} \sum_{\vec{k} \in 1.B}' |\hat{\phi}_{\vec{k}}|^2 \cos(k_x d)$$

función de partición

$$Z = \int \prod_{\vec{n}} d\phi_{\vec{n}}\, e^{-S}$$

S es una forma cuadrática

$$S = \sum_{\vec{n},\vec{n}'} \phi_{\vec{n}} A_{\vec{n},\vec{n}'} \phi_{\vec{n}'} \qquad A > 0 \text{ real.}$$

$$A_{\vec{n},\vec{n}'} = A_{\vec{n}-\vec{n}'} \quad \text{inv. por transl.}$$

↳ simetría $\phi_{\vec{n}} \to -\phi_{\vec{n}}$

$$\hat{A}_{\vec{k}} = \hat{A}^*_{-\vec{k}} \qquad \text{tq}$$

$$A_{\vec{n}} = \sum_{\vec{k}\in Z.B} \frac{1}{N} e^{i\vec{k}\cdot\vec{n}} \hat{A}_{\vec{k}}$$

$$S = \frac{1}{2N} \sum_{\vec{k}} |\hat{\phi}_{\vec{k}}|^2 \left[\hat{A}_{\vec{k}} + \hat{A}^*_{\vec{k}}\right]$$

$\mathrm{Re}\,\hat{A}_{\vec{k}}$

$$Z = \int \prod_{\vec r} d\phi_{\vec r}\; e^{-S}$$

$\phi_{\vec r} \to \hat\phi_{\vec k}$ es un cambio de variable

$$\int dx\, dy\, dz\; e^{-f(x,y,z)}$$

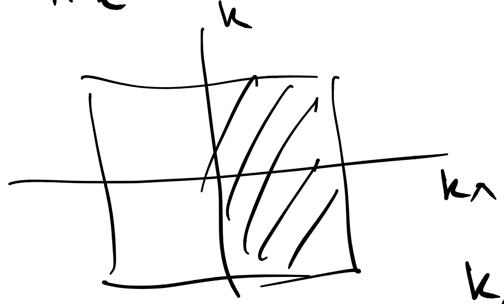
$\alpha = x + y - z$

$\beta = x - y$

γ ,

$$\int J\, d\alpha\, d\beta\, d\gamma\; e^{-\hat f(\alpha,\beta,\gamma)}$$

<u>obs</u> $\hat\phi_{\vec k}$ y $\hat\phi_{-\vec k}$ no son indep.



$k_x \geq 0$

$$\prod_{\vec r} d\phi_{\vec r} = \prod_{\substack{\vec k \\ \text{tq } k_x \geq 0}} d\hat\phi^{R}_{\vec k}\, d\hat\phi^{i}_{\vec k} \equiv \hat{\prod_{\vec k}} d\hat\phi_{\vec k}$$

$$Z = \int \prod_{\vec{k}}^{\Lambda} d\hat{\phi}_{\vec{k}}\, e^{-\frac{1}{N}\sum_{\vec{k}} (\hat{A}_{\vec{k}} + \hat{A}_{-\vec{k}}) |\hat{\phi}_{\vec{k}}|^2}$$

$$\int_{-\infty}^{\infty} dx_1\, dx_2\, dx_3 \dots e^{-\alpha_1 x_1^2 - \alpha_2 x_2^2 - \alpha_3 x_3^2 \dots}$$

$$|\hat{\phi}_{\vec{k}}|^2 = \hat{\phi}_{\vec{k}}^{Re\,2} + \hat{\phi}_{\vec{k}}^{i\,2}$$

$$\int dx\, dy\, e^{-\alpha(x^2+y^2)} = \frac{\pi}{\alpha}$$

$$\Rightarrow \boxed{Z = \prod_{\vec{k}}^{\Lambda} \frac{\pi N}{2 \mathrm{Re}\, \hat{A}_{\vec{k}}}}$$

Funciones de dos puntos:

$$\langle \phi_{\vec{r}_1} \phi_{\vec{r}_2} \rangle = \frac{1}{Z} \int \prod_{\vec{n}} d\phi_{\vec{n}}\, \phi_{\vec{r}_1} \phi_{\vec{r}_2}\, e^{-S}$$

$$\langle \phi_{\vec{r}_1} \rangle = 0$$

$$\langle \phi_{r_1} \phi_{r_2} \rangle_c = \langle \phi_{r_1} \phi_{r_2} \rangle - \langle \phi \rangle \langle \phi \rangle$$

$$\langle \phi_{\vec{r}_1}\phi_{\vec{r}_2}\rangle_S = \frac{1}{Z}\int \prod_{\vec{k}} d\hat{\phi}_{\vec{k}}\, e^{-S}\, \underline{\phi_{\vec{r}_1}\phi_{\vec{r}_2}}$$

$$= \frac{1}{Z}\frac{1}{N^2}\sum_{\vec{k}_1,\vec{k}_2}^{\prime} e^{i\vec{k}_1\cdot\vec{r}_1 + i\vec{k}_2\cdot\vec{r}_2} \int \prod_{\vec{k}} d\hat{\phi}_{\vec{k}}\, \hat{\phi}_{\vec{k}_1}\hat{\phi}_{\vec{k}_2}\, e^{-S_G}$$

Pero $\hat{\phi}_{\vec{k}_1}\hat{\phi}_{\vec{k}_2} = \underline{\phi^R_{\vec{k}_1}\phi^R_{\vec{k}_2}} + i\,\underline{\phi^R_{\vec{k}_1}\phi^i_{\vec{k}_2}} + i\,\underline{\phi^i_{\vec{k}_1}\phi^i_{\vec{k}_2}}$

$\quad + \underline{\phi^i_{\vec{k}_1}\phi^i_{\vec{k}_2}}$

si $\vec{k}_1$ y $\vec{k}_2$ cualesquiera

$$\int dx_1\,dx_2\,dx_3\,dx_4\; x_1 x_1\; e^{-a(x_1^2+x_2^2+x_3^2+x_4^2)}$$

$$x_1 \Longleftrightarrow -x_1$$

el resultado es $\neq 0$ únicamente si $\vec{k}_1 = -\vec{k}_2$!

$$\Rightarrow \langle \phi_{\vec{r}_1}\phi_{\vec{r}_2}\rangle = \frac{1}{N^2}\sum_{\vec{k}}^{\prime} e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)} \langle \hat{\phi}_{\vec{k}}\hat{\phi}_{-\vec{k}}\rangle$$

$$\gamma \langle \hat{\phi}_{\vec{k}} \hat{\phi}_{-\vec{k}} \rangle = \frac{N}{2\,\mathrm{Re}\,\hat{A}_k}$$

$$\frac{1}{\int dx\,dy\, e^{-\alpha(x^2+y^2)}} \int dx\,dy\,(x^2+y^2)\, e^{-\alpha(x^2+y^2)} = \frac{1}{\alpha}$$

$$\Rightarrow \boxed{\langle \phi_{\vec{r}_1} \phi_{\vec{r}_2} \rangle = \frac{1}{N^2} \sum_{\vec{k}}{}' \frac{N}{2\,\mathrm{Re}\,\hat{A}_{\vec{k}}} e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)}}$$