Regreso al Hubbard-Stratonovich.
y G.L.

Ising

$$H = -\frac{1}{2} \sum_{i,j}' J_{ij}\, \sigma_i \sigma_j \xrightarrow[\text{H.S. límite cont.}]{} \text{G.L.}$$

$$S_{GL} = \int d^d r \left[\frac{c}{2} (\vec{\nabla}\phi)^2 + \frac{a_2}{2} \phi^2 + \frac{a_4}{4} \phi^4 + \dots \right]$$

Simetrías : $\phi(\vec{r}) \to -\phi(\vec{r})$ $\quad \mathbb{Z}_2$
del parámetro de orden (ϕ) $\quad a_2 = a\,t$
$\quad t = T - T_c$

Simetrías espaciales: emergentes en el lím. del cont.

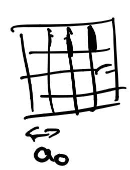
* traslación $\quad \vec{r} \to \vec{r} + \vec{d}$

$$\phi(\vec{r}) \to \phi(\vec{r} + \vec{d})$$

el sistema es homogéneo.

* rotación espacial.

$$\vec{\nabla}\phi \cdot \vec{\nabla}\phi \ \text{inv.} \quad \vec{r} \to R\,\vec{r}$$



$J_{ij} \sim$ función de $|i-j|$

$a_2 = a t$; $t = T - T_c$ ✓

en la literatura $t = \dfrac{T - T_c}{T_c}$

H.S. $\longrightarrow$ G.L.

Ising $\longrightarrow$ G.L. ϕ escalar

Modelo de Heisenberg ó XY $\quad \vec{S}_i = \begin{pmatrix} S_i^x \\ S_i^y \end{pmatrix}$ XY

$$H = -\frac{1}{2} \sum_{ij}' J_{ij} \, \underline{\vec{S}_i \cdot \vec{S}_j} \qquad \vec{S}_i = \begin{pmatrix} S_i^x \\ S_i^y \\ S_i^z \end{pmatrix} \text{ Heis.}$$

H.S. $\qquad$ simetría $O(n)$

$\vec{S}_i$ tiene n componentes ($n = 2$ para XY
$n = 3$ para Heisenberg)

$\hookrightarrow \vec{\phi}_i$ de n componentes.

$$-\infty < \phi_i^x, \phi_i^y, \phi_i^z < \infty$$

$$\int \prod_i d\vec{S}_i \; e^{\sum_i \vec{S}_i \cdot \vec{\phi}_i}$$

$$S\{\vec{\phi}\} = \int d^d\vec{r}\ \left\{ \frac{c}{2}\sum_{\mu=1,2,\ldots d}(\partial_\mu\vec{\phi})\cdot(\partial_\mu\vec{\phi}) + \frac{a_2}{2}\vec{\phi}\cdot\vec{\phi} + \frac{a_4}{4}(\vec{\phi}\cdot\vec{\phi})^2 + \ldots \right.$$

simetría $\vec{r} \to R\vec{r}$ en espacio de d dimensiones

$\vec{\phi} \to \tilde{R}\vec{\phi}$ $\tilde{R}$ matriz $n\times n$.

para $T > T_c$ $a_2 > 0$ $a_2 \sim at \leftarrow$

$\nu = \frac{1}{2}$

$T < T_c$ $a_2 < 0$

el caso $n=2$ (modelo XY)

$$\vec{S}_i = \begin{pmatrix} S_i^X \\ S_i^Y \end{pmatrix} \to \vec{\phi}(\vec{r}) = \begin{pmatrix} \phi^X(\vec{r}) \\ \phi^Y(\vec{r}) \end{pmatrix}$$

$$S_{XY}\{\vec{\phi}\} = \int d^d\vec{r}\ \left\{ \frac{c}{2}\left[(\vec{\nabla}\phi^X)^2 + (\vec{\nabla}\phi^Y)^2\right] + \frac{a_2}{2}(\phi^{X^2}+\phi^{Y^2}) + \frac{a_4}{4}(\phi^{X^2}+\phi^{Y^2})^2 + \ldots \right\}$$

$$\psi(\vec{r}) = \phi^X(\vec{r}) + i\phi^Y(\vec{r})$$

$$\bar{\psi}(\vec{r}) = \phi^X(\vec{r}) - i\phi^Y(\vec{r})$$

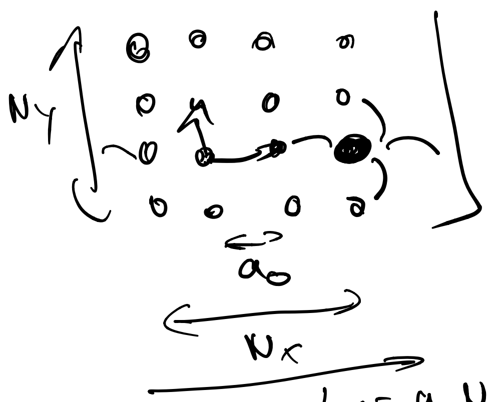
$$S_{XY}\{\psi\} = \int d^d\vec{r}\ \left\{ \frac{c}{2}|\vec{\nabla}\psi|^2 + \frac{a_2}{2}|\psi|^2 + \frac{a_4}{4}|\psi|^4 + \ldots \right\}$$

$$\vec{\nabla}\bar{\psi}\cdot\vec{\nabla}\psi$$

$$\Phi \to \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \Phi \qquad SO(2)$$

$$\psi \to e^{i\theta}\psi \qquad U(1)$$

→ Regreso al modelo Gaussiano.



Si la matriz de acople $A_{\vec{n},\vec{n}'}$ es inv. por traslación

→ Fourier

$$S = \sum_{\vec{n},\vec{n}'} \phi_{\vec{n}} A_{\vec{n},\vec{n}'} \phi_{\vec{n}'}$$

$$= \frac{1}{2N} \sum_{\vec{k}\in Z.B} |\hat{\phi}_{\vec{k}}|^2 \underbrace{\left[\hat{A}_{\vec{k}} + \hat{A}^*_{\vec{k}}\right]}_{2\mathrm{Re}\hat{A}_{\vec{k}}}$$

$$Z = \hat{\prod_{\vec{k}}} \frac{\pi N}{2 \mathrm{Re}\, \hat{A}_{\vec{k}}}$$

$$\langle \phi_{\vec{k}_1}, \phi_{\vec{k}_2} \rangle \neq 0 \text{ solamente si } \vec{k}_2 = -\vec{k}_1$$

$$\boxed{\langle \phi_{\vec{r}_1} \phi_{\vec{r}_2} \rangle = \frac{1}{N^2} \sum_{\vec{k}}' \frac{N}{2 \mathrm{Re}\, \hat{A}_{\vec{k}}} e^{i \vec{k} \cdot (\vec{r}_1 - \vec{r}_2)}}$$

$A_{\vec{r}-\vec{r}'} = ?$

si por ejemplo

$$S = \sum_{\vec{r}}' \left[\alpha \phi_{\vec{r}}^2 - \beta \left(\phi_{\vec{r}} \phi_{\vec{r}+a_0\hat{e}_x} + \phi_{\vec{r}} \phi_{\vec{r}+a_0\hat{e}_y} + \ldots \right) \right]$$

→ acople a los vecinos.

T.F.

$$S = \frac{1}{N} \sum_{\vec{k}}' |\hat{\phi}_{\vec{k}}|^2 \underbrace{\left[\alpha - \beta \left(\cos(k_x a_0) + \cos(k_y a_0) + \ldots \right) \right]}_{\hat{A}_{\vec{k}} + \hat{A}^*_{\vec{k}}}$$

$$\rightarrow \quad Z = \hat{\prod_{\vec{k}}} \frac{\pi N}{2 \left[\alpha - \beta \left(\cos k_x a_0 + \cos k_y a_0 + \ldots \right) \right]}$$

$$Y \quad \langle \phi_{\vec{r}_1} \phi_{\vec{r}_2} \rangle = \frac{1}{N} \sum_{\vec{k}} \frac{e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)}}{2[\alpha-\beta(\cos(k_x a_0)+\ldots)]}$$

obs para $|\vec{r}_1-\vec{r}_2|$ muy grande.

$\rightarrow |\vec{k}|$ pequeña.

$$\cos(k_x a_0) \sim 1-\frac{(k_x a_0)^2}{2}+\ldots$$

$$\langle \phi_{\vec{r}_1} \phi_{\vec{r}_2} \rangle \sim \frac{1}{N} \sum_{\vec{k}} \frac{e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)}}{2\left[(\alpha-\beta)+\frac{\beta a_0^2}{2}(k_x^2+k_y^2+\ldots)\right]}$$

$$\sim \frac{1}{m^2+\vec{k}^2}$$

$\rightarrow$ Límite al continuo :

manteniendo L_x, L_y etc finitos

$a_0 \rightarrow 0 \qquad L_x = a_0 N_x$

$N_x, N_y \ldots \rightarrow \infty$

$$\vec{k} = \frac{2\pi n_x}{L_x}\hat{e}_x + \frac{2\pi n_y}{L_y}\hat{e}_y + \ldots$$

$$-\frac{N_x}{2} < n_x \leq \frac{N_x}{2}$$

$n_x, n_y \ldots \in \mathbb{Z}$

$$N = N_x N_y \cdots = \frac{V}{a_0^d} \to \infty \; ; \; V \text{ finito}$$

$$V = L_x L_y \cdots$$

diccionario de traducción:

$$\delta_{\vec{r}_1,\vec{r}_2} \longrightarrow a_0^d\, \delta(\vec{r}_1 - \vec{r}_2)$$

$$a_0^d \sum_{\vec{r}} \longrightarrow \int d^d\vec{r}$$

ejemplo.

$$\sum_{\vec{k}} \frac{e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)}}{N} = \delta_{\vec{r}_1,\vec{r}_2}$$

$$\boxed{\frac{1}{V}\sum_{\vec{k}} e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)} = \delta(\vec{r}_1-\vec{r}_2)}$$

y

$$\sum_{\vec{r}} \frac{1}{N} e^{i(\vec{k}_1-\vec{k}_2)\cdot\vec{r}} = \delta_{\vec{k}_1,\vec{k}_2}$$

$$\int d\vec{r}\, e^{i(\vec{k}_1-\vec{k}_2)\cdot\vec{r}} = V\,\delta_{\vec{k}_1,\vec{k}_2}$$

$$\phi_{\vec{r}} \to \phi(\vec{r}) \equiv \frac{1}{a_0^d}\,\phi_{\vec{r}}$$

T.F.

$$\hat{\phi}_{\vec{k}} = \sum_{\vec{r}} e^{i\vec{k}\cdot\vec{r}} \phi_{\vec{r}} \to \int d\vec{r}\, e^{i\vec{k}\cdot\vec{r}} \phi(\vec{r})$$

$$y \quad \phi(\vec{r}) = \frac{1}{V} \sum_{\vec{k}} e^{i\vec{k}\cdot\vec{r}} \hat{\phi}_{\vec{k}}$$

i) $$\sum_{r} \phi^2_{\vec{r}} \to a_0^d \int d^d\vec{r}\, \phi^2(\vec{r})$$

$$= \frac{a_0^d}{V} \sum_{\vec{k}} |\hat{\phi}_{\vec{k}}|^2$$

ii) $$\sum_{\vec{r}} \phi_{\vec{r}} \phi_{\vec{r}\pm a_0\hat{e}_x} \to a^d \int d^d\vec{r}\, \phi(\vec{r})\, \phi(\vec{r} \pm a_0\hat{e}_x)$$

$$\sim a^d \int d^d\vec{r}\, \phi(\vec{r}) \left(\phi(\vec{r}) \underline{\pm a_0 \partial_x \phi(\vec{r})} + \underline{\frac{a_0^2}{2} \partial_x^2 \phi(\vec{r})} + \cdots \right)$$

iii) $$\sum_{r} \frac{1}{2} \left(\phi_{\vec{r}-a_0\hat{e}_x} \phi_{\vec{r}} + \phi_{\vec{r}} \phi_{\vec{r}+a_0\hat{e}_x} \right)$$

$$\hookrightarrow a^d \int d\vec{r} \left[\phi^2(\vec{r}) + a_0^2 \partial_x^2 \phi(\vec{r}) + \cdots \right]$$

$$= \frac{a^d}{V} \sum_{\vec{k}} |\hat{\phi}_{\vec{k}}|^2 \left[1 - a_0^2 k_x^2 + \cdots \right]$$

entonces $S = \sum_{\vec{n}}' \left[\alpha \phi_{\vec{n}}^2 - \beta \left(\phi_{\vec{n}} \phi_{\vec{n} + a_0 \hat{e}_1} + \dots\right)\right]$

$$\boxed{S = \int d\vec{r} \left[\frac{a_2}{2} \phi(\vec{r})^2 + \frac{c}{2} (\nabla \phi)^2 + \dots\right]}$$

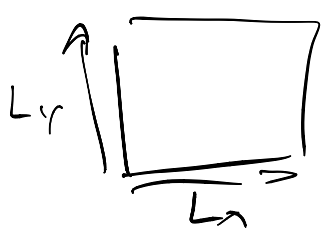
$$a_2 = 2 a_0^d (\alpha - \beta) \qquad c = \beta a_0^{d+2}$$

$$\boxed{S = \frac{1}{V} \sum_{\vec{k}}' |\phi_{\vec{k}}|^2 \left(\frac{a_2}{2} + \frac{c}{2} k^2\right)}$$

$$\langle \phi(\vec{r}_1) \phi(\vec{r}_2) \rangle = \frac{1}{N} \sum_{\vec{n}} \frac{e^{i \vec{k} \cdot (\vec{r}_1 - \vec{r}_2)}}{2[(\alpha - \beta \dots)]}$$

$$\boxed{= \frac{1}{V} \sum_{\vec{k}}' \frac{e^{i \vec{k} \cdot (\vec{r}_1 - \vec{r}_2)}}{a_2 + c k^2}}$$

Por último:



$V = L_x L_y \dots$

→ límite de tamaño infinito

$L_x, L_y \ldots \to \infty$, $V \to \infty$.

$$\Delta k_x \Delta k_y \ldots \quad \frac{2\pi}{L_x}\frac{2\pi}{L_y} \ldots = \frac{(2\pi)^d}{V}$$

Licciación de $V\to\infty$

$$\sum_{\vec{k}} \to \int \frac{d^d\vec{k}}{\Delta k_x \Delta k_y \ldots} \to \frac{V}{(2\pi)^d}\int d\vec{k}$$

$$\delta_{\vec{k}_1,\vec{k}_2} \to \Delta k_x \Delta k_y \ldots \delta(\vec{k}_1-\vec{k}_2) = \frac{(2\pi)^d}{V}\delta(\vec{k}_1-\vec{k}_2)$$

por ejemplo

$$\sum_{\vec{k}} \frac{e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)}}{V} = \delta(\vec{r}_1-\vec{r}_2)$$

$$\to \int \frac{d\vec{k}}{(2\pi)^d} e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)} = \delta(\vec{r}_1-\vec{r}_2)$$

$$\text{y} \quad \int d\vec{r}\, e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)} = \underbrace{V\delta_{\vec{k}_1,\vec{k}_2}} \to (2\pi)^d\delta(\vec{k}_1-\vec{k}_2)$$

$$\text{T.F.} \quad \hat{\phi}(\vec{k}) = \int d\vec{r}\, e^{i\vec{k}\cdot\vec{r}}\phi(\vec{r})$$

$$\phi(\vec r) = \frac{1}{(2\pi)^d}\int d\vec k \, e^{-i\vec k\cdot\vec r}\,\hat\phi(\vec k)$$

$$S = \frac{1}{V}\sum_{\vec k} |\hat\phi_{\vec k}|^2\left(\frac{a_2}{2} + \frac{c}{2}\vec k^2\right)$$

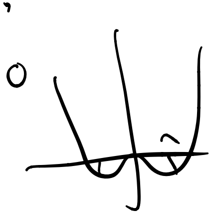
$$\hookrightarrow \int \frac{d\vec k}{(2\pi)^d}\, |\hat\phi(\vec k)|^2 \left(\frac{a_2}{2} + \frac{c}{2}\vec k^2\right)$$

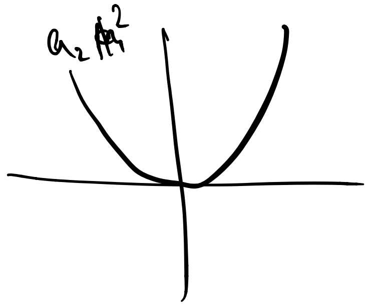
$$\boxed{\langle \phi(\vec r_1)\,\phi(\vec r_2)\rangle = \int \frac{d\vec k}{(2\pi)^d}\,\frac{e^{i\vec k\cdot(\vec r_1-\vec r_2)}}{a_2 + c\vec k^2}}$$

$$S_{gaussiana} = \int d^d\vec r\left[\frac{c}{2}(\nabla\phi)^2 + \frac{a_2}{2}\phi^2\right]$$

obs no hay $\frac{a_4}{4}\phi^4$ como en G.L.

* necesariamente $a_2 > 0$, $c > 0$
* para el modelo gaussiano no hay transición de fase.

G.L. $a_2 > 0$  $a_2 < 0$, $a_4 > 0$ 



→ punto crítico $a_2 \to 0$

La función de correlación

$$G(\vec{r}) \sim \text{cte} \int d^d\vec{k}\, \frac{e^{i\vec{k}\cdot\vec{r}}}{\vec{k}^2 + m^2} \qquad m^2 \sim a_2$$

$\vec{r} = \vec{r}_1 - \vec{r}_2$

con $m^2 = \frac{a_2}{c}$

$$G(\vec{r}) = \text{cte} \int d^d\vec{k}\, e^{i\vec{k}\cdot\vec{r}} \int_0^\infty d\alpha\, e^{-\alpha(\vec{k}^2 + m^2)}$$

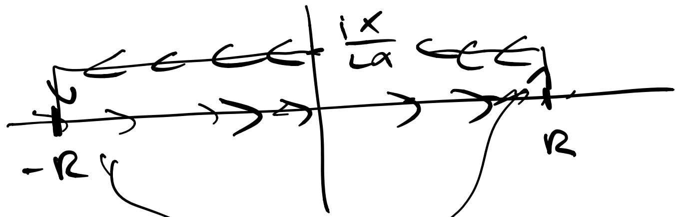
$$\forall x > 0 \qquad \frac{1}{x} = \int_0^\infty e^{-\alpha x}\, d\alpha$$

$$\Rightarrow G(\vec{r}) = \text{cte} \int_0^\infty d\alpha \int_{-\infty}^{\infty} d\vec{k}\, e^{i\vec{k}\cdot\vec{r} - \alpha\vec{k}^2 - \alpha m^2}$$

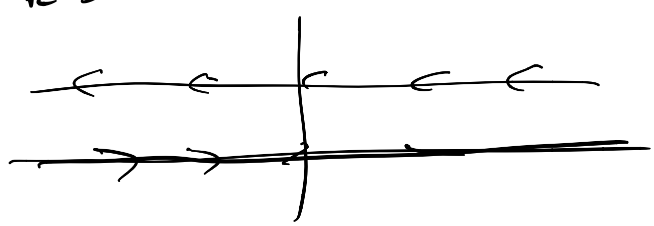
$$\int_{-\infty}^{\infty} dk_x\, e^{ik_x x - \alpha k_x^2} \times \int_{-\infty}^{\infty} dk_y\, e^{ik_y y - \alpha k_y^2} \cdots$$

$$\int_{-\infty}^{\infty} e^{i k_x x - \alpha k_x^2}\, dk_x = e^{-\frac{x^2}{4\alpha^2}} \int_{-\infty}^{\infty} e^{-\alpha\left(k_x - \frac{ix}{2\alpha}\right)^2} dk_x$$

k_x $\qquad \downarrow \sqrt{\frac{\pi}{\alpha}}$



$\lim_{R\to\infty} \to 0$



$$k_x \to \tilde{k}_x = k_x - \frac{ix}{2\alpha}$$

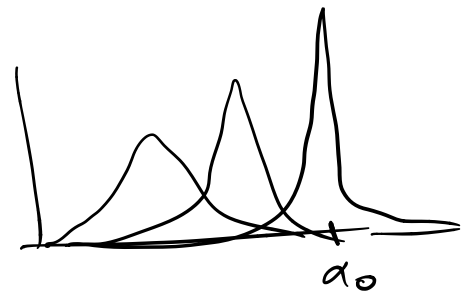
$$\sim \int_{-\infty}^{\infty} e^{-\alpha \tilde{k}_x^2}\, d\tilde{k}_x = \sqrt{\frac{\pi}{\alpha}}$$

$$\Rightarrow G(\bar{r}) = \text{cte} \int_0^{\infty} d\alpha \left(\frac{\pi}{\alpha}\right)^{d/2} \underline{e^{-\frac{r^2}{4\alpha} - m^2\alpha}}$$

→ Se puede evaluar en el límite $r \to \infty$

$$I_n = \int_0^\infty d\alpha \, e^{-f_n(\alpha)}$$

$n \nearrow$ $\quad e^{-f_n(\alpha)}$



α_0 mínimo de $f_n(\alpha)$

Saddle point approx.

$$f_n(\alpha) = f_n(\alpha_0) + \frac{1}{2} f_n''(\alpha_0)(\alpha-\alpha_0)^2 + \dots$$

$$\int_0^\infty d\alpha \, e^{-f(\alpha)} \sim \int_{-\infty}^\infty d\alpha \, e^{-f(\alpha)} \simeq \int_{-\infty}^\infty d\alpha \, e^{-f(\alpha_0)-\frac{1}{2}f''(\alpha_0)(\alpha-\alpha_0)^2}$$

$$= e^{-f(\alpha_0)} \sqrt{\frac{2\pi}{f''(\alpha_0)}} \left(1 + O\left(\frac{1}{n}\right) \dots\right)$$

aquí $f(\alpha) = \frac{n^2}{4\alpha} + m^2 \alpha + \frac{d}{2} \ln \alpha$

$$f'(\alpha) = -\frac{n^2}{4\alpha^2} + m^2 + \frac{d}{2\alpha}$$

α_0 es cuando $f'(\alpha_0) = 0$

$$\alpha_0 = \frac{1}{2m^2}\left[-\frac{d}{2} + \sqrt{\frac{d^2}{4} + r^2 m^2}\right] \longrightarrow \frac{r}{2m}$$

$$f''(\alpha) = \frac{r^2}{2\alpha^3} - \frac{d}{2\alpha^2}$$

$$f''(\alpha_0) = \frac{4m^3}{r} - \frac{4md}{r^2} \sim \frac{4m^3}{r}$$

así $G(r) \sim \text{cte}\; e^{-f(\alpha_0)} \sqrt{\frac{2\pi}{f''(\alpha_0)}}$

$$\Rightarrow \boxed{G(r) \sim \text{cte}' \frac{1}{r^{\frac{d-1}{2}}} e^{-mr}} \qquad r = |\vec{r}|$$

$$e^{-\frac{r}{\xi}}$$

$$m = \frac{1}{\xi}$$

ojo! esto es para $m \neq 0$ y $r \gg \xi$

hagamos lo mismo para el punto crítico $m=0$

$$G(\vec{r}) = \int d\vec{k} \frac{e^{i\vec{k}\cdot\vec{r}}}{\hat{c}^2 \vec{k}^2}$$ con $\hat{c}$ adimensional.

$$\text{dbs}\,[G(r)] \sim k^{d-2} \sim L^{2-d}$$

$$\Delta\phi \xRightarrow{T.F} -k^2\hat{\phi}(\vec{k})$$

$$G(\vec{r}) = \begin{cases} \frac{1}{\tilde{c}}|x| & \text{si } d=1 \\ -\frac{1}{2\pi\tilde{c}}\ln\frac{|\vec{r}|}{r_0} & \text{si } d=2 \leftarrow \\ \longrightarrow \frac{\text{cte}}{|\vec{r}|^{d-2}} & d>2 \end{cases}$$

función de correlación en el punto crítico.

es invariante de escala

$$S = \int d^d\vec{r}\;\tilde{c}\,(\vec{\nabla}\phi)^2 \qquad \begin{cases} \vec{r} \to d\vec{r} \\ \vec{\nabla} \to \frac{1}{d}\vec{\nabla} \\ \tilde{c} \to \tilde{c} \\ \phi \to d^{\frac{2-d}{2}}\phi \end{cases}$$

→ comparar con lo que dije en el curso 1

$$\langle \sigma_i\,\sigma_j\rangle \sim \frac{1}{|i-j|^{d-2+\eta}}\, e^{-\frac{|i-j|}{\xi}}$$

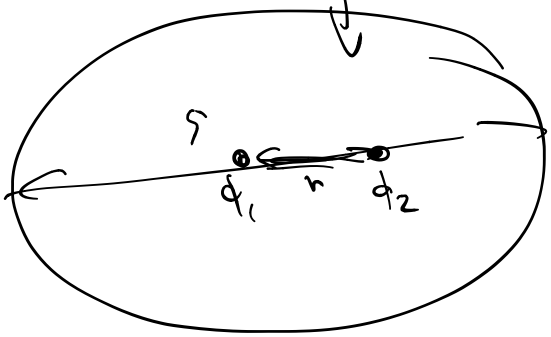
si $\xi \to \infty$ $\sim \dfrac{1}{|\vec{r}|^{d-2+\eta}}$

osea que aquí $\eta = 0$

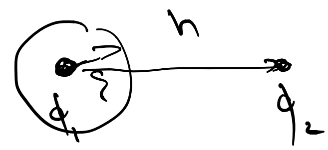
$-\frac{r}{\xi} \sim 1$

obs $\dfrac{1}{r^{d-2+\eta}} e^{-\frac{r}{\xi}}$ vs $\dfrac{1}{r^{\frac{d-1}{2}}} e^{-\frac{r}{\xi}}$

$r \sim \xi$ $\qquad$ $r >> \xi$



$\xi >> r$



función de cross-over.

$$\langle \phi(\vec{r}_1)\, \phi(\vec{r}_2) \cdots \phi(\vec{r}_M) \rangle = 0$$

si M impar.

$\phi \to -\phi$

si M par

$\to \langle \phi\, \phi \rangle \langle \phi\, \phi \rangle \cdots$

IV Validez de la aprox. de campo medio y criterio de Ginzburg.

$$Z \sim \text{cte} \int \mathcal{D}\phi \, e^{-S\{\phi\}}$$

$$S\{\phi\} = \int d^d\vec{r} \left[\frac{c}{2} (\vec{\nabla}\phi)^2 + \frac{a_2}{2}\phi^2 + \frac{a_4}{4}\phi^4 + \dots \right]$$

$$I = \int_{-L}^{L} dx \, e^{-f(x)}$$

$f(x)$ tiene un mínimo en x_0

$$f(x) \sim f(x_0)$$

$$I \sim e^{-f(x_0)}$$

$\phi = \phi_0$ mínimo de S $\quad \phi_0 = m = \begin{cases} 0 & a_2 > 0 \\ \pm m_0 & a_2 < 0 \end{cases}$

$$f(x) \simeq f(x_0) + \frac{1}{2} f''(x_0)(x-x_0)^2 + \dots$$

$$I \simeq e^{-f(x_0)} \int_{-\infty}^{\infty} dx \, e^{-\frac{f''(x_0)}{2}(x-x_0)^2} = e^{-f(x_0)} \sqrt{\frac{2\pi}{f''(x_0)}}$$

$$e^{-f(x_0)} \quad \text{vs} \quad \sqrt{\frac{2\pi}{f''(x_0)}}$$

$$f(x_0) \quad \text{vs} \quad \ln \sqrt{\frac{f''(x_0)}{2\pi}}$$

$$\sigma_i = m + \delta\sigma_i$$

$$\langle \sigma_i \rangle$$

$$\boxed{\cancel{\delta\sigma_i \, \delta\sigma_j}}$$

$$\rightarrow f(m) \sim \frac{a_2}{2} m^2 + \frac{a_4}{4} m^4 + \dots$$

$$S[\phi] = \int d^d x \left[\cancel{\frac{c}{2}(\nabla\phi)^2} + \frac{a_2}{2}\phi^2 + \frac{a_4}{4}\phi^4 + \dots \right]$$

mínimo $\phi = m =$ cte.