

Modelo gaussiano, límite del continuo, $a_0 \rightarrow 0$, V finito.

$$S = \frac{1}{V} \sum_{\vec{k}} |\hat{\phi}_{\vec{k}}|^2 \left(\frac{a_2}{2} + \frac{c}{2} \vec{k}^2 \right)$$

$$\langle \hat{\phi}_{\vec{k}_1} \hat{\phi}_{\vec{k}_2} \rangle = \frac{V}{a_2 + c\vec{k}^2} \delta_{\vec{k}_1, -\vec{k}_2}$$

factor de estructura:

$$\langle \hat{\phi}_{\vec{k}} \hat{\phi}_{-\vec{k}} \rangle = \langle |\hat{\phi}_{\vec{k}}|^2 \rangle = V S(\vec{k})$$

$S(\vec{k})$: factor de estructura,

$$\hat{\phi}_{\vec{k}} = \int d\vec{r} e^{i\vec{k} \cdot \vec{r}} \phi(\vec{r})$$

$$S(\vec{k}) = \frac{1}{V} \int d\vec{r}_1 d\vec{r}_2 e^{i\vec{k} \cdot (\vec{r}_1 - \vec{r}_2)} \langle \phi(\vec{r}_1) \phi(\vec{r}_2) \rangle$$

$\phi \sim$ variable magnética (para $\text{Im}(\sigma_i)$)

susceptibilidad magnética

$$\chi = \beta \sum_{ij} [\langle \sigma_i \sigma_j \rangle - \langle \sigma_i \rangle \langle \sigma_j \rangle]$$

$$\sum_{\sigma_i} \rightarrow \int d\vec{r}_1 d\vec{r}_2$$

$$\beta S(\vec{r}=\vec{0}) = \frac{\chi}{V}$$

$$S(\vec{r}=\vec{0}) = a_2^{-1} - \frac{\chi}{V\beta}$$

En la vecindad de un punto crítico.

$$\frac{\chi}{V} \sim |\tau - \tau_c|^{-\nu}$$

↳ volviendo a la teoría de r.l.

$$S = \int d^d \vec{r} \left[\frac{c}{2} (\nabla \phi)^2 + \frac{a_2}{2} \phi^2 + \frac{a_4}{4} \phi^4 + \dots \right]$$

$$a_2 \sim a(T - \tau_c)$$

$$T > \tau_c$$

aproximación gaussiana.

$$S \sim \int d^n r \left[\frac{c}{2} (\nabla \phi)^2 + \frac{a_2}{2} \phi^2 \right]$$

$$\frac{\chi}{V} \sim \beta a_2^{-1} \sim (T - T_c)^{-1} \rightarrow \underline{\underline{\beta = \Delta}}$$

$$\frac{\chi}{V} \sim (T - T_c)^{-\nu} \leftarrow$$

igual para $T < T_c$ $t = T - T_c$

$$\phi(\vec{r}) \sim m + \delta \phi$$

$$S = \int d^n r \left[\frac{c}{2} (\delta \phi)^2 + \underbrace{2a_1 t}_{T T} \phi^2 \right]$$

por ejemplo el oscilador armónico
en H.Q.

$$H = \frac{p^2}{2m} + \frac{1}{2} k x^2 + \lambda x^4$$

\Rightarrow teoría de perturbaciones

$$I = \int_{-\infty}^{\infty} dx e^{-f(x)}$$

De debe hacer si $f(x) = ax^2 + bx + c$
 si $f(x)$ es más complicada?

$\Rightarrow x_0$ es el mínimo de $f(x)$

$$f(x) = f(x_0) + \frac{1}{2}(x-x_0)^2 f''(x_0) + \dots$$

$$I \approx \int_{-\infty}^{\infty} dx e^{-f(x_0) - \frac{1}{2}(x-x_0)^2 f''(x_0)}$$

$$\sim e^{-f(x_0)} \int_{-\infty}^{\infty} dx e^{-\frac{1}{2}(x-x_0)^2 f''(x_0)}$$

$$\sim e^{-f(x_0)} \sqrt{\frac{2\pi}{f''(x_0)}}, \ln I \sim -f(x_0) - \frac{1}{2} \ln \left(\frac{f''(x_0)}{2\pi} \right)$$

$$Z \sim \int \mathcal{D}\phi \underline{e^{-S(\phi)}}$$

$$\sim \underline{e^{-S(\phi_0)}}$$

de S $\phi_0 = m$ donde ϕ_0 es el mínimo

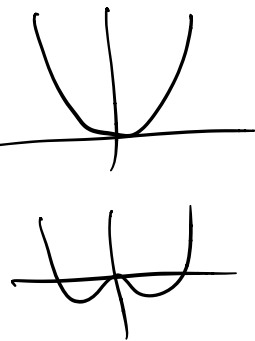
$$\phi(\vec{r}) = \underline{m + \delta\phi(\vec{r})} = m + \psi(\vec{r})$$

$$S(\phi) = \int d^d r \left[\frac{c}{2} (\nabla\phi)^2 + \frac{a_2}{2} \phi^2 + \frac{a_4}{4} \phi^4 \right]$$

$$a_2 = a t, \quad t = T - T_c$$

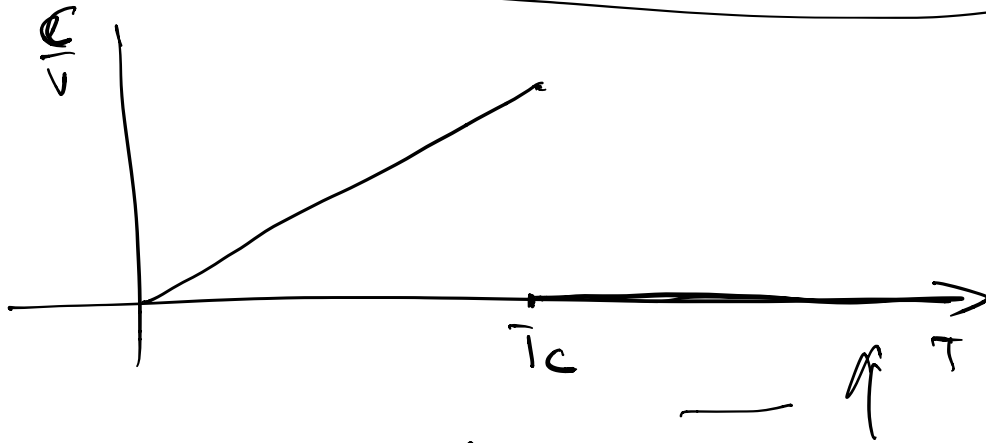
el mínimo de S : $\phi = m$

$$m = \begin{cases} 0 & T > T_c \\ \pm \sqrt{\frac{a(T_c T)}{a_4}} & T < T_c \end{cases}$$



$$\underline{\frac{S(m)}{V} = \frac{a}{2} (T - T_c) m^2 + \frac{a_4}{4} m^4}$$

$$\frac{1}{V} \rho = - \frac{T}{V} \frac{\partial^2 S}{\partial T^2} = \begin{cases} 0 & \text{si } T > T_c \\ \frac{T \alpha^2}{2 \alpha_4} & T < T_c \end{cases}$$

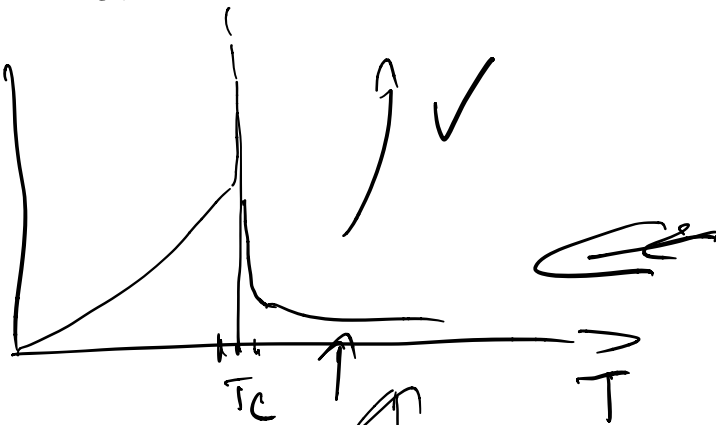


dos casos históricos.

He⁴ y la superconductividad "BCS"

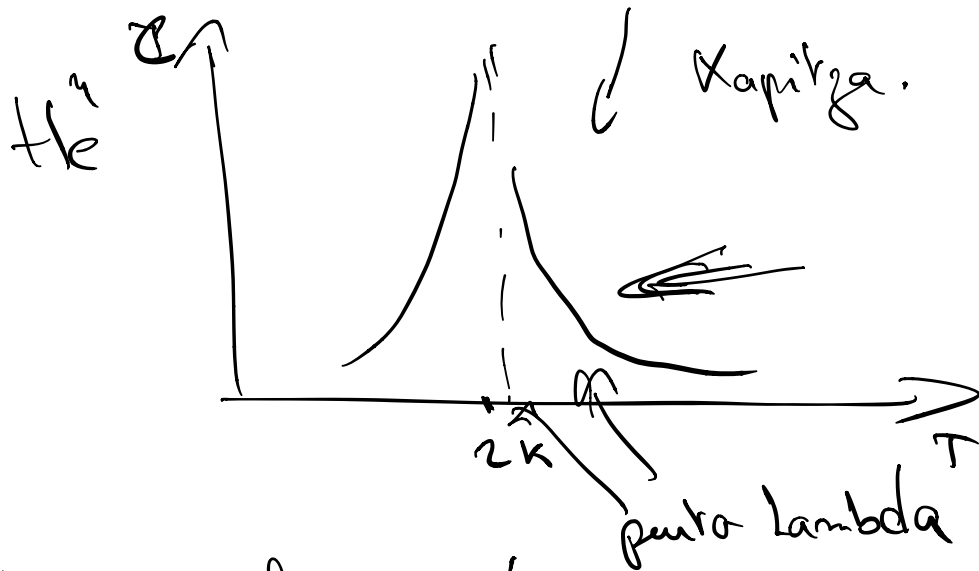
⇒ máxima de simetría SO(2) o U(1)
 espontánea

Supercond
 BCS.



BCS 1957

→ aprox. de campo medio.



retomamos la acción de G.L

$$\phi = m + \gamma(\tau)$$

$$S(m+\gamma) = \int_{t_0}^{t_1} \left[\frac{c}{2} \left[\dot{\gamma}(m+\gamma) \right]^2 + \frac{at}{2} (m+\gamma)^2 + \frac{a\gamma}{4} (m+\gamma)^4 \right]$$

$$S(m+\gamma) \approx S(m) + \int_{t_0}^{t_1} \left[\frac{c}{2} (\dot{\gamma})^2 + \frac{at + 3a\gamma m^2}{2} \gamma^2 \right]$$

$v \left(\frac{a_2 m^2}{2} + \frac{a_4 m^4}{4} \right) \quad + \dots$

$t < 0 \quad \downarrow$
 $t > 0 \quad \downarrow$ **peñis**

$$\Rightarrow S(m+\gamma) = \underline{S(m) + \int_{t_0}^{t_1} \left[\frac{c}{2} (\dot{\gamma})^2 + \frac{1}{3^2} \gamma^2 \right] + \dots}$$

$$z \sim c h e \int \omega d\phi e^{-S(\phi)}$$

$$\sim c h e e^{-S(m_0)} \times \int \omega d\phi e^{-\frac{c}{2} \int d^4x \left[(\vec{\nabla}\psi)^2 + \frac{1}{f^2} \psi^2 \right]}$$

$$z = \frac{c h e}{\mathcal{V}} e^{-S(m_0)} \frac{\prod_{\vec{k}} \left[c \left(\vec{k}^2 + \frac{1}{f^2} \right) \right]^{-1/2}}$$

$$= \underline{e^{-S(m_0)}} \frac{\prod_{\vec{k}} \left[c \left(\vec{k}^2 + \frac{1}{f^2} \right) \right]^{-1/2}}$$

$$[c_0] = 1^2$$

$$-\frac{1}{\mathcal{V}} \ln z = \frac{a t m^2}{2} + \frac{a_4 m^4}{4} + \frac{1}{2\mathcal{V}} \sum_{\vec{k}} \ln \left[c_0 \left(\vec{k}^2 + \frac{1}{f^2} \right) \right]$$

$$V \rightarrow \infty. \quad \frac{1}{V} \sum_{\vec{k}} \xrightarrow{f(x_0)} \int \frac{d^4k}{(2\pi)^4}$$

$$-\frac{1}{\mathcal{V}} \ln z = \frac{a t m^2}{2} + \frac{a_4 m^4}{4} + \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \ln \left[c_0 \left(\vec{k}^2 + \frac{1}{f^2} \right) \right]$$

$$\text{con } \frac{1}{\rho^2} = \begin{cases} \tau a & \text{si } \tau > 0 \quad (T > T_c) \\ -\tau a & \text{si } \tau < 0 \quad (T < T_c) \end{cases}$$

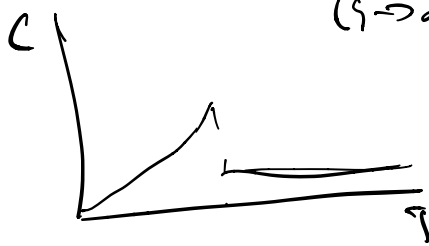
$$\frac{c}{v} = \begin{cases} 0 + \frac{\tau}{2} \int \frac{dk}{(2\pi)^d} \frac{1}{[c_d(k^2 + \frac{1}{\rho})]^2} \\ \frac{\tau a^2}{2a^4} + 2\tau \int \frac{dk}{(2\pi)^d} \frac{1}{[c_d(k^2 + \frac{1}{\rho})]^2} \end{cases}$$

Per $d > 4 \rightarrow$ divergenza per $k \rightarrow \infty$
(V.V.)

\Rightarrow regolarizza, $k_{\text{max}} \sim \frac{1}{\rho_{\text{min}}} \sim \frac{1}{\rho_0}$

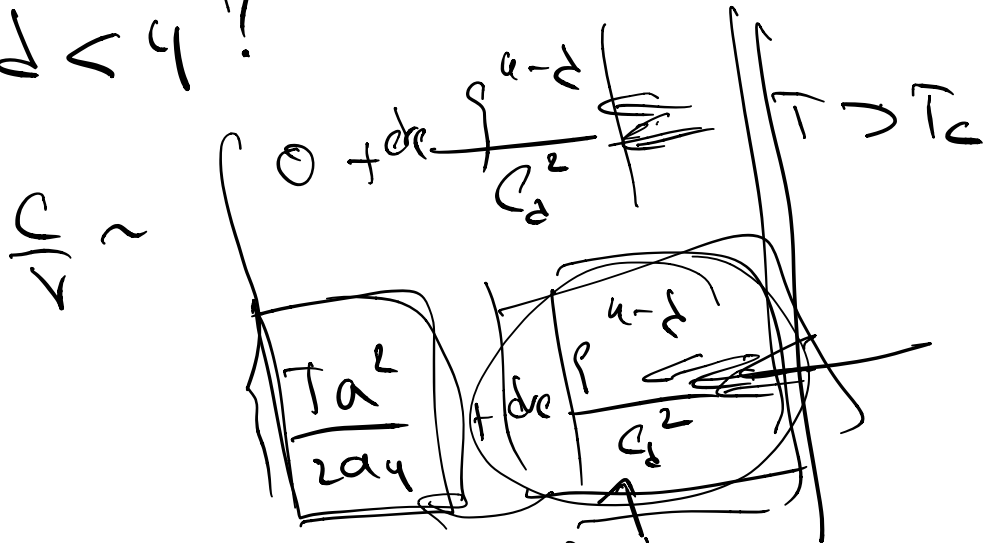
con $\rho_0 \sim a_0$

$$\frac{c}{v} \sim \begin{cases} \text{div } \rho_0^{4-d} & T < T_c \\ \frac{\tau a^2}{2a^4} + \text{div } \rho_0^{4-d} & \leftarrow \text{non diverge per } \tau \rightarrow T_c \quad (\rho \rightarrow \infty) \end{cases}$$

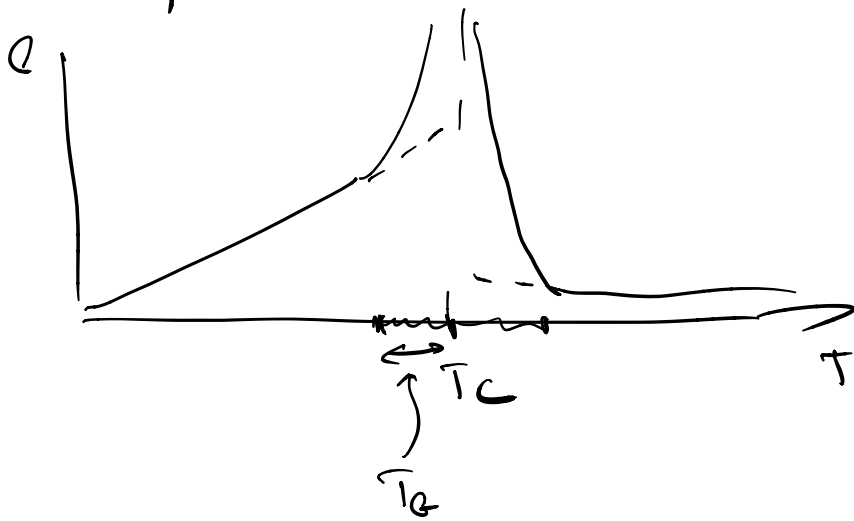


Para $d > 4 \rightarrow$ la aproximación de campo medio funciona bien!

¿ $d < 4$?



la contribución de las fluctuaciones diverge si $\gamma \rightarrow \infty$, o sea $T \rightarrow T_c$



⇒ criterio de Ginzburg

$\xi(\tau)$ longitud de correlación

$$\xi(\tau) \sim \xi_0 \left(\frac{t}{\tau_c} \right)^{-\nu} \quad t = T - \tau_c$$

aquí $\nu = \frac{1}{2}$

ξ_0 longitud microscópica

$$\underline{C_d \sim \xi_0^2}$$

la contribución de las fluctuaciones
se hacen importantes si

$$\frac{1}{\epsilon_d^2} \xi^{4-d} \gtrsim \frac{T_c a^2}{2a^4}$$

↑
valor de C en $T = T_c$

$$\Rightarrow \xi_0^{-d} \left(\frac{t}{\tau_c} \right)^{-\nu(4-d)} \gtrsim \frac{T_c a^2}{2a^4}$$

$$\Rightarrow \frac{|\bar{T} - T_c|}{T_c} \approx \left(\frac{2a_0}{T_c a^2} \right)^{\frac{1}{2}} \left(\frac{d}{v} \right)^{\frac{1}{2}}$$

$$\Rightarrow \bar{T} \approx T_c \left(\frac{2a_0}{T_c a^2} \right)^{\frac{1}{2}} \left(\frac{d}{v} \right)^{\frac{1}{2}}$$

si $\frac{d}{v}$ grande, \bar{T} pequeña
 $\frac{d}{v}$ pequeño, \bar{T} grande.

$\frac{d}{v}$ para varios modelos:

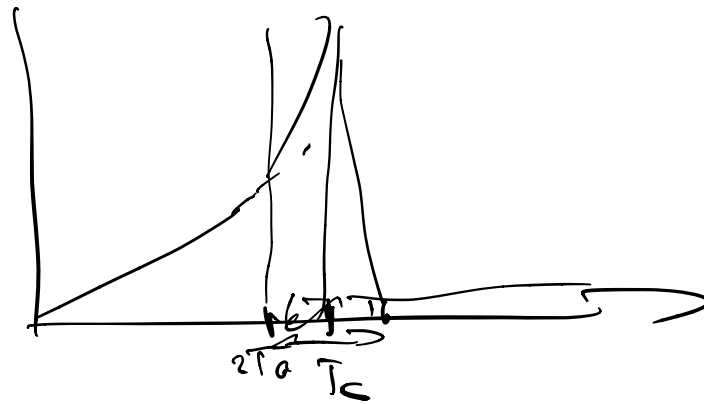
$$T_{avg} \approx \frac{1}{2} \sum_{i,j} S_{ij} \sigma_i \sigma_j$$

$f \sim$ alcance de S_{ij}

si interacción con los vecinos:

$$f \approx a_0$$

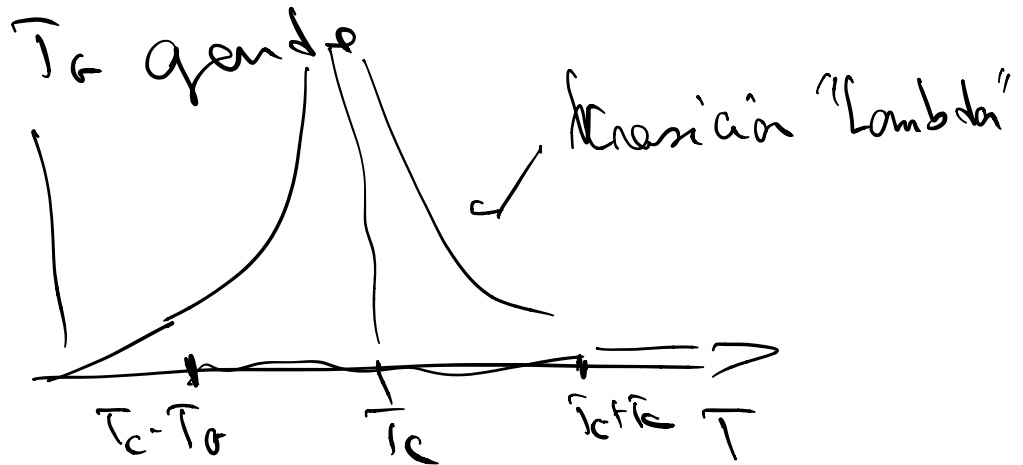
- \rightarrow mientras mayor el alcance de $S_{ij}(l)$
- \rightarrow mas pequeña será T_c



- para el He^4 longitud de onda térmica.

$$\epsilon \sim \frac{p^2}{2m} \sim \frac{\hbar^2 k^2}{2m} \quad k \sim \frac{2\pi}{\lambda}$$

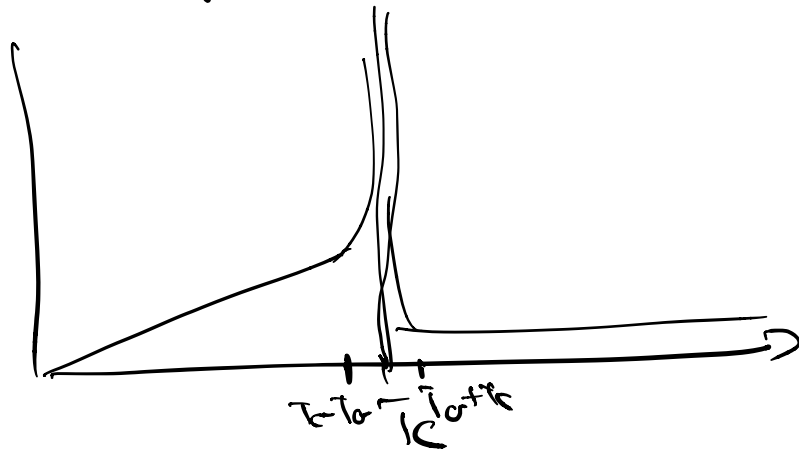
$$k_B T \sim \epsilon \sim \frac{\hbar^2 k_{Th}^2}{2m} \quad k_{Th} \sim \frac{2\pi}{\lambda_{Th}}$$



Superconductas BCS

$\xi_0 \rightarrow$ tamaño típico de los pares de Cooper. $\xi_0 \gg a_0$

$T_G \rightarrow$ pequeña



los exponentes críticos:

$$\left\{ \begin{array}{l} \gamma = \frac{1}{2}, \beta = \frac{1}{2} \\ \delta = 1 \end{array} \right.$$

($\beta = \frac{1}{4}$ para el punto tricrítico)

\rightarrow es conectado $d > 4$

\rightarrow no son conectos para $d < 4$

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{[c(k^2 + \frac{1}{r^2})]^2} \leftarrow$$

$\rightarrow \infty \rightarrow$ divergencia
para $k \rightarrow 0$ (divergencia I.R.)

II (OVI?)
Modelos de Goldstone
y teorema de Mermin-Wagner.

1) ruptura espontánea de simetría
continua

H en un grupo de simetría.

\times $\mathbb{Z}_2 \rightarrow \mathbb{Z}_2 \leftarrow \times$

$\left\{ \begin{array}{l} \leftarrow \text{Heisenberg} \rightarrow \text{SO}(3) \end{array} \right.$

$\left\{ \begin{array}{l} \leftarrow \text{X, He, Supercond} \rightarrow \text{SO}(2) \sim \text{U}(1) \end{array} \right.$

$\left\{ \begin{array}{l} \leftarrow \text{partículas en interacción} \\ \rightarrow \text{sign. de traslación.} \end{array} \right.$

ruptura espontánea de simetría:
 \rightarrow nueva fase.

partículas con interaccións:
 \rightarrow fase cristalina.

Heisenberg:
 \rightarrow fase ferromagnética.

metal
 \rightarrow fase supercond.

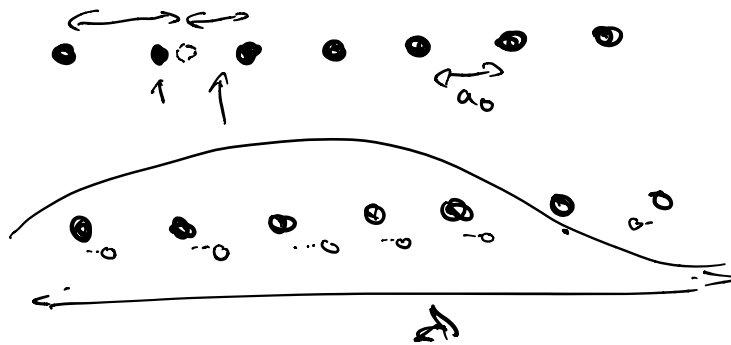
$He^4 \rightarrow$ fase superfluida.

\rightarrow aparecen excitaciones de baja energía.

$$E(k) \sim ck \text{ o } \alpha k^2$$

\rightarrow modos de fonones.

cristal:

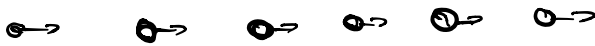


$$E(k) \sim \frac{1}{v} \omega ck$$

ω velocidad del sonido.

excitaciones: fonones.

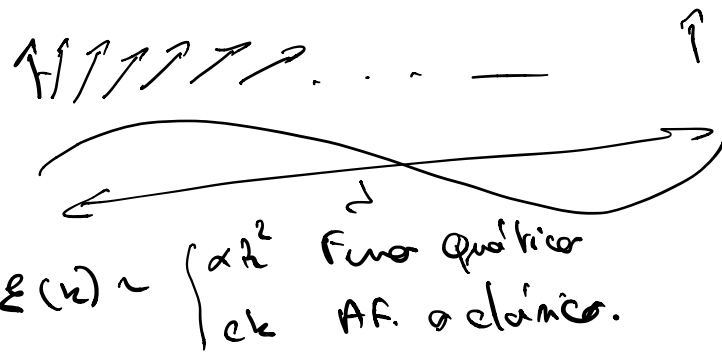
$$(k \rightarrow 0) \quad \omega \rightarrow 0$$



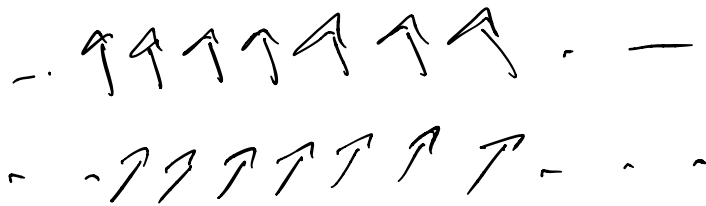
$$E(k) \xrightarrow[k \rightarrow 0]{\omega \rightarrow 0} 0$$

materiales magnéticos: Heisenberg





$E \rightarrow 0$ si $k \rightarrow 0$ ($\lambda \rightarrow \infty$)



$E=0$

aquí la excitacions se llaman ones de espín. (spinwaves).

$$G(r) \sim \int d^d k \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\hbar^2 + m^2} \sim \frac{e^{-mr}}{\hbar^2}$$

$$E(k) \approx \hbar^2 + m^2$$

$$E(k) \xrightarrow{k \rightarrow 0} 0 \Rightarrow m=0$$

\rightarrow correlacions de llarga abastada.

$E(k) \xrightarrow{k \rightarrow 0} 0$ ($\lambda \rightarrow \infty$) excitacions sin massa.