Tarea 1

1)

$$H = -J \sum_{\langle ij \rangle} \left[S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z \right]$$

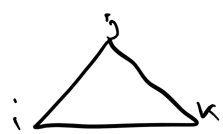
$\Delta \neq 1$ simetría $O(2) \times \mathbb{Z}_2$

$\Delta = 1 \Longrightarrow O(3)$

$\Delta > 1 \quad S_i^z = \pm 1$

$\Delta < 1$

2)



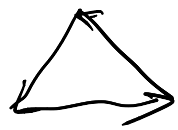
$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \qquad \vec{S}_i \cdot \vec{S}_i = 1$$

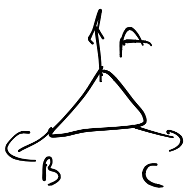
$$= \frac{J}{2} \left(\vec{S}_i + \vec{S}_j + \vec{S}_k \right)^2 - \frac{3}{2} J$$

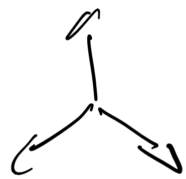
$$H = \frac{J}{2} \left(\vec{S}_{TOT} \right)^2 + cte$$

mínimo ($J>0$) $\vec{S}_{TOT} = \vec{0}$

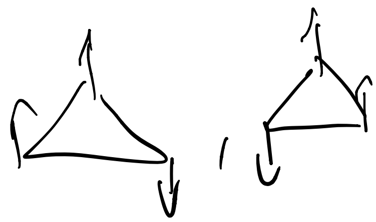
$\vec{S}_i + \vec{S}_j + \vec{S}_k = \vec{0}$

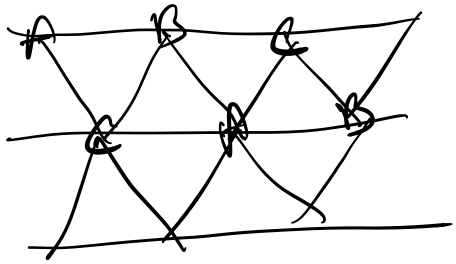


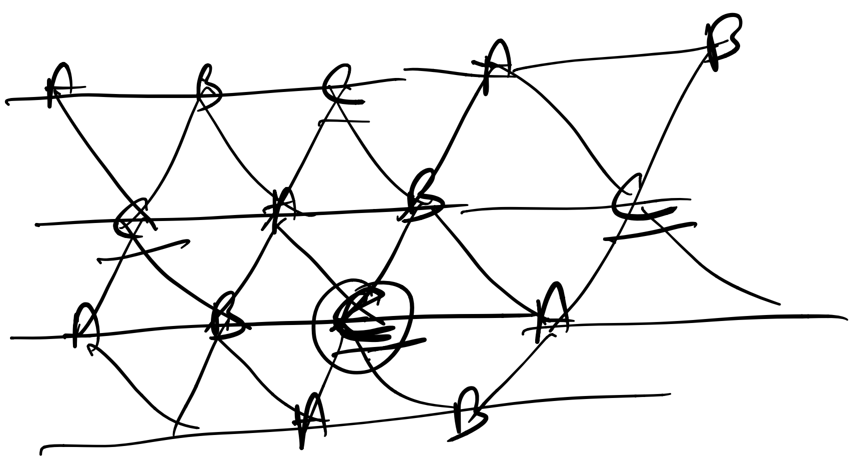




$\uparrow\uparrow\downarrow$ y $\downarrow\downarrow\uparrow$

 etc..





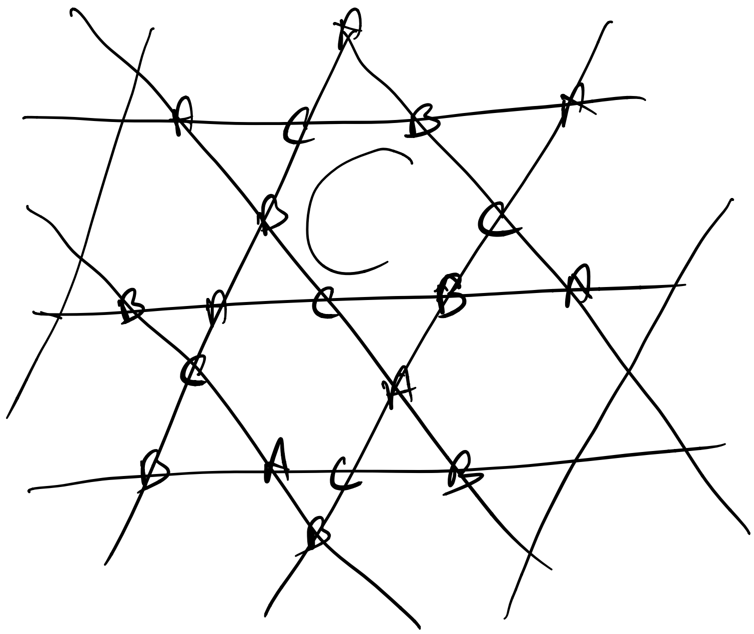
$\uparrow \to \downarrow$

A = $\uparrow$

B = $\downarrow$

C = $\uparrow$ o $\downarrow$

Wannier (1950)



4) $N \to \infty$ para la transición de fase.

→ Mean field o campo medio es exacto.

$$\sum_{i,j} \sigma_i \sigma_j = \sum_i \sigma_i \sum_j \sigma_j = \left(\sum_i \sigma_i\right)^2$$

$$e^{-\frac{J}{N}\left(\sum_i \sigma_i\right)^2} \xrightarrow[H.S]{}$$

$$S = \int d^D\vec{r}\,\Big[\frac{K}{2}(\nabla\phi)^2 + \frac{t}{2}\phi^2 + \boxed{\mu\phi^4 + \mu_6\phi^6 + \mu_8\phi^8 + \dots}\Big]$$

simetría $\mathbb{Z}_2$ $\phi \to -\phi$

$t \sim |T-T_c| \sim \xi^{-\frac{1}{\nu}}$

$D = 2$

$\vec{r} \to b\vec{r}\,' \qquad \vec{q} \to b^{-1}\vec{q}\,'$

$\phi \to b^{1-\frac{D}{2}}\phi' \,, \quad t \to b^{-2}t' \quad \to \underline{t' = b^2 t}$

$\mu \to b^{D-4}\mu' \quad \to \underline{\mu' = b^{4-D}\mu}$

$\mu_6 \to b^{-6+2D}\mu_6' \qquad \mu_6' = b^{-2D+6}\mu_6 \sim \underline{b^{-2}}\mu_6$

$\mu_8 \to b^{-8+3D}\mu_8' \qquad \mu_8' = b^{-3D-8}\mu_8 \sim \underline{b^{-4}\mu_8}$

si $D = 4 \quad \to \mu\phi^4$ invariante de escala

$\underline{K(\nabla\phi)^2}$

si $D > 4$ $\mu' \to 0$ $\to$ el campo medio funciona.

si $D < 4$ μ' aumenta $\to$ el campo medio no va a funcionar.

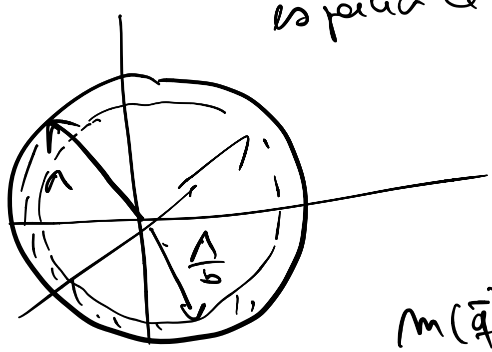
$\to D = 4$ es la dimensión crítica superior de μ. (nos interesa $D = 3$)

$\epsilon = 4 - D$ "parametro pequeño"

→ nos interesa el caso $D \lesssim 4$ (ϵ pequeño)

para $D \lesssim 4$ μ_6, μ_8 etc se van a 0 con el R.G. μ_6, μ_8 etc son irrelevantes

espacio de los "$\vec{q}$"



$\Lambda \sim \frac{1}{a_0}$

$$m(\vec{q}) = \begin{cases} \tilde{m}(\vec{q}) & \text{si } 0 < \vec{q} \le \frac{\Lambda}{b} \\ \sigma(\vec{q}) & \text{si } \frac{\Lambda}{b} < \vec{q} \le \Lambda \end{cases}$$

$$Z = \int \prod_{|\vec{q}| < \frac{\Lambda}{b}} d\tilde{m}(\vec{q}) \prod_{\frac{\Lambda}{b} < |\vec{q}| < \Lambda} d\sigma(\vec{q})\, e^{-S_0(m) - S_0(\sigma) - U}$$

$$U = \mu \int \frac{d\vec{q}_1\, d\vec{q}_2\, d\vec{q}_3\, d\vec{q}_4}{(2\pi)^{4d}} (2\pi)^D \delta(\vec{q}_1 + \vec{q}_2 + \vec{q}_3 + \vec{q}_4)\, m(\vec{q}_1) \cdots m(\vec{q}_4)$$

$$Z_{eff} = \int \mathcal{D}\tilde{m}(\vec{q})\, e^{-S_{eff}}$$

$$S_{eff}(m) = cte + S_0(m) - \ln \langle e^{-u} \rangle_\sigma$$

$$\ln \langle e^{-u} \rangle_\sigma = -\underline{\langle u \rangle_\sigma} + \frac{1}{2}\underline{\left(\langle u^2 \rangle_\sigma - \langle u \rangle_\sigma^2 \right) + \ldots}$$

$$\underline{t \to t + 12\mu \int_{\Lambda/b}^{\Lambda} \frac{d^D k}{(2\pi)^D} \frac{1}{t + K k^2}}$$

+ el cambio de escala

$$t' = b^2 \left[t + 12\mu \int \cdots \right]$$

$$\mu' = b^{(4-D)} \mu$$

$$b \simeq (1 + \delta l) \qquad b^2 \simeq (1 + 2\delta l)$$

$$b^\alpha = (1 + \alpha \delta l) \ldots$$

$$t' = t + \delta l \frac{dt}{dl}$$

$$\mu' = \mu + \delta l \frac{d\mu}{dl}$$

$$d^D k = \Lambda^D dl \, dS_D$$

$$\int_{\frac{\Lambda}{1+\delta l}}^{\Lambda} \frac{d^D k}{(2\pi)^D} \frac{1}{t + K k^2} = \frac{S_D}{(2\pi)^D} \frac{\Lambda^D \delta l}{t + K \Lambda^2}$$

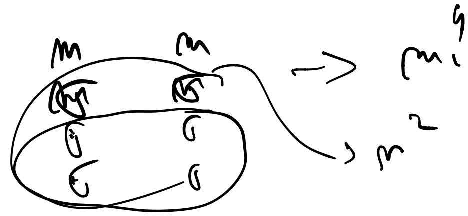
$$S_d = \frac{2\pi^{d/2}}{\Gamma\left(\frac{d}{2}\right)}$$

$$\boxed{\begin{aligned} \frac{dt}{dl} &= 2t + \frac{12\mu \frac{S_D}{(2\pi)^D}\Lambda^D}{t+\kappa\Lambda^2} \\ \frac{d\mu}{dl} &= (4-D)\mu \end{aligned}}$$

$$\langle u^2 \rangle$$

$$\phi^4 \cdot \phi^4 \to m^4 m^4 \to m^4$$

Cada m puede ser m $\hat{m}$ o



$$\frac{(4\cdot 3)^2}{2} = 72$$

$$\boxed{\begin{aligned} \frac{dt}{dl} &= 2t + \frac{12\mu \frac{S_D}{(2\pi)^D}\Lambda^D}{t+\kappa\Lambda^2} - A\mu^2 \\ \frac{d\mu}{dl} &= (4-D)\mu - \underbrace{\frac{36 \frac{S_d}{(2\pi)^d}\Lambda^D}{(t+\kappa\Lambda^2)^2}}_{B}\mu^2 \end{aligned}}$$

punto fijo. (invariante de escala)

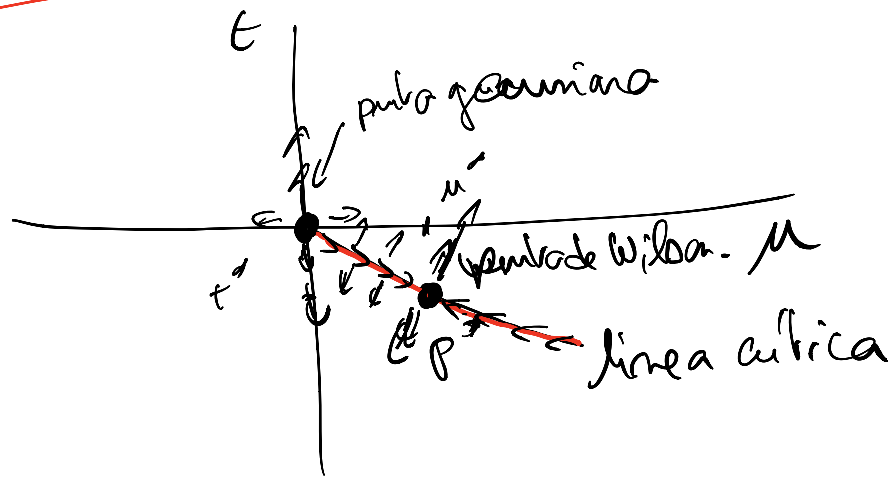
$$\left|\begin{aligned} t^* &= -6\mu^* \frac{S_D}{(2\pi)^D}\Lambda^D + \frac{A}{2}\mu^{*2} \\ \mu^* &= \frac{(t^*+\kappa\Lambda^2)^2}{36\frac{S_D}{(2\pi)^D}\Lambda^D}\underbrace{\frac{(4-D)}{A}}_{\epsilon} \end{aligned}\right|$$

$D \sim 4$, $\epsilon \ll 1$ $\qquad \frac{S_D}{(2\pi)^D} \to \frac{S_4}{(2\pi)^4}$

$t^* \sim \epsilon$ $\qquad t^* + \kappa\Lambda^2 \sim \kappa\Lambda^2$

$\mu^* \sim \epsilon$

$$\boxed{\begin{cases} t^* = -\dfrac{\kappa\Lambda^2}{6}\epsilon + \mathcal{O}(\epsilon^2) \\ \mu^* = \dfrac{\kappa^2}{36\frac{S_4}{(2\pi)^4}}\epsilon + \mathcal{O}(\epsilon^2) \end{cases}}$$



el punto de Wilson representa el punto crítico del modelo de Ising en dimensión $D = 4-\epsilon$

$t = t^{*} + \delta t$

$\mu = \mu^{*} + \delta\mu$

$$\frac{d}{dl}\begin{pmatrix}\delta t\\ \delta\mu\end{pmatrix} = \begin{pmatrix}\underline{2-\frac{\epsilon}{3}} & -\frac{A K^2 \epsilon (2\pi)^4}{18 S_4} \\ 0 & -\epsilon\end{pmatrix}\begin{pmatrix}\delta t\\ \delta\mu\end{pmatrix}$$

$$\frac{d}{dl}\delta\mu = -\epsilon\,\delta\mu \quad \Rightarrow \delta\mu \to 0$$

un autovalor negativo $(-\epsilon)$ dirección atractiva

" " positivo $(2-\frac{\epsilon}{3})$ " repulsiva

$$\frac{d}{dl}\delta t = (2-\frac{\epsilon}{3})\,\delta t$$

$$\delta t \sim (\delta l)^{2-\frac{\epsilon}{3}} \qquad \xi \sim (\delta l)^{-1}$$

$$\nu = \frac{1}{2-\frac{\epsilon}{3}} \simeq \frac{1}{2} + \frac{1}{12}\epsilon + O(\epsilon^2)$$

si $\epsilon = 1$ $\quad \nu \simeq 0{,}58 \quad (\nu_{3D} \simeq 0{,}63)$

complemento: el modelo $O(n)$

$$\vec{\phi} = \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix}$$

$$S = \int d^D \vec{r} \left[\frac{k}{2} \sum_\mu (\partial_\mu \vec{\phi})^2 + \frac{t}{2} \vec{\phi}^2 + u (\vec{\phi}^2)^2 \right]$$

$$\vec{\phi} = O \vec{\phi} \qquad O^\dagger O = \mathrm{Id}$$

las eq del R.G.

$$\frac{dt}{dl} = 2t + \frac{4u(n+2)K\Lambda^D}{t + K\Lambda^2} - Au^2$$

$$\frac{du}{dl} = (4-D)u - \frac{4(n+8)K\Lambda^D}{(t+K\Lambda^2)^2} u^2$$

$$u^* = \frac{k^2}{4(n+8)K} \epsilon + O(\epsilon^2)$$

$$t^* = -\frac{(n+2)}{2(n+8)} K\Lambda^2 \epsilon + O(\epsilon^2)$$

$$\nu = \frac{1}{2} + \frac{1}{4} \frac{n+2}{n+8} \epsilon + O(\epsilon^2)$$

5] puntos críticos y universalidad

P_1, P_2 etc. puntos fijos

fine tuning.

S.O.C. = self organized criticality.

→ Universalidad: cada punto fijo corresponde a una clase de universalidad.

→ valores específicos para los exp. críticos

Cada punto crítico es una teoría de campos inv. de escala.

→ campos fundamentales $\Phi_i(\vec{r})$ locales

$$\langle \Phi_i(\vec{r}_1)\, \Phi_i(\vec{r}_2) \rangle \sim \frac{cte}{|\vec{r}_1 - \vec{r}_2|^{2x_i}}$$

x_i dimensión de escala del camp Φ_i

$[\Phi_i] = L^{-x_i}$

si Dalimos de e ← punto crítico.

la accion de c ~~es~~ S_0

$$\underline{S = S_0 + \int d^D r\, g_i \Phi_i}$$

$[\Phi_i] = L^{-x_i}$ $\qquad$ $[g_i] = \underline{L^{x_i - D}}$

$L \to bL$ $\qquad$ $g_i \to g_i' \, b^{x_i - D}$

$$g_i' = g_i \, b^{D - x_i}$$

si $\underline{x_i > D}$ $\quad g_i' \to 0$ → valores a S_0

si $\underline{x_i < D}$ $\quad g_i' \nearrow$ → relevante → nos alejamos de S_0

el campo "termico" $\qquad t = |T - T_c|$

si movemos $T - T_c$

$$S_0 \to S = S_0 + \int d^D \vec{r}\, t\, \phi_E(\vec{r})$$

$[t] = L^{-1/\nu}$ $\qquad t \sim \xi^{-1/\nu}$, $\boxed{x_E = D - \frac{1}{\nu}}$

la noción de O.P.E. (operator product expansion)

$\underline{\Phi_i(\vec r)}$ son $\underline{\underline{\text{todos}}}$ los campos locales asociados

a C.

$$\Phi_i(\vec r)\;\Phi_j(\vec r') \longrightarrow \underset{\text{campo local}}{\bullet\,\vec r}\;\sum_k \Phi_k(\vec r)$$

$\vec r' \to \vec r$

$$\Phi_i(\vec r)\,\Phi_j(\vec r') \xrightarrow[\lim |\vec r - \vec r'| \to 0]{} \sum_k{}' \frac{\Phi_k\left(\frac{\vec r+\vec r'}{2}\right) C_{ijk}}{|\vec r - \vec r'|^{x_i+x_j-x_k}}$$

C_{ijk} = las constantes de O.P.E de C.

o las constantes de estructura.

6] R.C. con el O.P.E

$$S = S_0 - \sum_i{}' g_i \underline{\sum_{\vec r}{}' a_0^{x_i} \Phi_i(\vec r)}$$

$[g_i] = 0$

$\rightarrow$ continuo $\quad \sum_{\vec r}{}' \rightarrow \int \frac{d^D \vec r}{a_0^D}$

$$S = \underline{S_0} - \underbrace{\int d^D\vec{r} \sum_i a^{x_i - D} g_i \Phi_i(\vec{r})}_{S_{pert.}}$$

$$Z_g = \underset{\uparrow}{Z_0} \left< \underline{e^{-S_{pert}}} \right>_0 = Z_0 \left(1 - \left< S_{pert} \right>_0 + \frac{1}{2} \left< S_{pert}^2 \right>_0 + \cdots \right)$$

$$= Z_0 \left[1 - \sum_i g_i \int \frac{d^D\vec{r}}{a^{D-x_i}} \left< \phi_i(\vec{r}) \right> + \frac{1}{2} \sum_{i,j} g_i g_j \iint \frac{d^D\vec{r}_1 \, d^D\vec{r}_2}{\underline{a_0^{2D-x_i-x_j}}} \left< \phi_i(\vec{r}_1) \phi_j(\vec{r}_2) \right> + \cdots \right]$$

$$\vec{r}_1 \to \vec{r}_2 \qquad \phi_i(\vec{r}_1) \phi_j(\vec{r}_2) \to \sum_k \frac{c_{ijk} \phi_k(\vec{r}_1)}{|\vec{r}_1 - \vec{r}_2|^{x_i + x_j - x_k}}$$

hacemos la integral en $\vec{r}_2$ únicamente cuando $\underline{a_0 < |\vec{r}_2 - \vec{r}_1| < a_0(1+\delta l)}$

queda $\underline{\frac{1}{2} S_d \, \delta l \sum_{ijk}} \int d^D\vec{r}_1 \frac{\left< \phi_k(\vec{r}_1) \right>}{\underline{a_0^{D-x_k}}}$

$$Z = Z_0 \left[1 - \sum_i \hat{g}_i \int \langle \phi_i \rangle \frac{d^D r_i}{a_0^{\ldots}} + \cdots \right.$$

$$\hat{g}_k = g_k - \frac{1}{2} S_d \sum_{ij}{}' C_{ijk}\, g_i g_j\, \delta \ell$$

$+$ el re-escale $\underline{a \to a(1+\delta\ell)}$

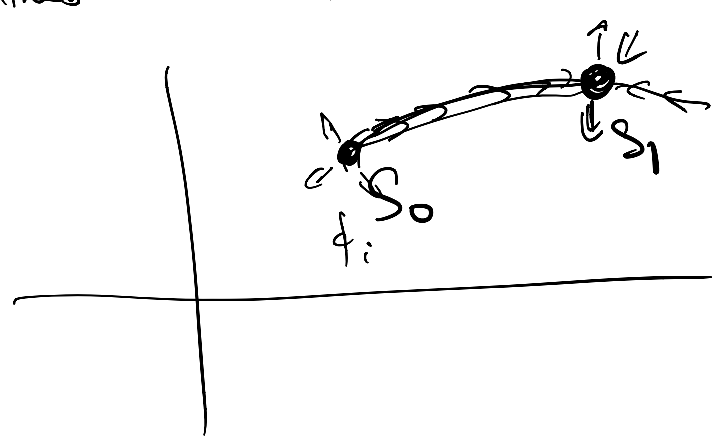
$$\Rightarrow \frac{d g_k}{d\ell} = (d - x_k)\, g_k - \frac{1}{2} S_d \sum_{ij}{}' C_{ijk}\, g_i g_j + \mathcal{O}(g^3)$$

redefinimos los $g_k \to \frac{2}{S_d} g_k$

$$\Rightarrow \boxed{\frac{d g_k}{d\ell} = (d - x_k)\, g_k - \sum_{ij}{}' C_{ijk}\, g_i g_j} + \mathcal{O}(g^3)$$

ecuaciones del R.G. a segundo orden.

Fórmula de Polyakov.



obs $g_1 g_2$ monos.

$$C_{ijk} = C_{jik}$$

$$C_{12k} g_1 g_2 + C_{21k} g_2 g_1$$

$$= 2 C_{12k} g_1 g_2$$

$$C_{11k} g_1^2$$