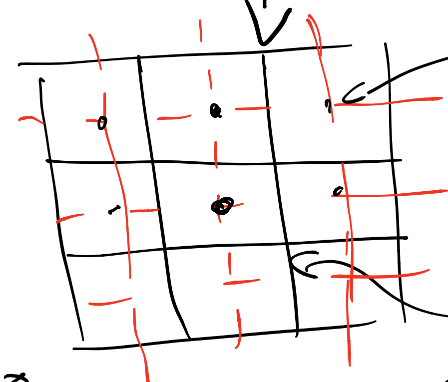
Modelo XY

$$Z = \int_0^{2\pi} \prod_i d\theta_i \; e^{\tilde{\beta} \sum_{i,\mu} \underbrace{\cos(\Delta_\mu \theta)}}$$

$$\theta_{i+a_0\hat{e}_\mu} - \theta_i$$

$$\tilde{\beta} = \frac{J}{k_B T}$$

$\Big\downarrow$ Transf. de dualidad



$$Z \propto \int_{-\infty}^{\infty} \prod_{j^*} d\varphi_{j^*} \sum_{\{m(j^*)=-\infty\}}^{+\infty} e^{\left(-\frac{1}{2\tilde{\beta}} \sum_{j^*,\mu} (\Delta_\mu \varphi_{j^*})^2 + 2\pi i \sum_{j^*} m(j^*)\varphi_{j^*}\right)}$$

→ gas de partículas de carga $m(j^*)$ interactuando con el potencial de Coulomb.

las partículas representan los vórtices.

$$Z \propto \int \prod d\varphi_{j^*} \, e^{-\frac{1}{2\tilde{\beta}} \sum_{j^*,\mu} (\Delta_\mu \varphi_{j^*})^2} \sum_{\{m(j^*)\}} e^{2\pi i \sum_{j^*} m(j^*) \varphi_{j^*}}$$

aquí la $\sum_{\{m(j^*)\}} \Rightarrow \sum_{m(j^*)=0,\pm 1}$

para cada j^*, $m = 0, \pm 1, \pm 2, \ldots$

$$Z \simeq \int \prod_{j^*} d\varphi_{j^*} \, e^{-\frac{1}{2\tilde{\beta}} \sum_{j^*,\mu} (\Delta_\mu \varphi_{j^*})^2} \Big(1 + \sum_{j^*, k^*}' e^{2\pi i \varphi_{j^*}} e^{-2\pi i \varphi_{k^*}}$$

$$+ \sum_{j^*, k^*, l^*, n^*}' e^{2\pi i \varphi_{j^*}} e^{2\pi i \varphi_{k^*}} e^{-2\pi i \varphi_{l^*}} e^{-2\pi i \varphi_{n^*}} \Big)$$

$$+ \ldots$$

$$Z \simeq \int \prod d\varphi_j \, e^{-\frac{1}{2\tilde{\beta}} \cdots + \lambda \sum_{j^*} e^{2\pi i \varphi_{j^*}} + \lambda \sum_{k^*} e^{-2\pi i \varphi_{k^*}} + \ldots}$$

$\Rightarrow$ límite al continuo

$$\sum_{j^*,\mu} (\Delta_\mu \varphi_{j^*})^2 \Rightarrow \int d^2\vec{r} \, (\vec{\nabla} \varphi)^2$$

$$Z \propto \int \mathcal{D}\varphi(\vec{r})\, e^{-\frac{1}{2\hat{\beta}} \int d^2\vec{r} \left[(\nabla\varphi)^2 + g\cos(2\pi\varphi(\vec{r})) \right] + \cdots}$$

S_0 inv. de escala

acción de Sine-Gordon.

eq. de movimiento

$$\Delta\varphi + 2\pi g \sin(2\pi\varphi) = 0$$

eq. de Sine-Gordon.

$$\cos(2\pi\varphi) \to m^2\varphi^2$$

$$\Delta\varphi + m^2\varphi = 0 \quad \text{Helmholtz}$$

↳ Klein-Gordon con la métrica de Lorentz.

$$S = S_0 + \int d^2\vec{r}\, g\, \Phi_{\cos} \to R.G.$$

→ dimensión de escala de $\Phi_{\cos} = \cos(2\pi\varphi(\vec{r}))$

a partir de

$$\langle e^{2\pi i \varphi(\vec{0})}\, e^{-2\pi i \varphi(\vec{r})} \rangle_{S_0} \sim \frac{1}{|\vec{r}|^{2\pi\hat{\beta}}}$$

$$[\cos(\ldots)] = L^{-\pi\hat{\beta}}$$

$$\to [g] = L^{\pi\hat{\beta}-2}$$

R.G.

$$\frac{dg}{dl} = \left(2-\pi\hat{\beta}\right) g + \dots$$

$\rightarrow$ g es relevante si $2-\pi\hat{\beta} > 0$

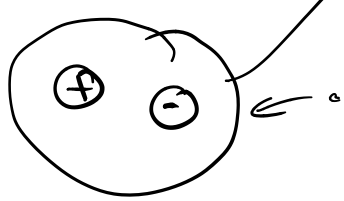
$$\Rightarrow 2 - \frac{\pi J}{k_B T} > 0 \Rightarrow T > \frac{\pi J}{2 k_B} = T_{BKT} = T_{KT}$$

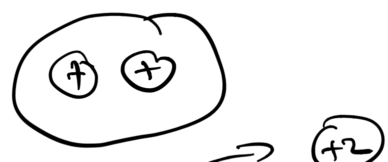
O.P.E

$$\Phi_{\cos}\cdot\Phi_{\cos} \rightarrow \underbrace{cte_1 \left(\nabla\varphi\right)^2}_{g} + cte_2 \cancel{\cos(4\pi\varphi)}$$

$e^{+i}\cdot e^{-i}$

$e^{+i}\, e^{+i}$



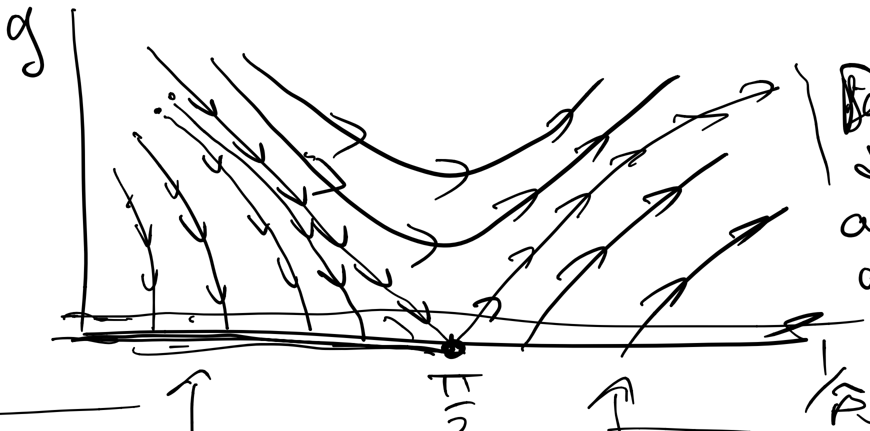


$$S = +\frac{1}{2\tilde{\beta}}\int d^2r \left[\left(\nabla\varphi\right)^2 + g\cos(2\pi\varphi) + g_2\cos(4\pi\varphi) + g_3\cos(6\pi\varphi) + \dots\right]$$

la dim de escala de $\cos(n 2\pi\varphi)$ aumenta como n^2 $\rightarrow$ son campos cada vez menos relevantes eq de R.G.

$$\frac{d}{dl}\left(\frac{1}{\hat{\rho}}\right) = A g^2 + \dots$$

$$\frac{d}{dl} g = (2-\pi\hat{\rho})\, g + O(g^3)$$

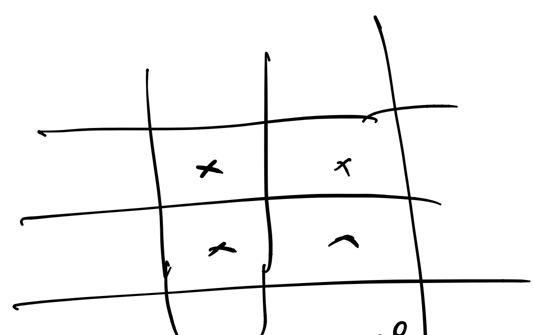


Fase desorden a corto alcance.

modelo gaussiano, crítico.
Con un $\frac{1}{\hat{\rho}}$ variable
→ con exponentes variables

g finito.

Fase $T < T_{BKT}$
QLRO

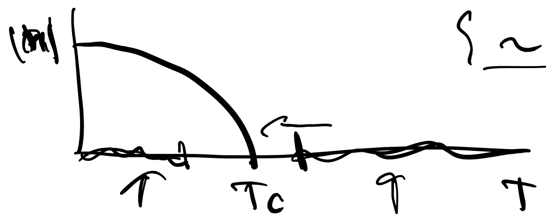


Campos de He^4 ρ_s

VI Temas Diversos

1) Escaleo en tamaño finito

Ising



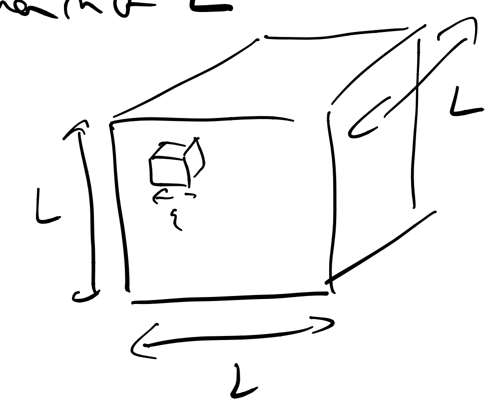
$\underline{\xi \sim t^{-\nu}}$ $\qquad t = |T - T_c|$

$\frac{\chi}{N} \Rightarrow \chi_v \sim t^{-\gamma}$

$\frac{C}{N} \sim c \sim t^{-\alpha}$

$N \to \infty$

→ tamaño L



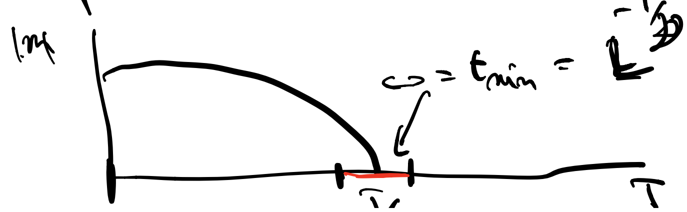
$\xi \ll L$

→ como si $L \to \infty$

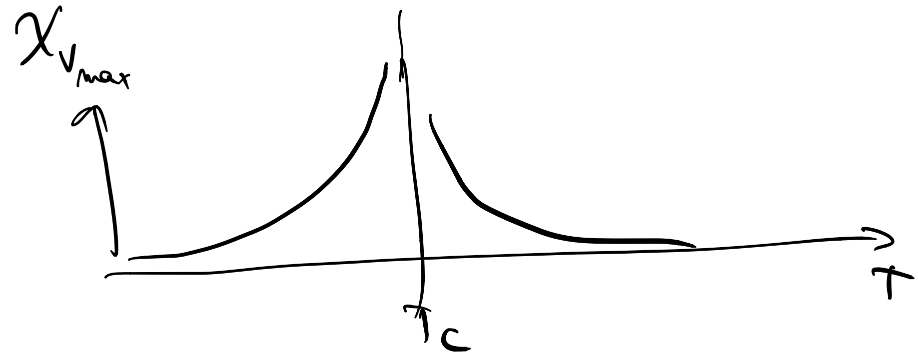
si $t^{-\nu} \ll L$

pero si, $t^{-\nu} \sim L$ o $t \sim L^{-1/\nu}$ osea $\xi \sim L$

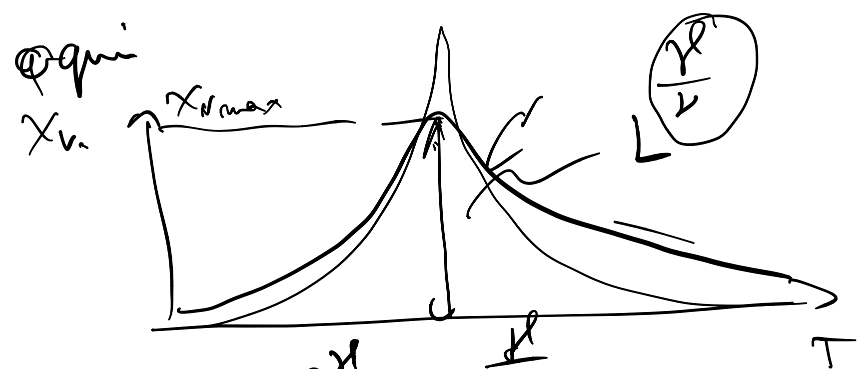
→ nos sirve para poder estudiar el comportamiento del sistema para escalas mas grandes



→ Como tema en $\xi = L$ $\quad t \neq 0 \quad T \neq T_c$

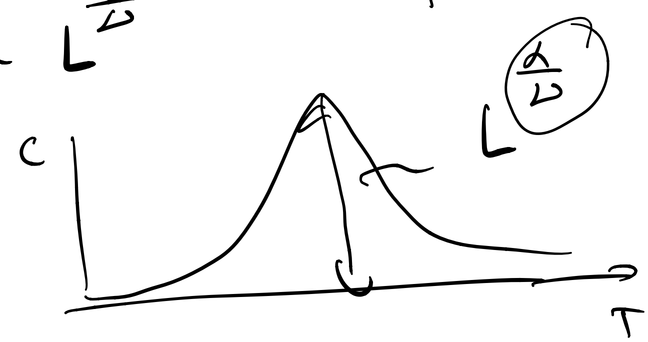


↳ aquí

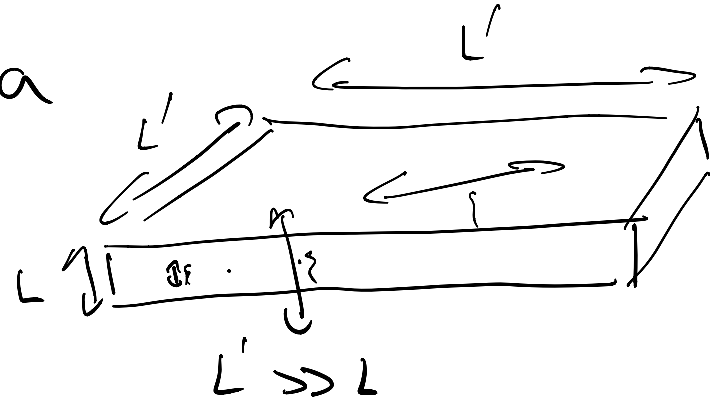


$$\chi_{V max} = t_{min}^{-\gamma} \sim L^{\gamma/\nu}$$

$$C_{max} \sim L^{\alpha/\nu}$$



Si ahora



$L' \gg L$

$\xi \ll L \rightarrow$ comportamiento 3-D

si $l \ll \xi \ll L' \rightarrow$ comportamiento 2-D

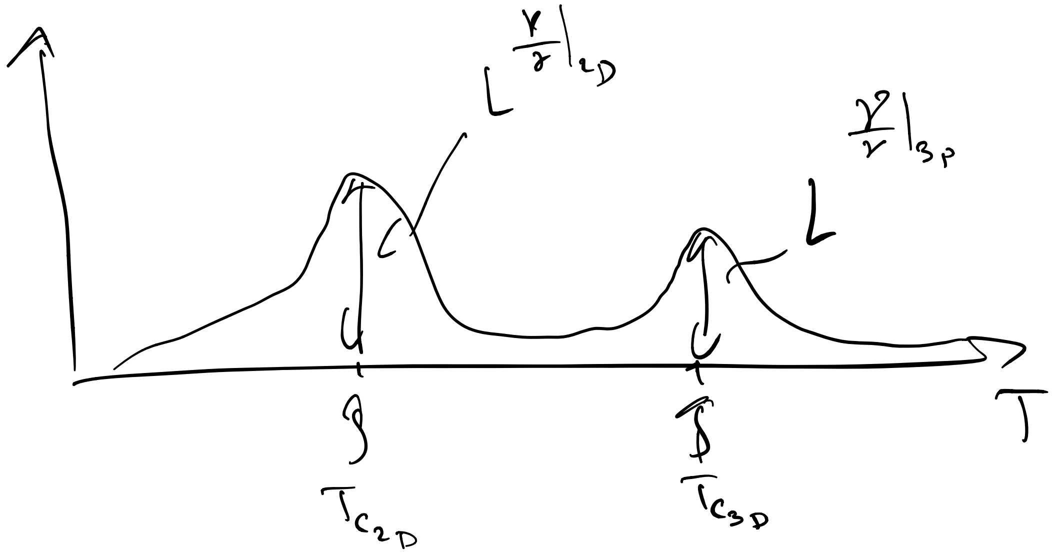
Pero $T_{c_{3D}} \neq T_{c_{2D}}$

* si apuntamos a $T_{c_{3D}} \sim T$

$$\chi_{v_{max}} \sim L^{\frac{\gamma_{3D}}{\nu_{3D}}}$$

* si apuntamos a $T_{c_{2D}} \sim T$

$$\chi_{v_{max}} \sim L'^{\frac{\gamma_{2D}}{\nu_{2D}}}$$



2) Sobre las fluctuaciones cuánticas

gases

estadística de M.B.

estadísticas cuánticas

B.E

F.D.

$\frac{p^2}{2m} \sim E \sim k_B T$

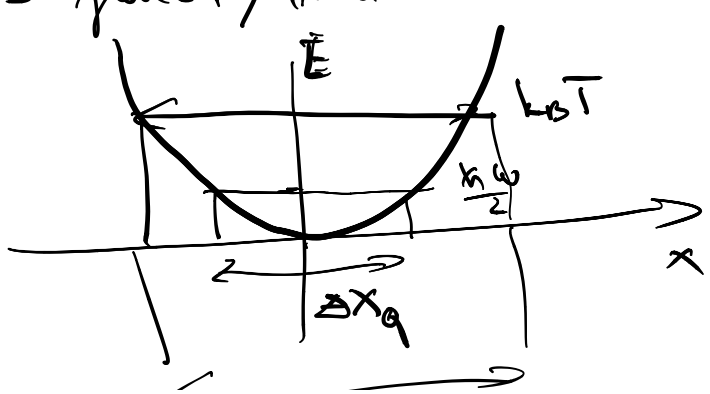
$p = \hbar k \sim \frac{h}{\lambda} \Rightarrow \lambda(T)$

$\lambda \sim d$

criticalidad para $T \sim T_c \neq 0$

→ Cuando tomar en cuenta las fluctuaciones cuánticas? o incertidumbres.

vs fluct. / incert. de origen térmico.



$\longleftarrow \Delta x_{Th}$

Δx_{Th} vs Δx_Q

$\underline{k_B T}$ vs $\underline{\hbar\omega}$

en general, para un sist. a temp. T

Sólo las fluct. cuánticas de energías $\underline{E \geq k_B T}$

Se van a observar.

$E \sim cte\, P^z$ $\quad z$ exponente dinámico.

$E = cP \qquad z = 1$

$E = \frac{P^2}{2m} \qquad z = 2$

$\Rightarrow P^z \geq cte\, k_B T \quad$ pero $P \sim \frac{h}{\lambda}$

λ longitud de onda

$\Rightarrow \lambda^{-z} \geq cte\, T \Rightarrow \lambda \leq \hat{cte}\, T^{-1/z}$

en la vecindad de T_c

$\lambda_{max} \sim T_c^{-1/z}$ pero $\xi \sim t^{-\nu} \to \infty$

entonces $t^{-\nu} \gg T_c^{-1/z}$ ó $\boxed{t \ll T_c^{\nu/z}}$

→ $\xi \gg \lambda_{max}$ → ~~la~~ ~~es~~ escalas de orden

ξ → nunca más las incert/fluct. cuánticas

$T_c \neq 0$

→ que pasa si $T_c \to 0$???

→ aquí sí, la naturaleza cuántica del sist. es importante!

→ **Transición de Fase Cuántica**

Q.P.T.

TFC ???

sistema cuántico $\hat{H}|\Psi_n\rangle = E_n|\Psi_n\rangle$

$\{|\Psi_n\rangle\}$ forman una base del esp. de estados.

$|\Psi_0\rangle$ es el estado fund. (no degenerado)

$$E_0 < E_1 \leq E_2 \leq E_3 - - -$$

una TFC. → es un cambio en la naturaleza de $|\Psi_0\rangle$.

$$Z = \mathrm{tr}\left\{e^{-\beta\hat{H}}\right\} = \sum_n \langle\Psi_n|e^{-\beta\hat{H}}|\Psi_n\rangle$$

$$= \sum_n e^{-\beta E_n}$$

$\mathcal{O}$ observable

$$\langle\mathcal{O}\rangle = \frac{1}{Z}\mathrm{tr}\left\{e^{-\beta\hat{H}}\mathcal{O}\right\}$$

$$= \frac{1}{Z}\sum_n e^{-\beta E_n}\langle\Psi_n|\mathcal{O}|\Psi_n\rangle$$

si $T \to 0$? $(\beta\to\infty)$?

$$Z = \sum_n e^{-\beta E_n} = e^{-\beta E_0} + \sum_{n=1} e^{-\beta E_n}$$

$$= e^{-\beta E_0}\left(1 + \sum_{n=1} e^{-\beta(E_n - E_0)}\right)$$

$$\langle\mathcal{O}\rangle = \frac{e^{-\beta E_0}\langle\Psi_0|\mathcal{O}|\Psi_0\rangle + \sum_{n=1}\langle\Psi_n|\mathcal{O}|\Psi_n\rangle e^{-\beta E_n}}{e^{-\beta E_0}\left(1+\sum_{n=1} e^{-\beta(E_n-E_0)}\right)}$$

$$\Rightarrow \langle O \rangle = \frac{\langle \psi_0|O|\psi_0\rangle + \sum_{n=1} \cancel{\langle \psi_n|O|\psi_n\rangle e^{-\beta\overbrace{(E_n-E_0)}^{>0}}}}{\left(1 + \sum_{n=1} \cancel{e^{-\beta\overbrace{(E_n-E_0)}^{>0}}}\right)}$$

$$\lim_{\beta\to\infty} \langle O \rangle = \langle \psi_0|O|\psi_0\rangle$$

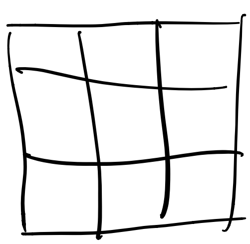
$$\hat H(g) \text{ tq } \hat H(g)|\psi_0\rangle_{(g)} = E_0(g)|\psi_0\rangle_{(g)}$$

Una T.F.C. es cuando para un g_c $|\psi_0\rangle_g$ cambia de naturaleza.

Ejemplo:

El modelo de Ising en campo transverso (T.F.I.M.)

Espines $\frac{1}{2}$ en una red cuadrada



$\leftarrow i$ $\quad \vec S_i$ tq $\vec S_i \cdot \vec S_i = \hbar^2 \frac{1}{2}\left(\frac{1}{2}+1\right)$

S_i^z ~~des~~ autovalores $+\frac{\hbar}{2}$, $-\frac{\hbar}{2}$

S_i^x, S_i^y idem.

$$\boxed{\hat{H} = -J\sum_{\langle ij\rangle} S^z_i S^z_j - h\sum_i S^x_i}$$

$$[S^a_l, S^b_k] = i\hbar\epsilon^{abc} S^c_l \,\delta_{l,k}$$

Simetría $\mathbb{Z}_2$ → rotación de π alrededor del eje "x"

$$\forall i \qquad S^x_i \to S^x_i$$
$$S^z \to -S^z$$
$$S^y \to -S^y$$

si $h=0$?

$|\psi_0\rangle \overset{?}{=}$? $\qquad |+\rangle_i$, $|-\rangle_i$ los autoestados de S^z_i

$$S^z_i |+\rangle_i = +\frac{\hbar}{2}|+\rangle_i$$
$$S^z_i |-\rangle_i = -\frac{\hbar}{2}|-\rangle_i$$

$$|\psi_0\rangle_1 = \prod_{\otimes} |+\rangle_i \qquad \uparrow\uparrow\uparrow\uparrow \quad \uparrow\uparrow\uparrow\uparrow$$

$$|\psi_0\rangle_2 = \prod_{\otimes} |-\rangle_i \qquad \downarrow\downarrow\downarrow \quad \downarrow\downarrow\downarrow \quad \downarrow\downarrow\downarrow$$

Cada $|\Psi_0\rangle_\alpha$ rompe la simetría $\mathbb{Z}_2$

$\alpha = 1, 2$

$$\forall i \quad {}_1\langle\Psi_0| S_j^z |\Psi_0\rangle_1 = +\frac{\hbar}{2}$$

$$ {}_2\langle\Psi_0| S_j^z |\Psi_0\rangle_2 = -\frac{\hbar}{2}$$

otro extremo

$J = 0$

$$\underline{\hat{H} \sim -h \sum_j \underline{S_i^x}}$$

$$|\rightarrow\rangle_i \ \text{y}\ |\leftarrow\rangle_i$$

$$|\rightarrow\rangle_i = \underline{\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_i + |\downarrow\rangle_i\right)}$$

$$|\leftarrow\rangle_i = \underline{\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_i - |\downarrow\rangle_i\right)}$$

$$S_i^x |\rightarrow\rangle_i = +\frac{\hbar}{2}|\rightarrow\rangle_i$$

$$S_i^x |\leftarrow\rangle_i = -\frac{\hbar}{2}|\leftarrow\rangle_i$$

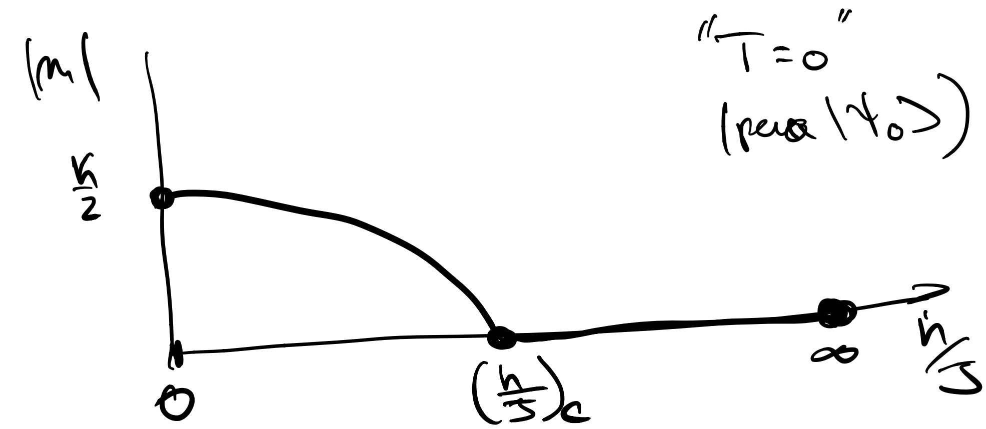
$$|\Psi_0\rangle = \prod_{\otimes} |\rightarrow\rangle_i \quad \text{único.}$$

$|\Psi_0\rangle$ <u>no</u> rompe la sim. $\mathbb{Z}_2$

$\forall j \quad \langle \Psi_0 | S_j^z | \Psi_0 \rangle = 0$

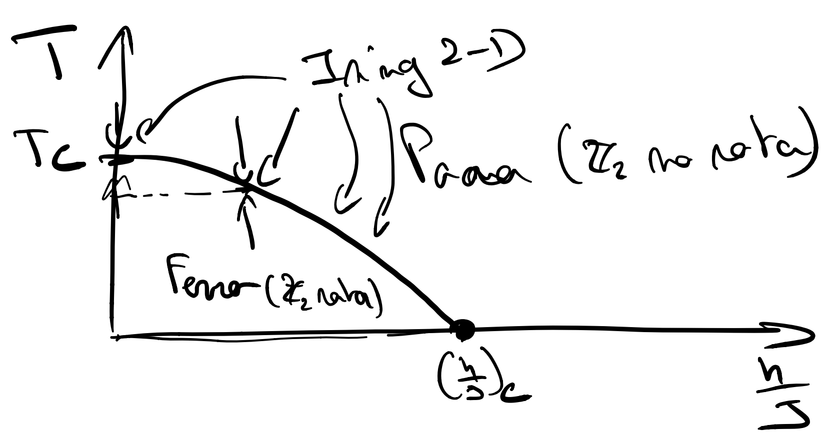
ya que $\langle \rightarrow | S^z | \rightarrow \rangle = 0$

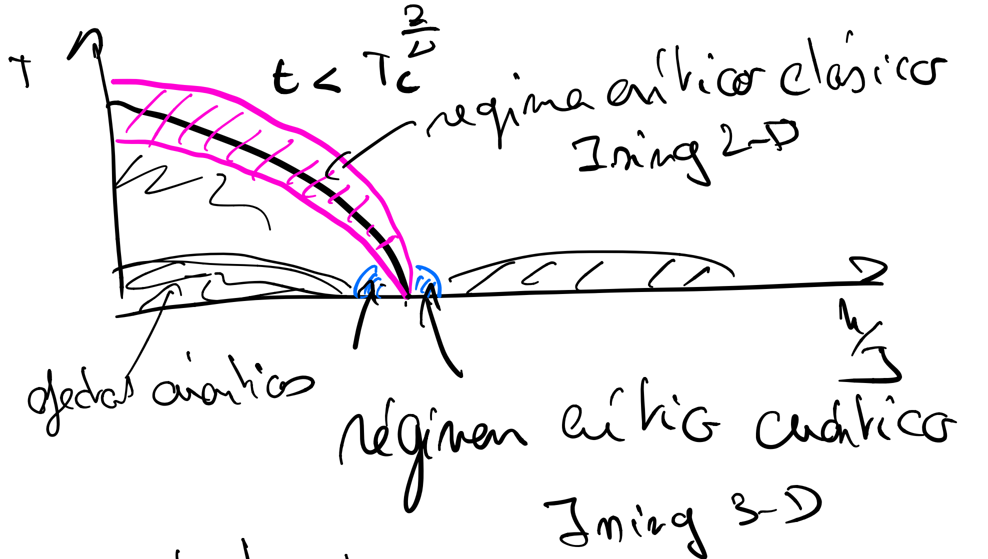
$m = \langle \Psi_0 | S_j^z | \Psi_0 \rangle$



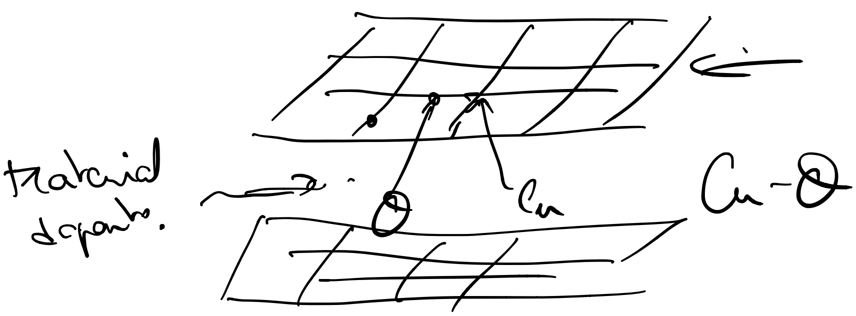
"g" $= \frac{h}{J}$

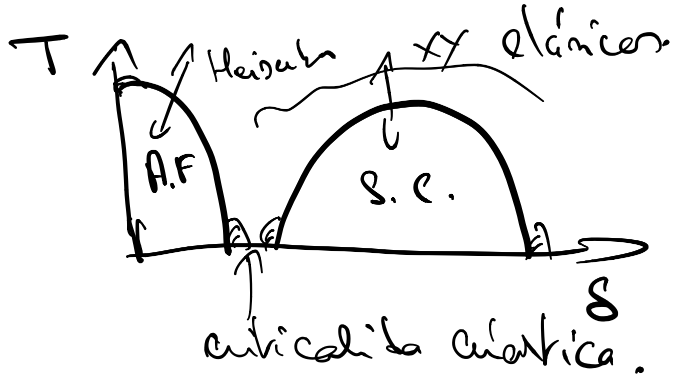
T.F.C.



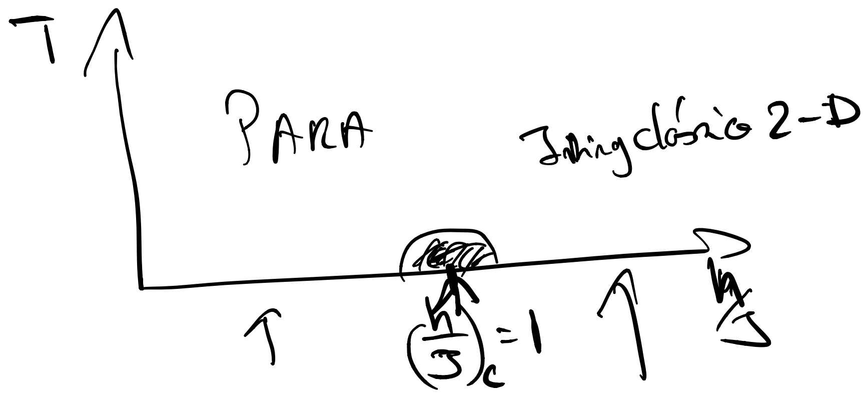


Superconductividad a altas Tc.





$$H = \sum_i S_i^z S_{i+1}^z - h \sum_i S_i^x$$



$\langle \psi_0 | S_i^z | \psi_0 \rangle \neq 0$

$\xi \sim T^{-\nu}$

$\xi \sim \frac{1}{J} \sim L$ vs L'