Física de Partículas

Decaimientos y Dispersiones

Carlos Sandoval (UNAL/UAN, Colombia)

Latin American alliance for Capacity buildiNG in Advanced physics
LA-CoNGA physics

Cofinanciado por el programa Erasmus+ de la Unión Europea

Experimental probes

- Bound states: Static properties such as mass, spin, parity, magnetic moments
- Particle decays: Allowed and forbidden decays / Conservation laws
- Particle scattering: Production of new massive particles / Study of particle interaction cross sections / High energies to study short distances

Force	Typical Lifetime [s]	Typical cross-section [mb]
Strong	10^{-23}	10
Electromagnetic	10^{-20}	10^{-2}
Weak	10^{-8}	10^{-13}

Fermi's Golden Rule

- Particle decays and particle scattering are transitions between quantum mechanical states
- In QM the transition rate between states i and j is:

$$\Gamma_{fi} = 2\pi \, |T_{fi}|^2 \rho(E_i)$$

where T_{fi} is the transition matrix element and ρ is the density of states

Decay rates

- Lifetime of a particle (average or mean)

- Decay rate (Γ): probability per unit time that the particle of interest will decay

- If we had $N(t)$ particles, $N\Gamma dt$ particles would decay in the next instant dt

$$\mathrm{d}N = -\Gamma N \, \mathrm{d}t$$

- It follows that

$$N(t) = N(0)e^{-\Gamma t}$$

- We can see that the mean lifetime:

$$\tau = \frac{1}{\Gamma}$$

Decay rates

- Lifetime of a particle (average or mean)

- Decay rate (Γ): probability per unit time that the particle of interest will decay

- Rate of decays

$$\frac{dN}{dt} = -\Gamma N(t)$$

- Activity

$$A(t) = \left|\frac{dN}{dt}\right| = \Gamma N(t)$$

Decay rates

- Particles can decay in several ways (decay modes, channels)
- The total decay rate is the sum of the individual decay rates

$$\Gamma = \sum_j \Gamma_j .$$

- Branching ratios: relative frequency of a particular decay mode:

$$BR(j) = \frac{\Gamma_j}{\Gamma}$$

- Decaying states do not correspond to a single energy – they have a width:

$$\Delta E\, \tau \sim \hbar \quad \xrightarrow{\text{yields}} \quad \Delta E \sim \frac{\hbar}{\tau} = \hbar\Gamma$$

Decaying states in QM

- For a decaying state the probability density must decay exponentially: $\psi(t) = \psi(0)\mathrm{e}^{-iE_0t}\mathrm{e}^{-t/2\tau}$ $\quad |\psi(t)|^2 = |\psi(0)|^2\,\mathrm{e}^{-t/\tau}$

- The energies present in the wavefunction are given by the Fourier transform of $\psi(t)$:

$$f(\omega) = f(E) = \int_0^\infty \psi(t)\mathrm{e}^{iEt}\,\mathrm{d}t = \int_0^\infty \psi(0)\mathrm{e}^{-t(iE_0+\frac{1}{2\tau})}\mathrm{e}^{iEt}\,\mathrm{d}t$$

$$= \int_0^\infty \psi(0)\mathrm{e}^{-t(i(E_0-E)+\frac{1}{2\tau})}\,\mathrm{d}t = \frac{i\psi(0)}{(E_0-E)-\frac{i}{2\tau}}$$

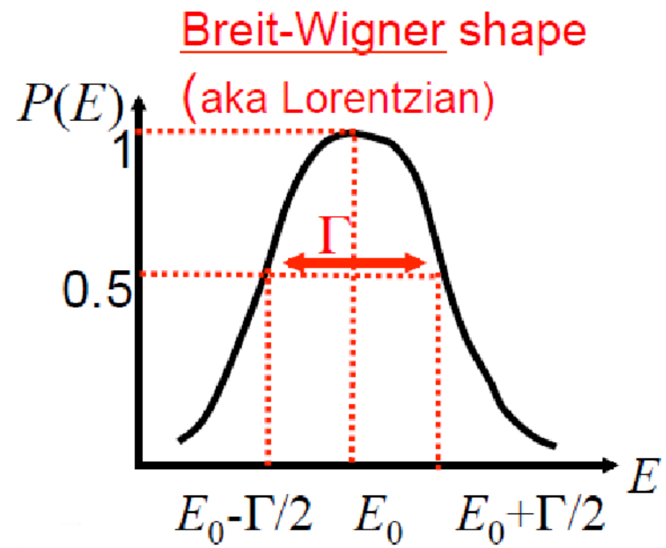
- So the probability of finding a state with energy E:

$$P(E) = |f(E)|^2 = \frac{|\psi(0)|^2}{(E_0-E)^2+\frac{1}{4\tau^2}}$$

- The probability density function for finding the particle with energy E is

$$p(E) \propto \frac{1}{(E_0 - E)^2 + \frac{\Gamma^2}{4}}$$

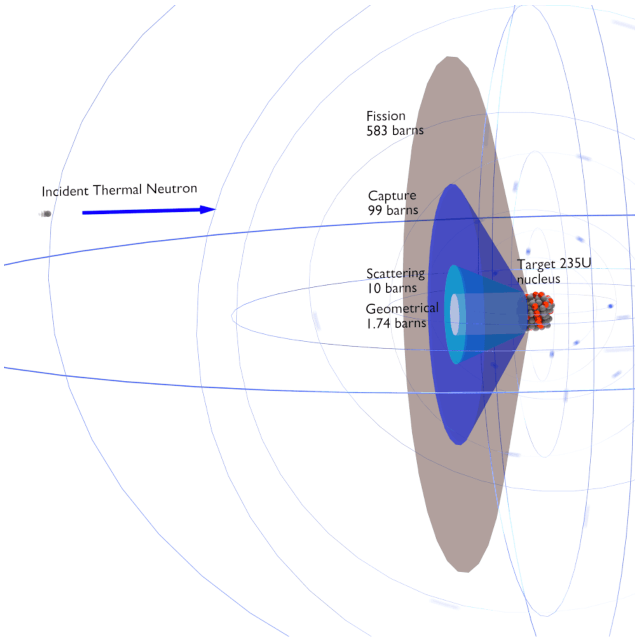
- E is the energy of the system
- E_0 is the characteristic rest-mass of the unstable particle
- The probability density function has a Lorentzian, peaked, line shape: ***Breit-Wigner***
- Full-width at half max (FWHM) of the peak equal to Γ: ***width***
- Long-lived particles: narrow width, well defined energies



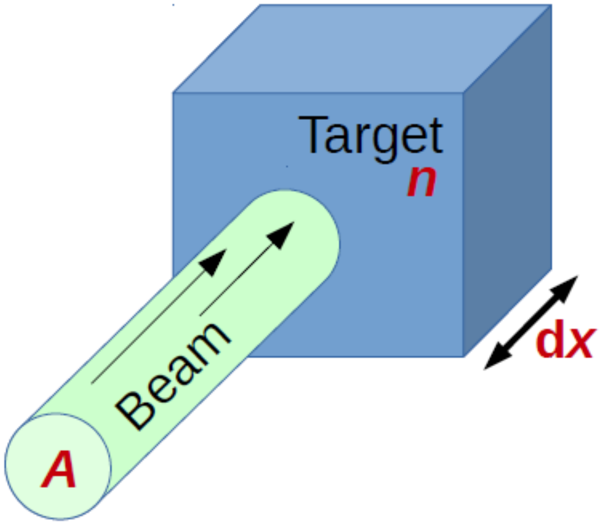
- ***Cross section***: "strength" of a particular interaction between two particles
- Effective target area presented to the incoming particle, units: barns ($1\ \text{barn} = 10^{-28}\text{m}^2$)
- Interaction rate per target particle:

$$\Gamma = \phi\sigma$$

- ϕ is the ***flux***: number of particles passing through unit area per second

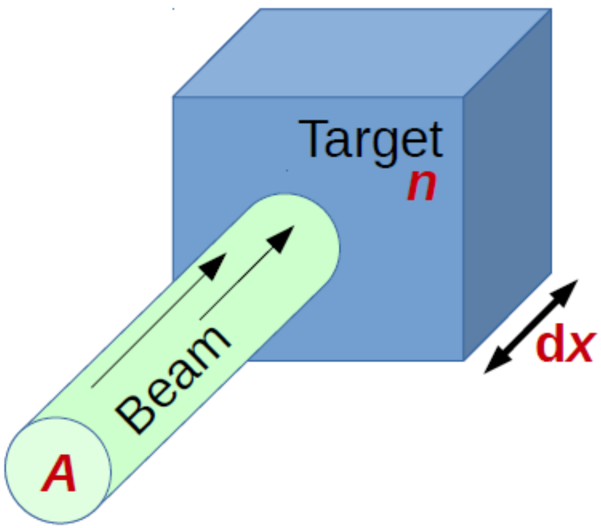


- Consider a beam of N particles per unit time with area A
- The beam hits a target of n nuclei per unit volume and thickness dx



Scattering

- Consider a beam of N particles per unit time with area A
- The beam hits a target of n nuclei per unit volume and thickness dx



- Number of target particles in area A:

$$N_T = n \cdot A \cdot dx$$

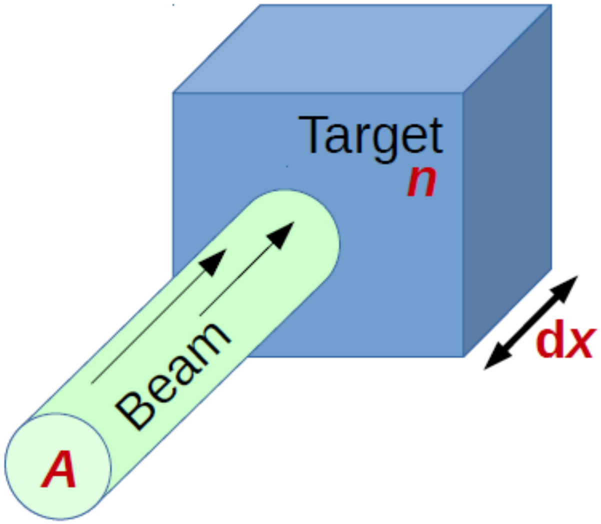
- Effective area of interaction:

$$\sigma N_T = \sigma n A dx$$

- Incident flux:

$$\phi = N/A$$

- Consider a beam of N particles per unit time with area A
- The beam hits a target of n nuclei per unit volume and thickness dx



- Number of target particles in area A:

$$N_T = n \cdot A \cdot dx$$

- Effective area of interaction:

$$\sigma N_T = \sigma n A dx$$

- Incident flux:

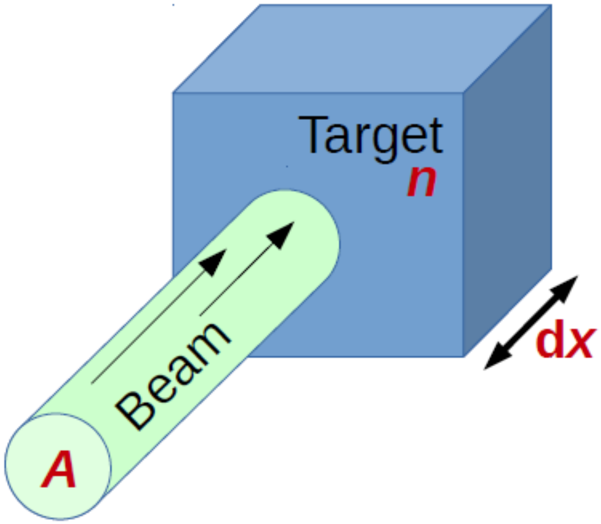
$$\phi = N/A$$

- Number of particles scattered per unit time

$$-dN = \phi \sigma N_T = \frac{N}{A} \sigma n A dx$$

Scattering

- Consider a beam of N particles per unit time with area A
- The beam hits a target of n nuclei per unit volume and thickness dx



- Number of target particles in area A:

$$N_T = n \cdot A \cdot dx$$

- Effective area of interaction:

$$\sigma N_T = \sigma n A dx$$

- Incident flux:

$$\phi = N/A$$

- So the cross section is proportional to the scattering rate:

$$\sigma = \frac{-dN}{nNdx}$$

Attenuation of a beam

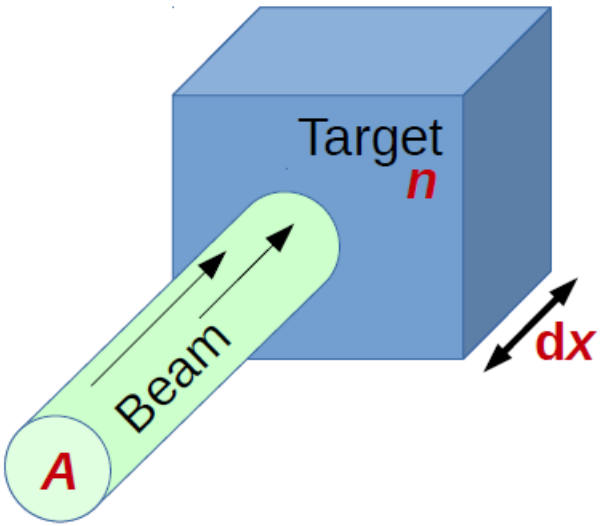
Beam attenuation in a target of thickness L:

- Thick target $\sigma n L \gg 1$:

$$\int_{N_0}^{N} -\frac{dN}{N} = \int_0^L \sigma n dx$$

$$N = N_0 e^{-\sigma n L}$$

the beam attenuates exponentially



- Thin target $\sigma n L \ll 1$:

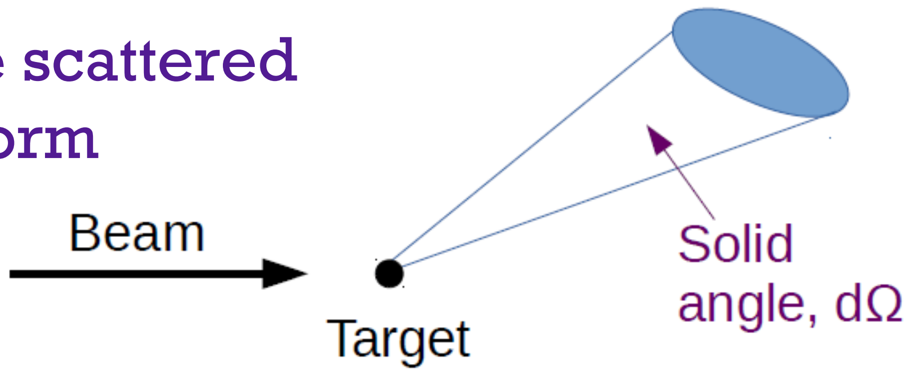
$$e^{-\sigma n L} \sim 1 - \sigma n L$$

$$N = N_0(1 - \sigma n L)$$

- *Mean free path* between interactions: $1/\sigma n$ (also referred to as *interaction length*)

Differential cross section

- The angular distribution of the scattered particles is not necessarily uniform



- Number of particles scattered per unit time into $d\Omega$ is

$$dN = d\sigma \phi N_T$$

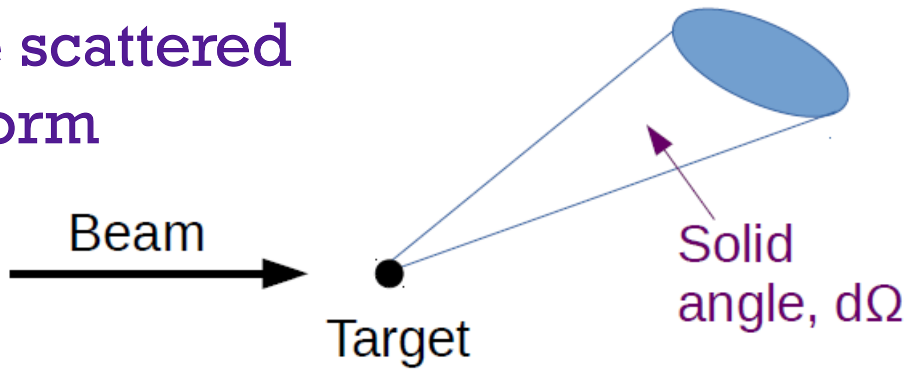
- The differential cross-section:

$$\frac{d\sigma}{d\Omega} = \frac{dN}{d\Omega \phi N_T}$$

is the number of particles scattered per unit time and solid angle, divided by the incident flux and by the number of target nuclei defined by the beam area

Differential cross section

- The angular distribution of the scattered particles is not necessarily uniform



- Number of particles scattered per unit time into $d\Omega$ is

$$dN = d\sigma \phi N_T$$

- The differential cross-section:

$$\frac{d\sigma}{d\Omega} = \frac{dN}{d\Omega \phi N_T}$$

- Most experiments do not cover 4π solid angle, and in general we measure $d\sigma/d\Omega$
- Angular distributions provide more information than the total cross-section about the mechanism of the interaction

Scattering in QM

- Consider a beam of particles scattering from a fixed potential $V(r)$
- The scattering rate is characterised by the interaction cross-section $\sigma = \Gamma/\phi$
- We can calculate the cross section using Fermi's golden rule

Scattering in QM

- Consider a beam of particles scattering from a fixed potential $V(r)$
- The scattering rate is characterised by the interaction cross-section $\sigma = \Gamma/\phi$
- We can calculate the cross section using Fermi's golden rule

- In first order perturbation theory, and using plane wave solutions:

$$\psi(\mathbf{x}, t) = Ae^{i(\mathbf{p}\cdot\mathbf{x} - Et)}$$

we need:
- Wave function normalisation
- Matrix element in perturbation theory
- Incident flux
- Density of states

- In first order perturbation theory, and using plane wave solutions:

$$\psi(\mathbf{x}, t) = Ae^{i(\mathbf{p}\cdot\mathbf{x}-Et)}$$

- Wave function normalisation: Normalise wave-functions to one particle in a box of side a

$$\int_0^a \int_0^a \int_0^a \psi^* \psi \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z = 1$$

$$A^2 = 1/a^3$$

- In first order perturbation theory, and using plane wave solutions:

$$\psi(\mathbf{x}, t) = A e^{i(\mathbf{p}\cdot\mathbf{x} - Et)}$$

- Wave function normalisation: Normalise wave-functions to one particle in a box of side a

$$\int_0^a \int_0^a \int_0^a \psi^* \psi \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z = 1$$

$$A^2 = 1/a^3$$

- The matrix element contains the physics of the interaction. In perturbation theory (first order):

$$T_{fi} = \langle f | \hat{H} | i \rangle$$

Scattering in QM

- Incident flux: consider a target of area A and a beam of particles with velocity v. Any incident particle within a volume vA will cross the target area every second

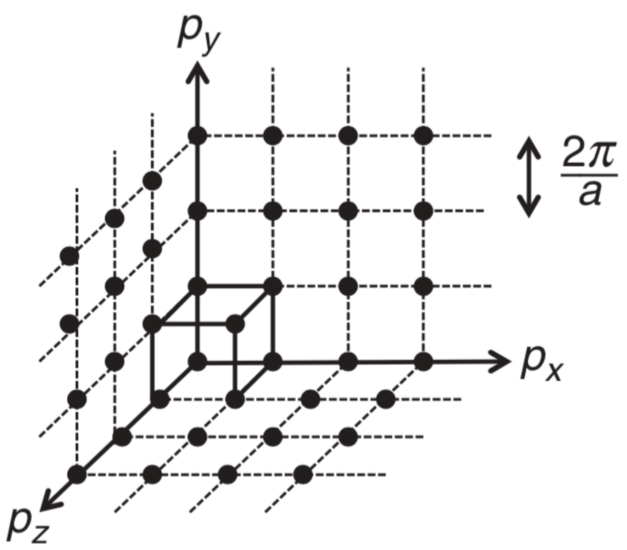
$$\phi = \frac{vA}{A} n = vn = \frac{v}{a^3}$$

- Density of states (or phase space): the normalisation of the wave function implies periodic boundary conditions, which implies the momentum components are quantised:

$$(p_x, p_y, p_z) = (n_x, n_y, n_z)\frac{2\pi}{a}$$

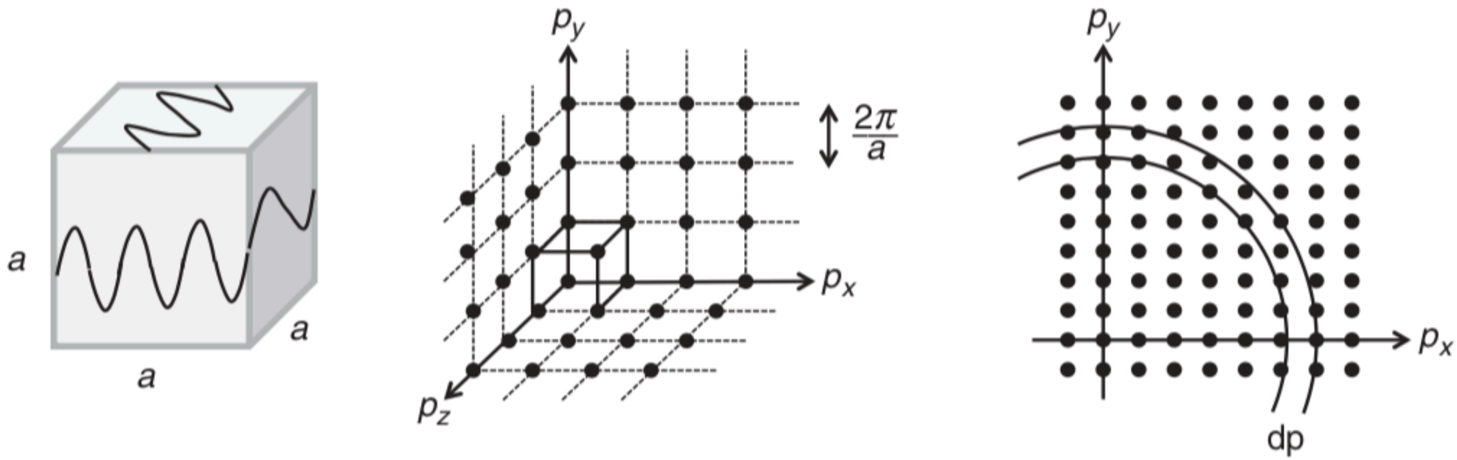
each state in momentum space occupies a cubic volume of

$$\mathrm{d}^3\mathbf{p} = \mathrm{d}p_x\mathrm{d}p_y\mathrm{d}p_z = \left(\frac{2\pi}{a}\right)^3 = \frac{(2\pi)^3}{V}$$



- Density of states (or phase space): the number of states dn with magnitude of momentum in the range $p \to p + dp$ is the volume (in momentum space) divided by the volume of a single state:

$$\mathrm{d}n = 4\pi \mathrm{p}^2 \mathrm{dp} \times \frac{V}{(2\pi)^3}$$



- Density of states (or phase space): the number of states dn with magnitude of momentum in the range $p \rightarrow p + dp$ is the volume (in momentum space) divided by the volume of a single state:

$$\mathrm{d}n = 4\pi \mathrm{p}^2 \mathrm{dp} \times \frac{V}{(2\pi)^3}$$

and the density of states:

$$\rho(E) = \frac{\mathrm{d}n}{\mathrm{d}E} = \frac{\mathrm{d}n}{\mathrm{dp}} \left| \frac{\mathrm{dp}}{\mathrm{d}E} \right|$$

$$\frac{\mathrm{d}n}{\mathrm{dp}} = \frac{4\pi \mathrm{p}^2}{(2\pi)^3} V.$$

Putting everything together:

$$\sigma = \frac{\Gamma}{\phi} = \frac{2\pi T_{fi}^2 \rho(E)}{\phi}$$

$$T_{fi} = \langle f | \hat{H} | i \rangle$$

$$= \int \psi_f^* \hat{H} \psi_i \, d^3\vec{r}$$

$$= \int A e^{-i\vec{p}_f\cdot\vec{r}} V(\vec{r}) A e^{i\vec{p}_i\cdot\vec{r}} d^3\vec{r}$$

$$= A^2 \int e^{-i\vec{q}\cdot\vec{r}} V(\vec{r}) d^3\vec{r} \quad ; \quad \vec{q} = \vec{p}_f - \vec{p}_i$$

$$a^3 = 1/V$$

Putting everything together:

$$\sigma = \frac{\Gamma}{\phi} = \frac{2\pi T_{fi}^2 \rho(E)}{\phi}$$

$$|T_{fi}|^2 = \frac{1}{V^2} \left| \int e^{-i\vec{q}\cdot\vec{r}} V(\vec{r})\, d^3\vec{r} \right|^2$$

$$\phi = \frac{v_0}{V} \quad ; \quad \rho(E) = \frac{dn}{dp} \left| \frac{dp}{dE} \right|$$

$$= d\Omega\, p^2 \frac{V}{(2\pi)^3} \frac{E}{p}$$

Putting everything together:

$$\sigma = \frac{\Gamma}{\phi} = \frac{2\pi T_{fi}^2 \rho(E)}{\phi}$$

$$d\sigma = 2\pi \frac{1}{\cancel{V^2}} \left| \int e^{-i\vec{q}\cdot\vec{r}} \cdot V(\vec{r})\, d^3\vec{r} \right|^2 d\Omega\, p^{\cancel{2}} \frac{\cancel{V}}{(2\pi)^3} \frac{E}{\cancel{p}} \frac{\cancel{V}}{V_0}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{(2\pi)^2 V_0} \left| \int e^{-i\vec{q}\cdot\vec{r}} \cdot V(\vec{r})\, d^3\vec{r} \right|^2 p\, E$$

If $v{\sim}c{\sim}1$, $p{\sim}E$, **Born approximation**:

$$\frac{d\sigma}{d\Omega} = \frac{E^2}{(2\pi)^2} \left| \int e^{-i\vec{q}\cdot\vec{r}} V(\vec{r}) d^3\vec{r} \right|^2$$

Yukawa potential

- Consider relativistic elastic scattering from a Yukawa potential
$$V(\vec{r}) = \frac{g\,\mathrm{e}^{-mr}}{r}$$
- Our matrix element then:
$$\begin{aligned}
\int \mathrm{e}^{-\mathrm{i}\vec{q}.\vec{r}}\, V(\vec{r})\, \mathrm{d}^3\vec{r} &= \int_0^\infty \int_0^{2\pi} \int_0^\pi V(r)\, \mathrm{e}^{iqr\cos\theta}\, r^2 \sin\theta\, \mathrm{d}\theta\, \mathrm{d}\phi\, \mathrm{d}r \\
&= \int_0^\infty \int_{-1}^{+1} 2\pi V(r)\, \mathrm{e}^{iqr\cos\theta}\, r^2\, \mathrm{d}(\cos\theta)\, \mathrm{d}r \\
&= \int_0^\infty 2\pi V(r) \left(\frac{\mathrm{e}^{iqr} - \mathrm{e}^{-iqr}}{iqr}\right) r^2\, \mathrm{d}r \\
&= \int_0^\infty 2\pi g \frac{\mathrm{e}^{-mr}}{r} \left(\frac{\mathrm{e}^{iqr} - \mathrm{e}^{-iqr}}{iqr}\right) r^2\, \mathrm{d}r \\
&= \int_0^\infty 2\pi g\, \mathrm{e}^{-mr} \left(\frac{\mathrm{e}^{iqr} - \mathrm{e}^{-iqr}}{iq}\right) \mathrm{d}r \\
&= \int_0^\infty \frac{2\pi g}{iq} \left(\mathrm{e}^{-r(m-iq)} - \mathrm{e}^{-r(m+iq)}\right) \mathrm{d}r \\
&= \frac{2\pi g}{iq} \left(\frac{1}{m - \mathrm{i}q} - \frac{1}{m + \mathrm{i}q}\right) = \frac{2\pi g}{iq} \frac{2iq}{m^2 + q^2} \\
&= \frac{4\pi g}{m^2 + q^2}
\end{aligned}$$

where we chose the z-axis along r: $\vec{q}.\vec{r} = -qr\cos\theta$

- Consider relativistic elastic scattering from a Yukawa potential

$$V(\vec{r}) = \frac{g\,\mathrm{e}^{-mr}}{r}$$

- Our matrix element then:

$$\begin{aligned}\int \mathrm{e}^{-i\vec{q}.\vec{r}}\, V(\vec{r})\, \mathrm{d}^3\vec{r} &= \int_0^\infty \int_0^{2\pi} \int_0^\pi V(r)\, \mathrm{e}^{iqr\cos\theta}\, r^2 \sin\theta\, \mathrm{d}\theta\, \mathrm{d}\phi\, \mathrm{d}r \\ &= \int_0^\infty \int_{-1}^{+1} 2\pi V(r)\, \mathrm{e}^{iqr\cos\theta}\, r^2\, \mathrm{d}(\cos\theta)\, \mathrm{d}r \\ &= \int_0^\infty 2\pi V(r) \left(\frac{\mathrm{e}^{iqr} - \mathrm{e}^{-iqr}}{iqr}\right) r^2\, \mathrm{d}r \\ &= \int_0^\infty 2\pi g \frac{\mathrm{e}^{-mr}}{r} \left(\frac{\mathrm{e}^{iqr} - \mathrm{e}^{-iqr}}{iqr}\right) r^2\, \mathrm{d}r \\ &= \int_0^\infty 2\pi g\, \mathrm{e}^{-mr} \left(\frac{\mathrm{e}^{iqr} - \mathrm{e}^{-iqr}}{iq}\right) \mathrm{d}r \\ &= \int_0^\infty \frac{2\pi g}{iq} \left(\mathrm{e}^{-r(m-iq)} - \mathrm{e}^{-r(m+iq)}\right) \mathrm{d}r \\ &= \frac{2\pi g}{iq}\left(\frac{1}{m-iq} - \frac{1}{m+iq}\right) = \frac{2\pi g}{iq}\,\frac{2iq}{m^2+q^2} \\ &= \frac{4\pi g}{m^2+q^2}\end{aligned}$$

where we chose the z-axis along r: $\vec{q}.\vec{r} = -qr\cos\theta$

$$|M_{if}|^2 = \frac{16\pi^2 g^2}{(m^2+q^2)^2}$$

Rutherford scattering

- Consider relativistic elastic scattering from a Coulomb potential

$$V(\vec{r}) = -\frac{Z\alpha}{r}$$

$$|M_{if}|^2 = \frac{16\pi^2 Z^2 \alpha^2}{q^4}$$

($m = 0$ and $g = Z\alpha$ in the Yukawa potential)

- Consider relativistic elastic scattering from a Coulomb potential

$$V(\vec{r}) = -\frac{Z\alpha}{r}$$

$$|M_{if}|^2 = \frac{16\pi^2 Z^2 \alpha^2}{q^4}$$

($m = 0$ and $g = Z\alpha$ in the Yukawa potential)

$$\vec{q} = \overrightarrow{p_f} - \overrightarrow{p_i}$$

$$|\vec{q}|^2 = 2|\vec{p}|^2(1 - \cos\theta) = 4E^2 \sin^2\frac{\theta}{2}$$

Rutherford scattering

- Consider relativistic elastic scattering from a Coulomb potential

$$V(\vec{r}) = -\frac{Z\alpha}{r}$$

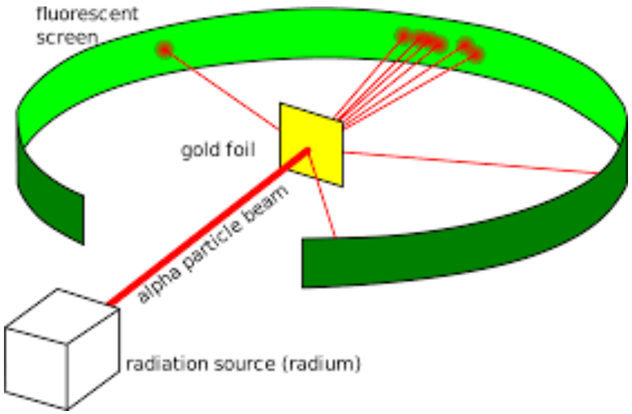
$$|M_{if}|^2 = \frac{16\pi^2 Z^2 \alpha^2}{q^4}$$

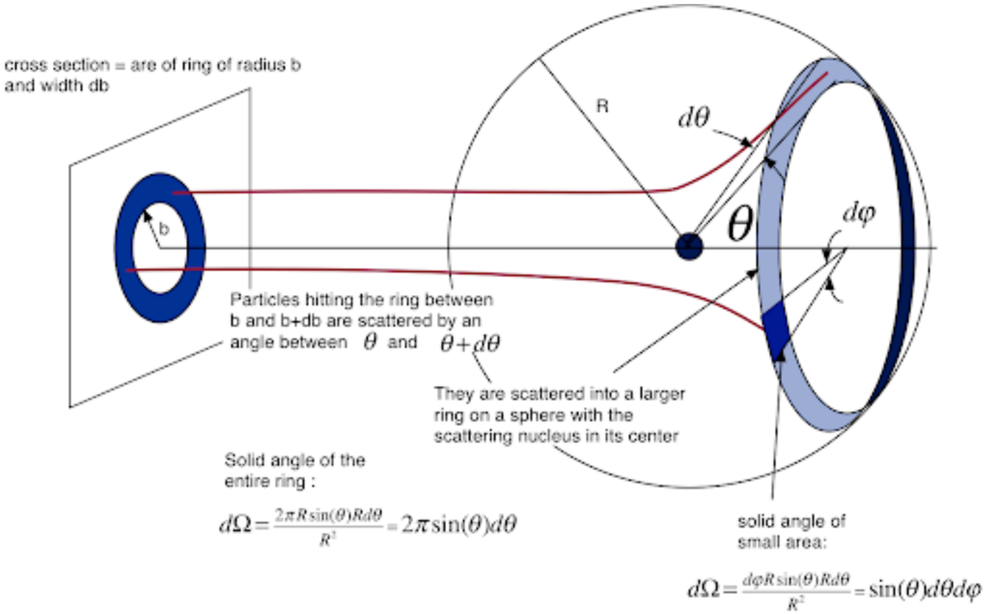
the differential cross section then:

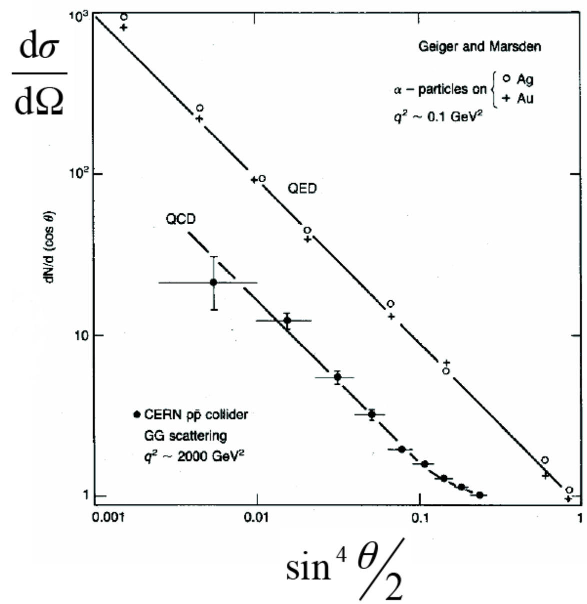
$$\frac{d\sigma}{d\Omega} = \frac{E^2}{(2\pi)^2}|\mathcal{M}|^2 = \frac{E^2}{(2\pi)^2}\frac{16\pi^2 Z^2\alpha^2}{16E^4\sin^4\frac{\theta}{2}}$$

$$\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2}{4E^2\sin^4\frac{\theta}{2}}$$

- *Fixed target* experiment
- Alpha particles shot at a target
- Metal foil as target (Au and Ag)



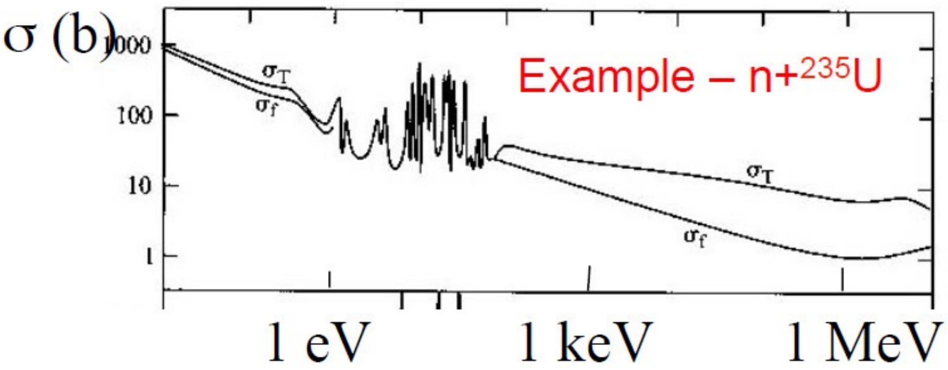




Resonant scattering

- Some particle interactions occur via an intermediate resonant state which then decays

$$a + b \to O \to c + d$$



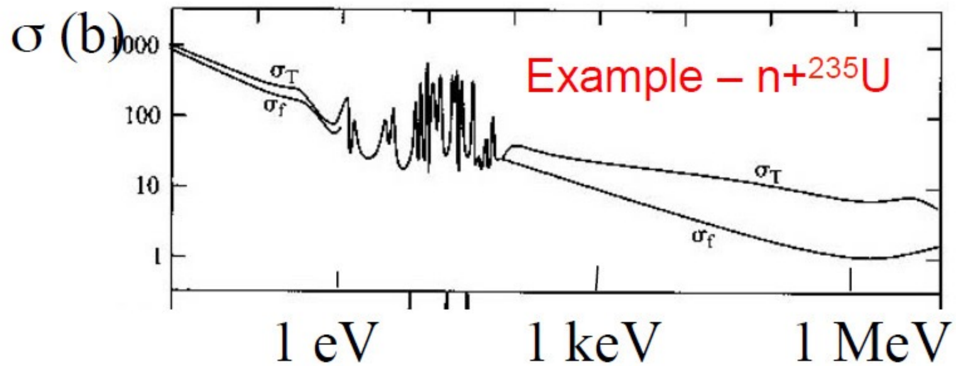
- The matrix element is given by second order perturbation theory

$$T_{fi} = \langle f|V|i\rangle + \sum_{j \neq i} \frac{\langle f|V|j\rangle\langle j|V|i\rangle}{E_i - E_j}$$

Resonant scattering

- Some particle interactions occur via an intermediate resonant state which then decays

$$a + b \to O \to c + d$$



- The matrix element is given by second order perturbation theory

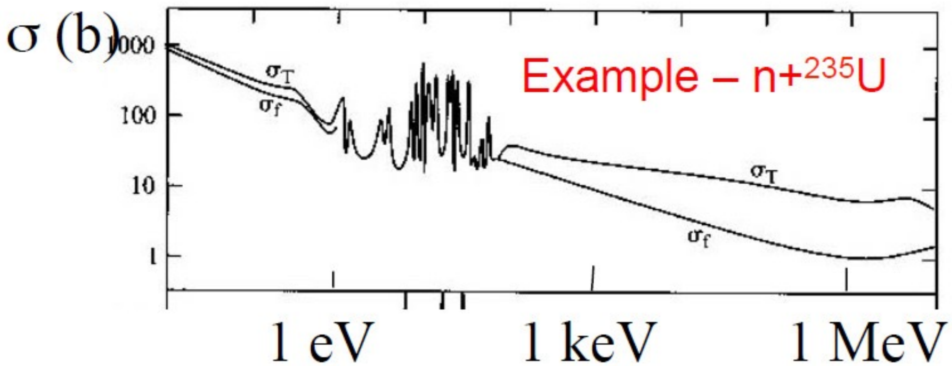
$$T_{fi} = \langle f|V|i\rangle + \sum_{j\neq i} \frac{\langle f|V|j\rangle\langle j|V|i\rangle}{E_i - E_j}$$

- Two stage picture:

Production: $a + b \to O$ Decay: $O \to c + d$

- Some particle interactions occur via an intermediate resonant state which then decays

$$a + b \to O \to c + d$$



- The matrix element is given by second order perturbation theory

$$T_{fi} = \langle f|V|i\rangle + \sum_{j\neq i} \frac{\langle f|V|j\rangle\langle j|V|i\rangle}{E_i - E_j}$$

- Two stage picture:

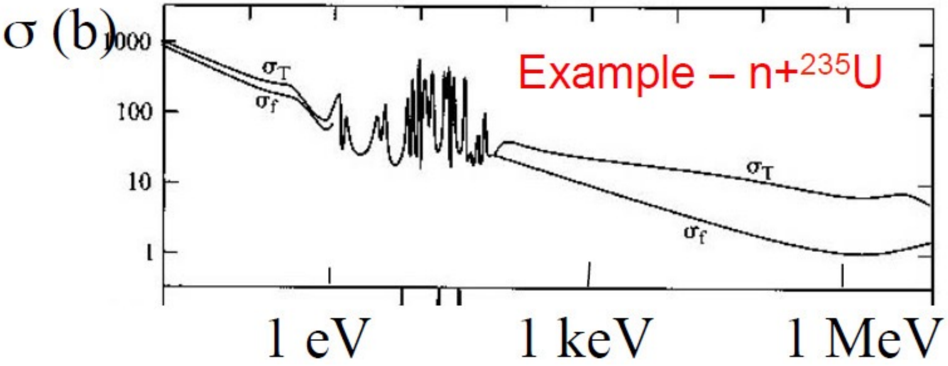
Production: $a + b \to O$ Decay: $O \to c + d$

- Near the *resonance* $(E \sim E_0 \sim M_O)$ – 2nd order effects are large

Resonant scattering

- Some particle interactions occur via an intermediate resonant state which then decays

$$a + b \to O \to c + d$$



- The matrix element is given by second order perturbation theory

$$T_{fi} = \langle f|V|i\rangle + \sum_{j\neq i} \frac{\langle f|V|j\rangle\langle j|V|i\rangle}{E_i - E_j}$$

- Near the *resonance* ($E \sim E_0 \sim m_O$) – 2nd order effects are large
- To account for the fact that O is unstable:

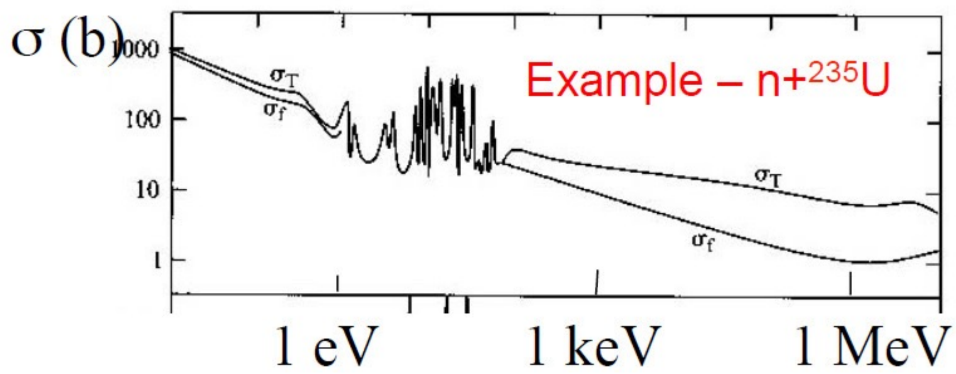
$$\psi \propto e^{-imt} \quad \to \quad \psi \propto e^{-imt} e^{-\Gamma t/2}$$

$$m \to m - i\Gamma/2$$

Resonant scattering

- Some particle interactions occur via an intermediate resonant state which then decays

$$a + b \rightarrow O \rightarrow c + d$$



- The matrix element is given by second order perturbation theory

$$T_{fi} = \langle f|V|i\rangle + \sum_{j\neq i} \frac{\langle f|V|j\rangle\langle j|V|i\rangle}{E_i - E_j}$$

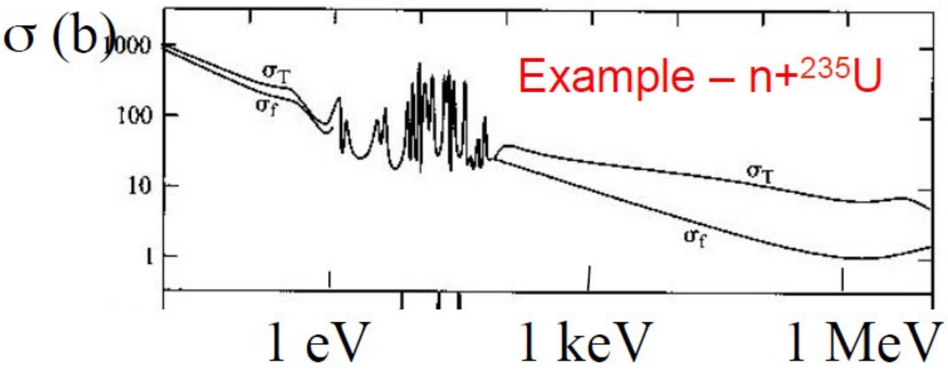
- The matrix element squared is then:

$$|T_{fi}|^2 = \frac{|T_{fO}|^2 |T_{Oi}|^2}{(E - E_O)^2 + \frac{\Gamma^2}{4}}$$

Resonant scattering

- Some particle interactions occur via an intermediate resonant state which then decays

$$a + b \to O \to c + d$$



- So we have for the cross section:

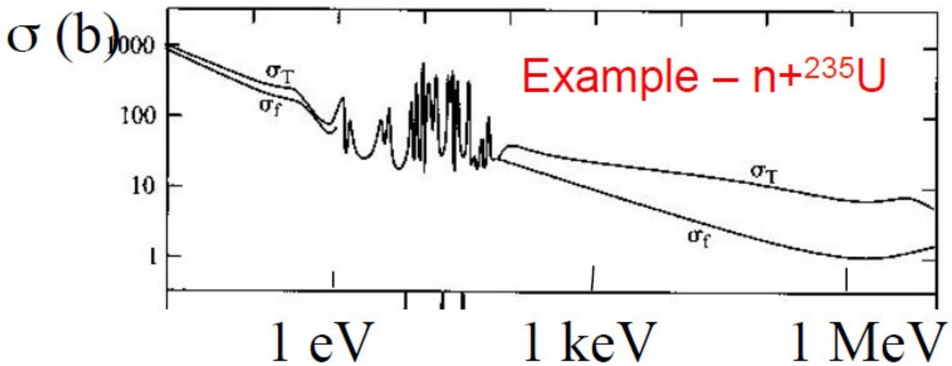
$$\sigma = \frac{\pi}{p_i^2} \frac{\Gamma_{O \to i} \Gamma_{O \to f}}{(E - E_O)^2 + \frac{\Gamma^2}{4}}$$

this is the **Breit-Wigner** cross section

Resonant scattering

- Some particle interactions occur via an intermediate resonant state which then decays

$$a + b \rightarrow O \rightarrow c + d$$



- So we have for the cross section:

$$\sigma = \frac{\pi}{p_i^2} \frac{\Gamma_{O \rightarrow i} \Gamma_{O \rightarrow f}}{(E - E_O)^2 + \frac{\Gamma^2}{4}}$$

this is the **Breit-Wigner** cross section

- p_i^2 is calculated in the centre-of-mass frame
- E is the centre-of-mass energy,
- E_O is the rest mass of the resonance
- $\Gamma_{O \rightarrow x}$ are partial widths and Γ the full width of the resonance

- We should also include information about spin:

$$\sigma = \frac{g\pi}{p_i^2}\frac{\Gamma_{O\to i}\Gamma_{O\to f}}{(E-E_O)^2+\frac{\Gamma^2}{4}}$$

with: $$g = \frac{2J_O+1}{(2J_a+1)(2J_b+1)}$$

is the ratio of the number of spin states for the resonant state to the total number of spin states for the $a+b$ system

- It is the probability that $a+b$ collide in the correct spin state to form the resonance O

Resonant scattering

- We can use measurements of cross sections to infer other information

- Total cross section:

$$\sigma_{tot} = \sum_f \sigma(i \rightarrow f)$$

$$\sigma_{tot} = \frac{g\pi}{p_i^2} \frac{\Gamma_{O\rightarrow i}\Gamma}{(E - E_O)^2 + \frac{\Gamma^2}{4}}$$

Resonant scattering

- We can use measurements of cross sections to infer other information

- Total cross section:

$$\sigma_{tot} = \sum_{f} \sigma(i \to f)$$

$$\sigma_{tot} = \frac{g\pi}{p_i^2} \frac{\Gamma_{O \to i}\Gamma}{(E - E_O)^2 + \frac{\Gamma^2}{4}}$$

- Elastic cross section:

$$\sigma_{el} = \sigma(i \to i)$$

$$\sigma = \frac{g\pi}{p_i^2} \frac{\Gamma_{O \to i}\Gamma_{O \to i}}{(E - E_O)^2 + \frac{\Gamma^2}{4}}$$

Resonant scattering

- We can use measurements of cross sections to infer other information

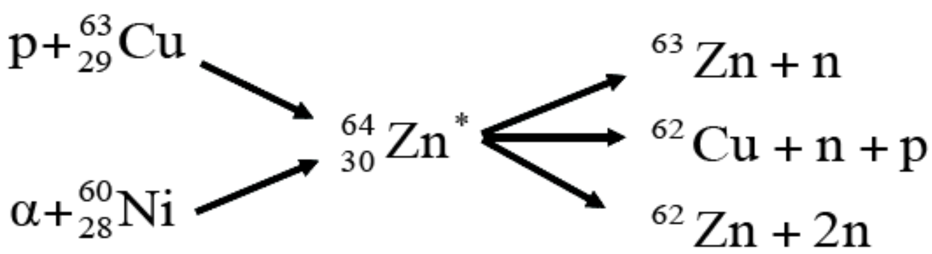
- On peak resonance ($E = E_0$)

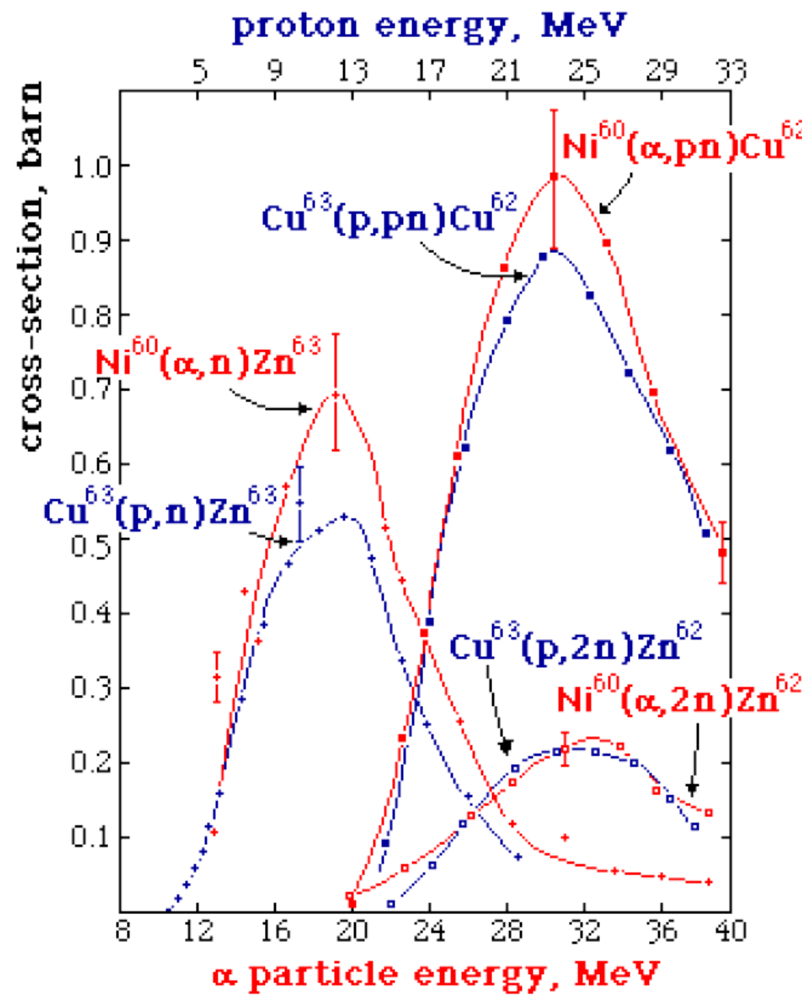
$$\sigma_{peak} = \frac{g4\pi}{p_i^2}\frac{\Gamma_{O\to i}\Gamma_{O\to f}}{\Gamma^2}$$

$$\sigma_{peak-el} = \frac{g4\pi}{p_i^2}\frac{\Gamma_{O\to i}\Gamma_{O\to i}}{\Gamma^2} = \frac{g4\pi}{p_i^2}BR(i)^2$$

$$\sigma_{peak-tot} = \frac{g4\pi}{p_i^2}\frac{\Gamma_{O\to i}}{\Gamma} = \frac{g4\pi}{p_i^2}BR(i)$$

Resonances (nuclear physics)



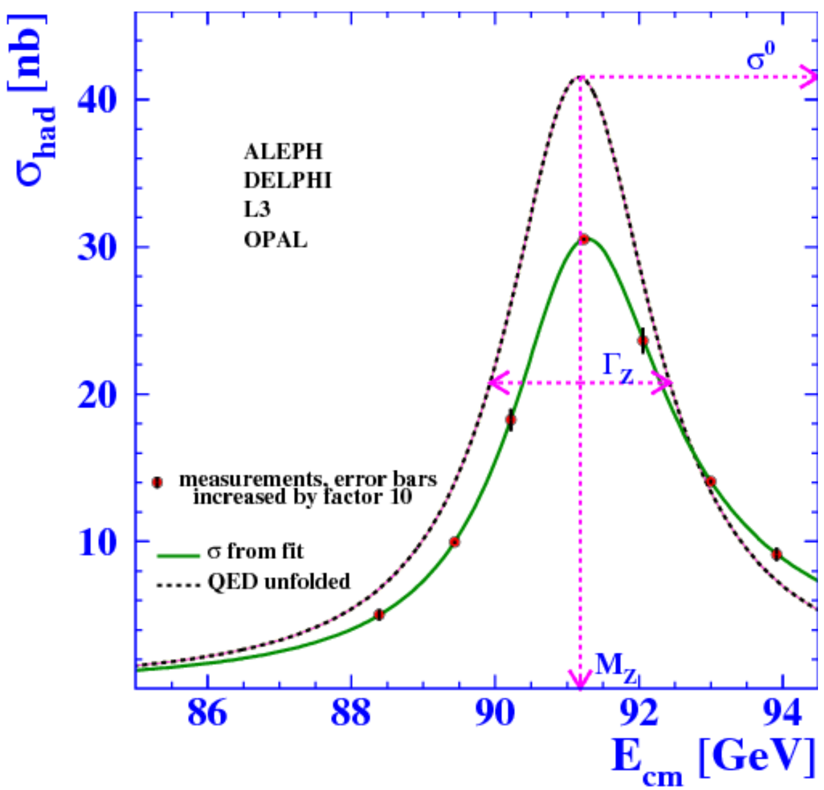


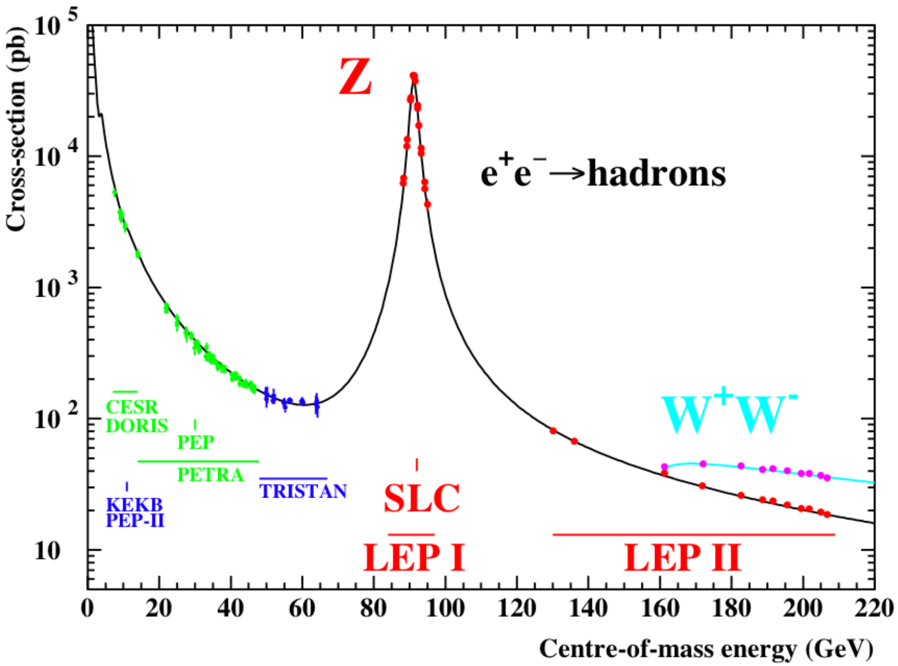
- Production independent of decay
- We can see the 3 resonances from the 2 production mechanisms
- Notation in nuclear physics:

$$a + B \rightarrow c + D = B(a,c)D$$

- Z boson at LEP

$$m_Z = 91.1875 \pm 0.0021\ \text{GeV}$$





- Total decay width

$$\Gamma_Z = 2.4952 \pm 0.0023\ \text{GeV}$$

- Peak cross section

Example

- $\pi^- p$ **scattering: Resonance at** $p_\pi^{\text{lab}} \sim 0.3$ GeV, $E_{\text{cm}} = 1.25$ GeV. $\sigma_{peak-tot} = 72\text{mb}$, $\sigma_{peak-el} = 28\text{mb}$. **Find** g **and** J_0 ($J_p = \frac{1}{2}, J_\pi = 0$)