

Física de Partículas

Modelo GWS

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Latin American alliance for
Capacity building in Advanced physics

LA-CoNGA physics



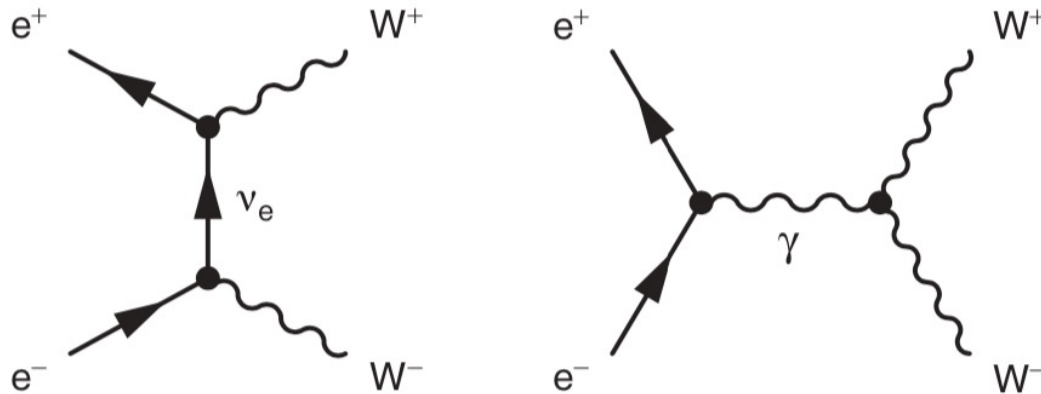
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W-pair production

- Since W bosons are charged (EM charge), they couple to the photon
- W pairs can be produced in electron-positron colliders or hadron colliders

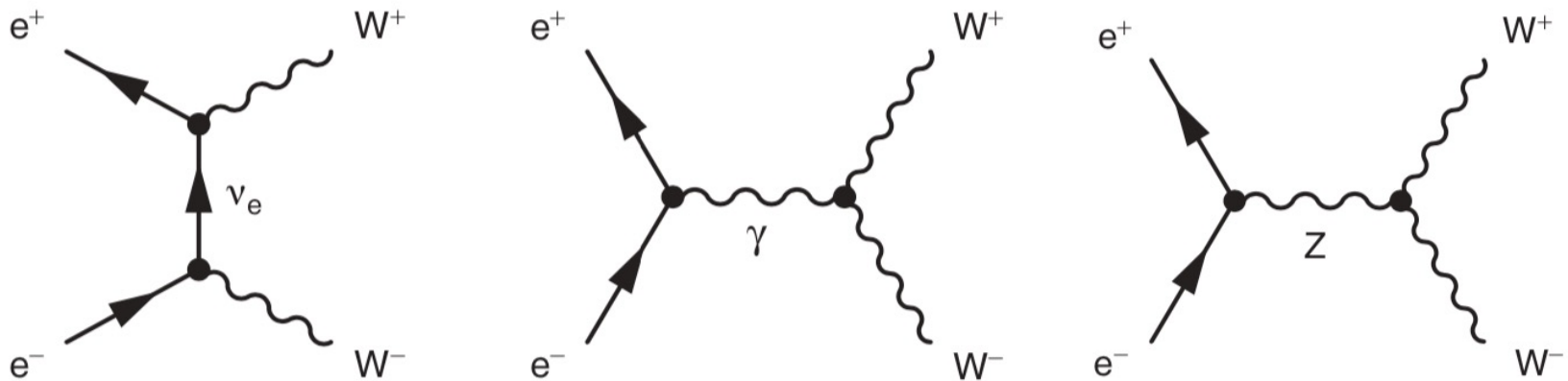


- However: the cross section diverges if only these diagrams are considered
- An additional gauge boson: the neutral Z boson



W-pair production

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- An additional gauge boson: the neutral Z boson

$$|\mathcal{M}_\nu + \mathcal{M}_\gamma + \mathcal{M}_Z|^2 < |\mathcal{M}_\nu + \mathcal{M}_\gamma|^2$$

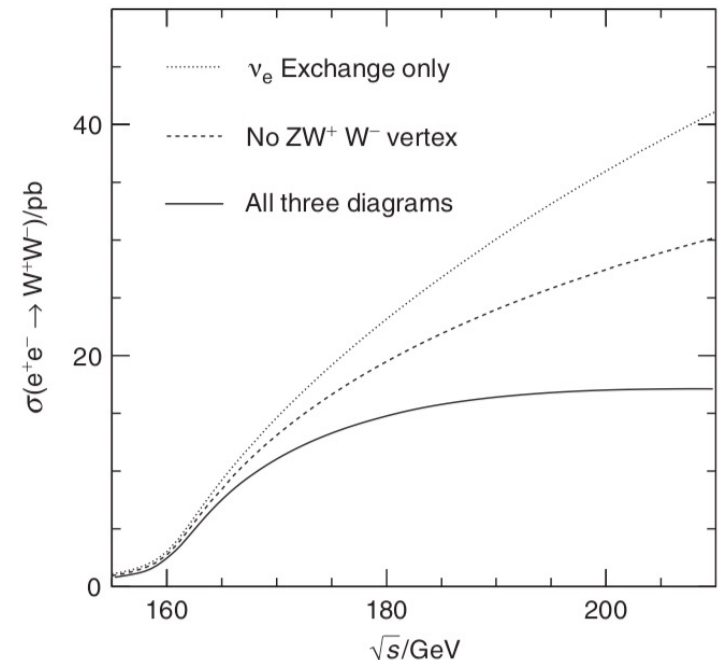


W-pair production

- Since W bosons are charged (EM charge), they couple to the photon
- W pairs can be produced in electron-positron colliders or hadron colliders
- An additional gauge boson: the neutral Z boson

$$|\mathcal{M}_\nu + \mathcal{M}_\gamma + \mathcal{M}_Z|^2 < |\mathcal{M}_\nu + \mathcal{M}_\gamma|^2$$

- This only works if the couplings are all related:
Electroweak Unification





Weak interaction gauge group

- QED and QCD are associated to U(1) and SU(3) local gauge symmetries
- The charged weak interaction can be associated with an SU(2) local gauge symmetry:

$$\varphi(x) \rightarrow \varphi'(x) = \exp [ig_W \alpha(x) \cdot \mathbf{T}] \varphi(x) \quad \mathbf{T} = \frac{1}{2} \boldsymbol{\sigma}$$

- 3 gauge fields: W_1, W_2, W_3
- The wave function must be a doublet: $\varphi(x) = \begin{pmatrix} \nu_e(x) \\ e^-(x) \end{pmatrix}$
- However: the weak interaction couples only to LH particles and RH anti-particles
- So RH particles and LH anti-particles are put in singlets (remain unaffected by local gauge transformation)



Weak interaction gauge group

- QED and QCD are associated to U(1) and SU(3) local gauge symmetries
- The charged weak interaction can be associated with an SU(2)_L local gauge symmetry

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L, \quad \begin{pmatrix} u \\ d' \end{pmatrix}_L, \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L, \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L$$
$$e_R^-, \quad \mu_R^-, \quad \tau_R^-, \quad u_R, \quad c_R, \quad t_R, \quad d_R, \quad s_R, \quad b_R$$

- We get an interaction term:

$$ig_W T_k \gamma^\mu \mathbf{W}_\mu^k \varphi_L = ig_W \frac{1}{2} \sigma_k \gamma^\mu \mathbf{W}_\mu^k \varphi_L \quad \varphi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$



Weak interaction gauge group

- The weak charged current corresponding to the exchange of the physical W bosons are:

$$j_{\pm}^{\mu} = \frac{1}{\sqrt{2}} (j_1^{\mu} \pm i j_2^{\mu}) = \frac{g_W}{\sqrt{2}} \bar{\varphi}_L \gamma^{\mu} \frac{1}{2} (\sigma_1 \pm i \sigma_2) \varphi_L$$

$$= \frac{g_W}{\sqrt{2}} \bar{\varphi}_L \gamma^{\mu} \sigma_{\pm} \varphi_L$$

- And the W bosons are linear combinations of W_1, W_2 :

$$W_{\mu}^{\pm} = \frac{1}{\sqrt{2}} (W_{\mu}^{(1)} \mp i W_{\mu}^{(2)})$$

- Explicitly:

$$j_{+}^{\mu} = \frac{g_W}{\sqrt{2}} \bar{\varphi}_L \gamma^{\mu} \sigma_{+} \varphi_L = \frac{g_W}{\sqrt{2}} (\bar{\nu}_L \ \bar{e}_L) \gamma^{\mu} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$= \frac{g_W}{\sqrt{2}} \bar{\nu}_L \gamma^{\mu} e_L \equiv \frac{g_W}{\sqrt{2}} \bar{\nu} \gamma^{\mu} \frac{1}{2} (1 - \gamma^5) e$$

$$j_{-}^{\mu} = \frac{g_W}{\sqrt{2}} \bar{\varphi}_L \gamma^{\mu} \sigma_{-} \varphi_L = \frac{g_W}{\sqrt{2}} (\bar{\nu}_L \ \bar{e}_L) \gamma^{\mu} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$= \frac{g_W}{\sqrt{2}} \bar{e}_L \gamma^{\mu} \nu_L \equiv \frac{g_W}{\sqrt{2}} \bar{e} \gamma^{\mu} \frac{1}{2} (1 - \gamma^5) \nu$$



Weak interaction gauge group

- The symmetry of the weak interaction results in the charged weak currents:

$$j_+^\mu = \frac{g_W}{\sqrt{2}} \bar{\nu} \gamma^\mu \frac{1}{2} (1 - \gamma^5) e \quad j_-^\mu = \frac{g_W}{\sqrt{2}} \bar{e} \gamma^\mu \frac{1}{2} (1 - \gamma^5) \nu$$

with the already familiar V-A structure

- The third gauge boson implies a neutral current:

$$j_3^\mu = g_W \bar{\varphi}_L \gamma^\mu \frac{1}{2} \sigma_3 \varphi_L$$

$$j_3^\mu = g_W \frac{1}{2} \begin{pmatrix} \bar{\nu}_L & \bar{e}_L \end{pmatrix} \gamma^\mu \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$= g_W \frac{1}{2} \bar{\nu}_L \gamma^\mu \nu_L - g_W \frac{1}{2} \bar{e}_L \gamma^\mu e_L$$

$$j_3^\mu = I_W^{(3)} g_W \bar{f} \gamma^\mu \frac{1}{2} (1 - \gamma^5) f \quad I_W^{(3)} = \pm 1/2$$



- At this point it would be natural to assume the weak neutral current W_3 corresponds to the Z boson
- However: the Z boson does couple to RH particles and LH anti-particles
- Also, W and Z bosons would have the same coupling strength – not seen experimentally
- The solution: Unify QED and the weak force – Electroweak model



Electroweak model (GWS model)

- The Glashow, Weinberg and Salam model: EM and weak interactions as different manifestations of a single unified electroweak force (Nobel Prize 1979)
- The local U(1) symmetry of QED is replaced by a local U(1)_Y symmetry (weak hypercharge):

$$\psi(x) \rightarrow \psi'(x) = \hat{U}(x)\psi(x) = \exp\left[ig' \frac{Y}{2}\zeta(x)\right] \psi(x)$$
$$g' \frac{Y}{2} \gamma^\mu B_\mu \psi$$

- The photon and the Z boson can be written as combinations of the neutral gauge bosons:

$$A_\mu = +B_\mu \cos \theta_W + W_\mu^{(3)} \sin \theta_W$$

$$Z_\mu = -B_\mu \sin \theta_W + W_\mu^{(3)} \cos \theta_W$$



Electroweak model (GWS model)

- Underlying symmetry: $SU(2)_L \times U(1)_Y$
- Photon properties must be the same we found for QED – couplings are related:

$$e = g_W \sin \theta_W$$

- Z boson coupling:

$$g_Z = \frac{g_W}{\cos \theta_W} \equiv \frac{e}{\sin \theta_W \cos \theta_W}$$

- And the Z boson current:

$$j_Z^\mu = g_Z (c_L \bar{u}_L \gamma^\mu u_L + c_R \bar{u}_R \gamma^\mu u_R)$$

the Z boson couples to both LH and RH particles, but differently



Electroweak model (GWS model)

- Underlying symmetry: $SU(2)_L \times U(1)_Y$
- Analogous to the charged weak current, the Z boson current can be written as:

$$j_Z^\mu = g_Z \bar{u} \gamma^\mu \left[c_L \frac{1}{2} (1 - \gamma^5) + c_R \frac{1}{2} (1 + \gamma^5) \right] u$$

$$= g_Z \bar{u} \gamma^\mu \frac{1}{2} \left[(c_L + c_R) - (c_L - c_R) \gamma^5 \right] u.$$

$$j_Z^\mu = \frac{1}{2} g_Z \bar{u} \left(c_V \gamma^\mu - c_A \gamma^\mu \gamma^5 \right) u$$

- Z boson interaction vertex:

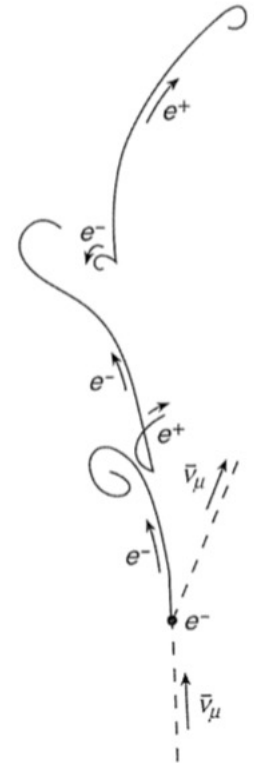
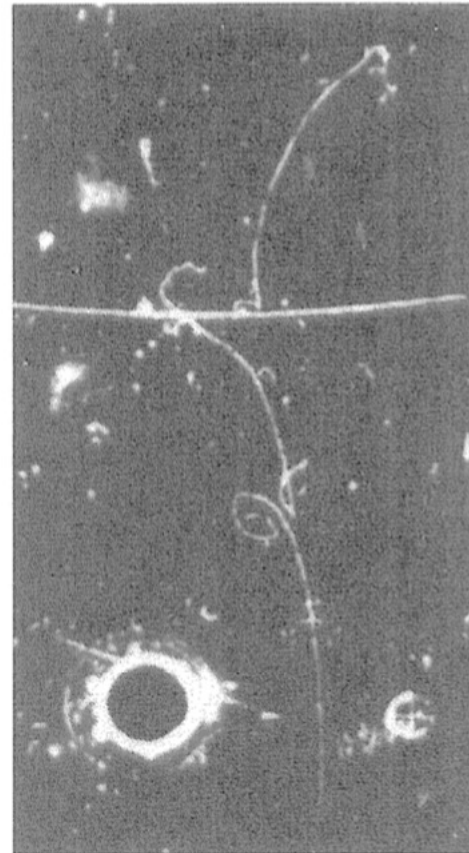
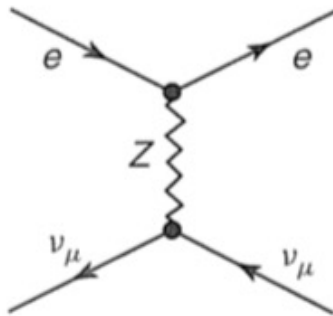
$$-i \frac{1}{2} g_Z \gamma^\mu \left[c_V - c_A \gamma^5 \right]$$

f	c_V	c_A
ν_e, ν_μ, ν_τ	$\frac{1}{2}$	$\frac{1}{2}$
e^-, μ^-, τ^-	$-\frac{1}{2} + 2 \sin^2 \theta_w$	$-\frac{1}{2}$
u, c, t	$\frac{1}{2} - \frac{4}{3} \sin^2 \theta_w$	$\frac{1}{2}$
d, s, b	$-\frac{1}{2} + \frac{2}{3} \sin^2 \theta_w$	$-\frac{1}{2}$



Evidence for the GWS Model

- Discovery of Neutral Currents (1973): Only possible Feynman diagram (no W^\pm diagram). Indirect evidence for Z.



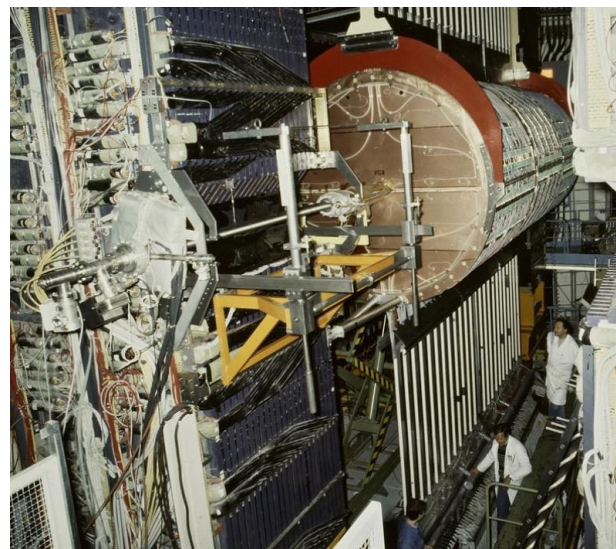


Evidence for the GWS Model

- First direct observation in $p\bar{p}$ collisions at $\sqrt{s} = 540 \text{ GeV}$ via decays into $p\bar{p} \rightarrow Z + X$
- Precision measurements (LEP):

$$\sin^2 \theta_W = 0.23146 \pm 0.00012$$

$$\frac{\alpha}{\alpha_W} = \frac{e^2}{g_W^2} = \sin^2 \theta_W \sim 0.23$$

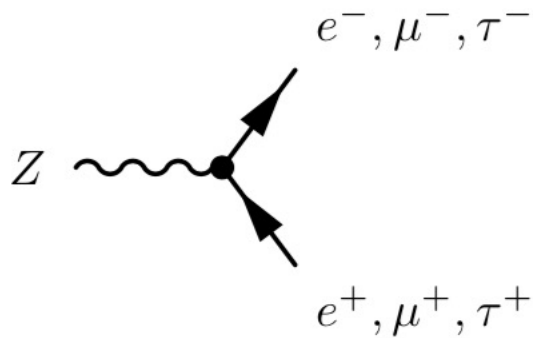


UA1 Experiment at CERN Used Super Proton Synchrotron (now part of LHC)

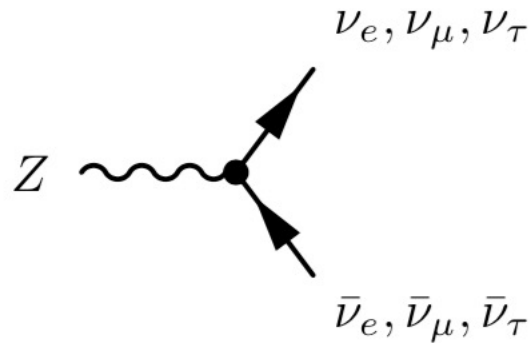


The weak Neutral Current vertex

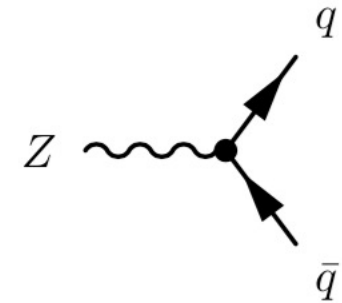
$$Z \rightarrow e^+e^-, \mu^+\mu^-, \tau^+\tau^-$$



$$Z \rightarrow \nu_e\bar{\nu}_e, \nu_\mu\bar{\nu}_\mu, \nu_\tau\bar{\nu}_\tau$$



$$Z \rightarrow q\bar{q}$$



- The Z boson does not change flavour
- The Z coupling is a mixture of EM and weak couplings – depends on $\sin^2 \theta_W$

So far in the model the gauge bosons remain massless



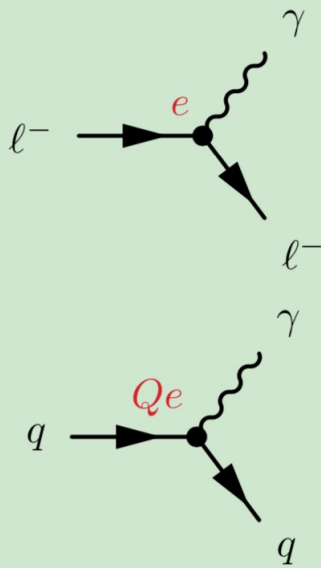
Where we are so far

- The main elements of the Standard Model of particle physics have been described
- There are 12 fundamental spin-half fermions (particles and anti-particles): Dirac equation
- The interactions between particles are described by the exchange of spin-1 gauge bosons : local gauge principle
- Underlying gauge symmetry of the Standard Model is $U(1)_Y \times SU(2)_L \times SU(3)_C$: EM and weak interactions described by the unified electroweak theory
- The predictions of the Electroweak theory were confronted with precision measurements at LEP



Where we are so far

Electromagnetic (QED)

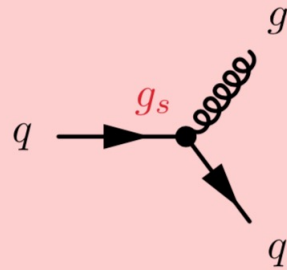


$$\alpha = \frac{e^2}{4\pi}$$

$q = u, d, s, c, b, t$

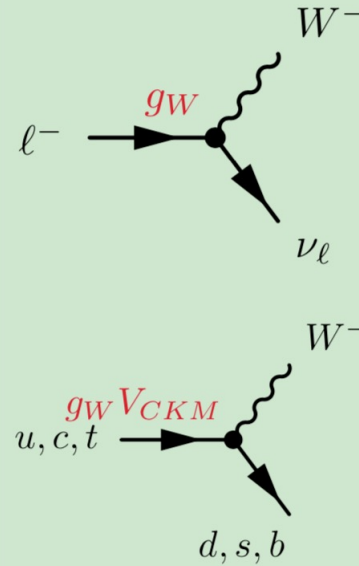
+ antiparticles

Strong (QCD)



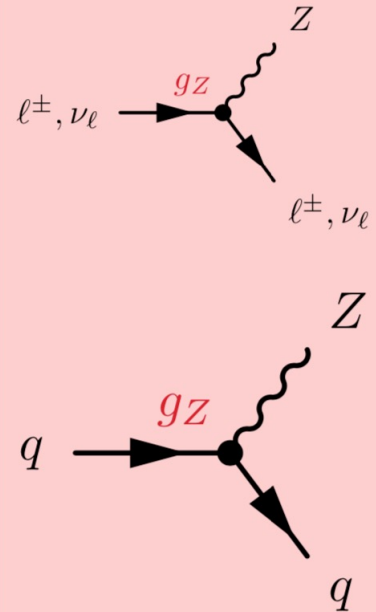
$$\alpha_s = \frac{g_s^2}{4\pi}$$

Weak CC



$$\alpha_W = \frac{g_W^2}{4\pi}$$

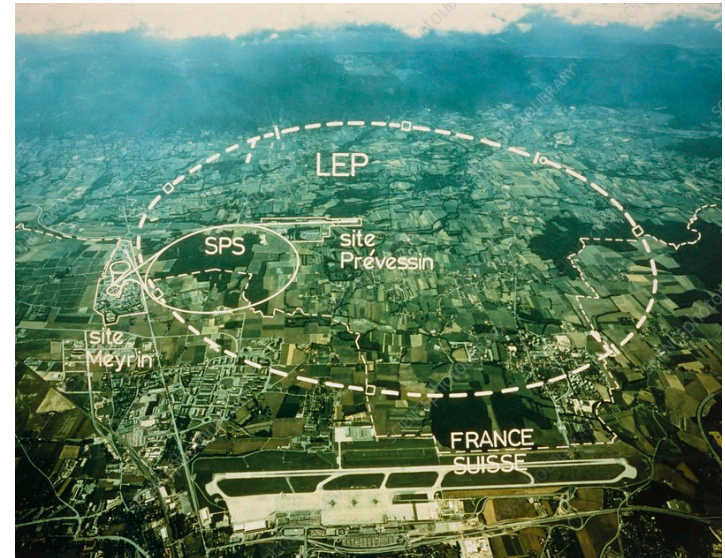
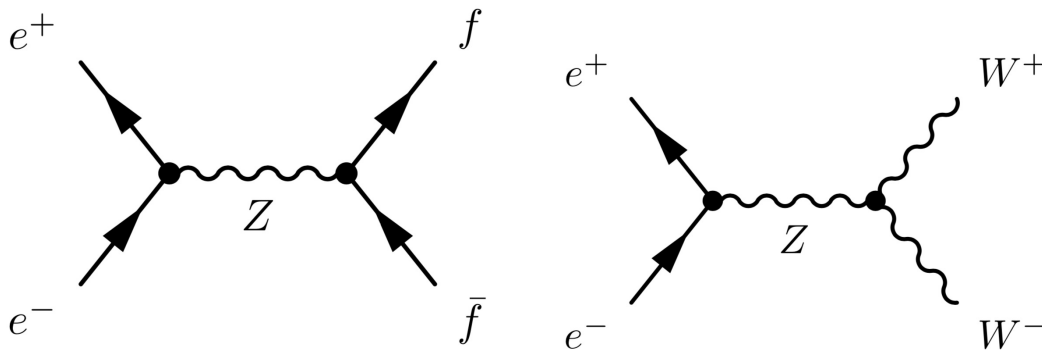
Weak NC



$$g_Z = \frac{g_W}{\cos \theta_W}$$



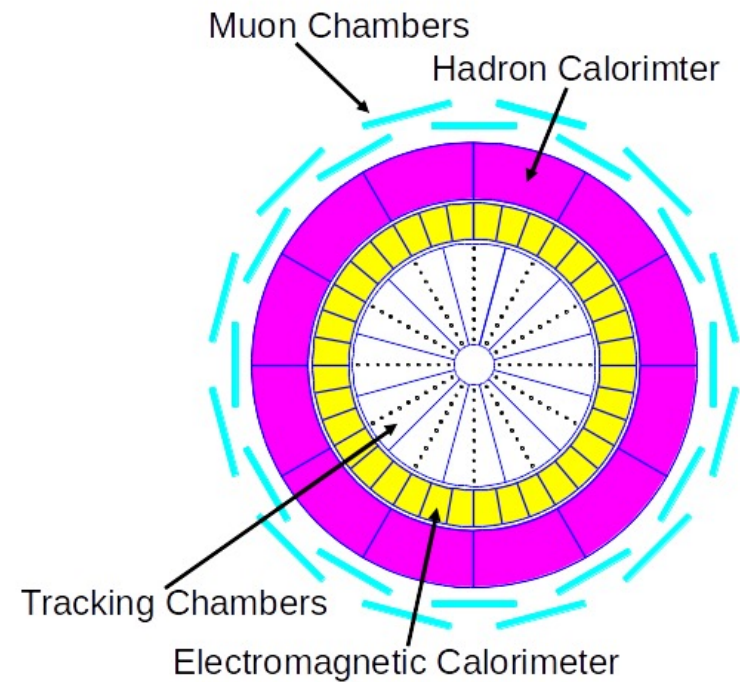
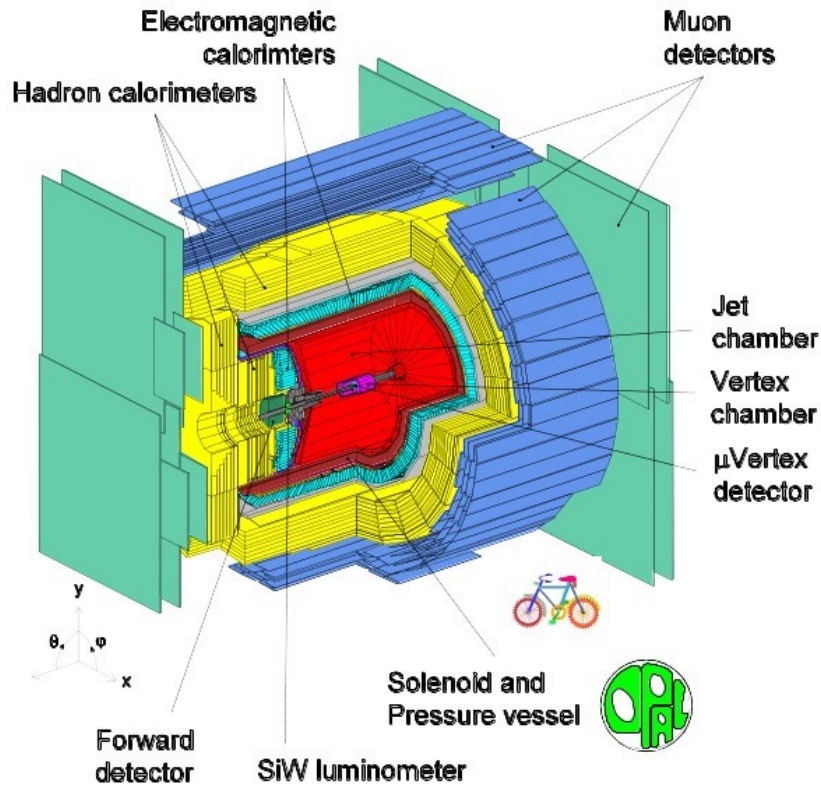
- Large Electron Positron collider at CERN (1989-2000)
- Designed as a Z and W boson factory



- Highest energy e^+e^- collider ever built: $\sqrt{s} = 90\text{-}209$ GeV
- Circumference: 27 km (LEP tunnel used for the LHC)
- The 4 experiments combined: 16×10^6 Z events, 30×10^3 W events



- One of the 4 experiments at LEP

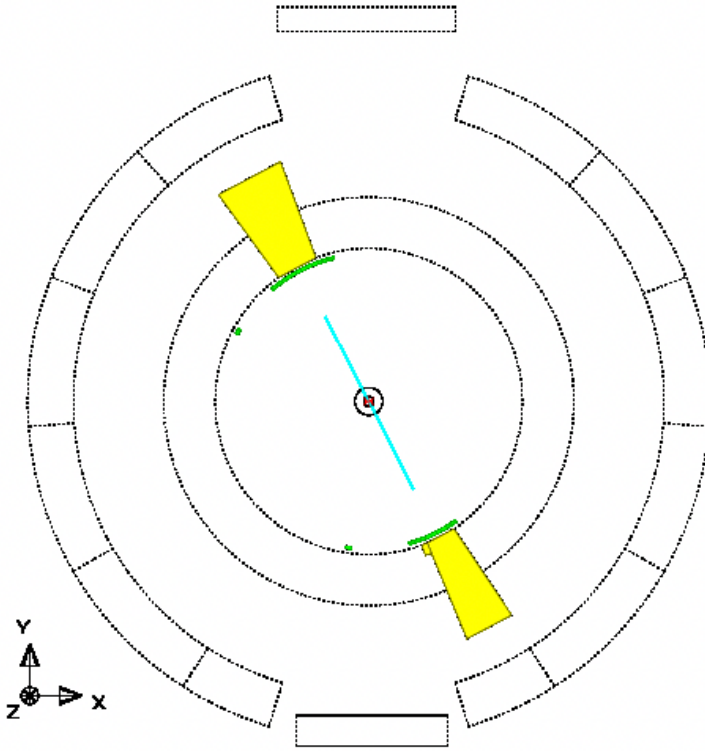




Typical Z events

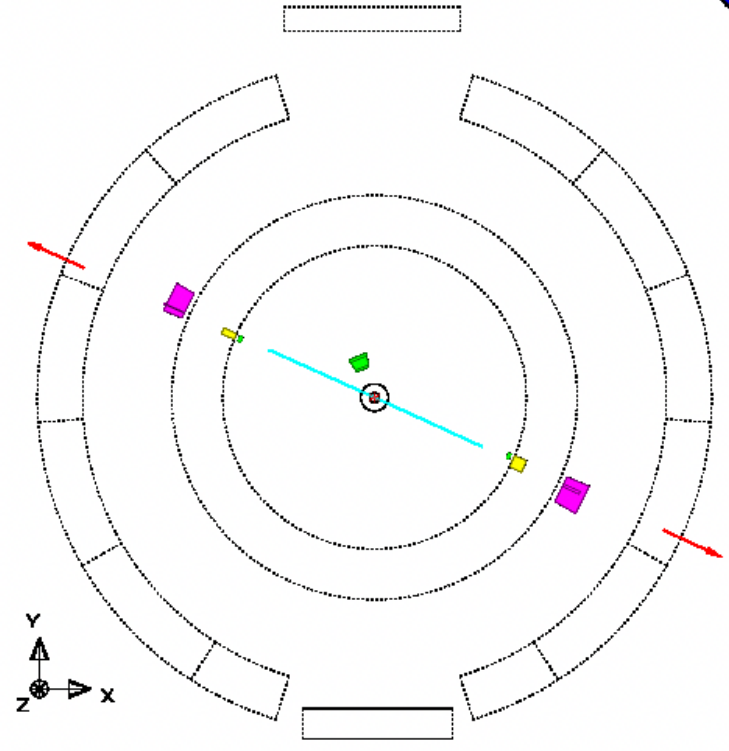
$$e^+e^- \rightarrow Z \rightarrow e^+e^-$$

Run:event 4093: 1150 Ctrk(N= 2 Sump= 93.0) Ecal(N= 8 SumE= 87.5)
Ebeam 45.682 Vix (-0.04, 0.08, 0.33) Hcal(N= 0 SumE= 0.0) Muon(N= 0)



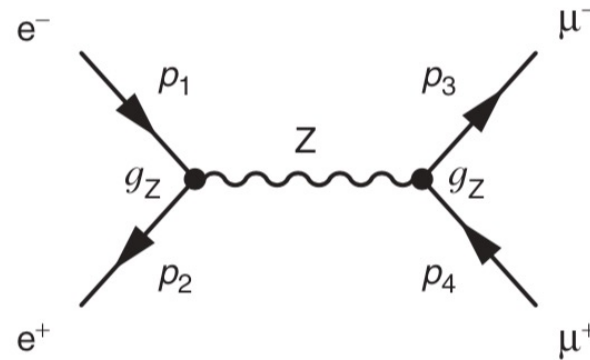
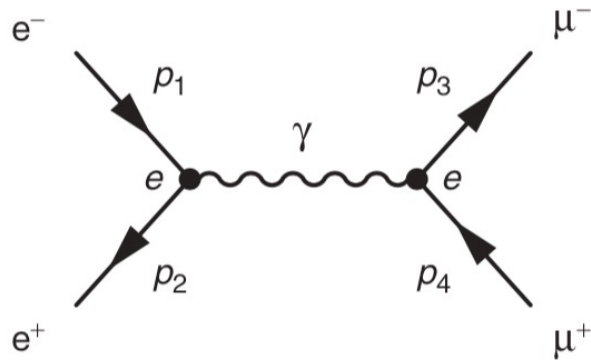
$$e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-$$

Run:event 4093: 4550 Ctrk(N= 2 Sump= 85.8) Ecal(N= 4 SumE= 1.6)
Ebeam 45.682 Vix (-0.04, 0.08, 0.33) Hcal(N= 4 SumE= 4.0) Muon(N= 2)





Z resonance



- We already calculated the QED process

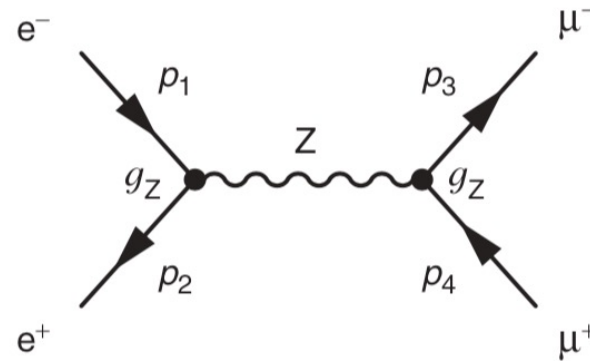
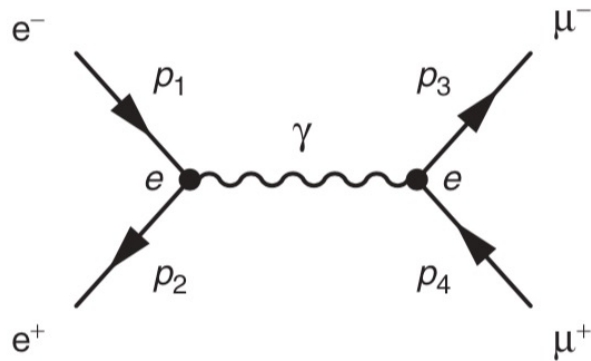
- The matrix elements:

$$\mathcal{M}_\gamma \propto \frac{e^2}{q^2} \quad \mathcal{M}_Z \propto \frac{g_Z^2}{q^2 - m_Z^2}$$

- The QED process dominates at low centre of mass energy ($q^2 = s$)
- In the region $\sqrt{s} \sim m_Z$ the Z-boson process dominates



Z resonance



- The Z boson is not a stable particle: propagator modified

$$\frac{1}{q^2 - m_Z^2} \rightarrow \frac{1}{q^2 - m_Z^2 + im_Z\Gamma_Z}$$

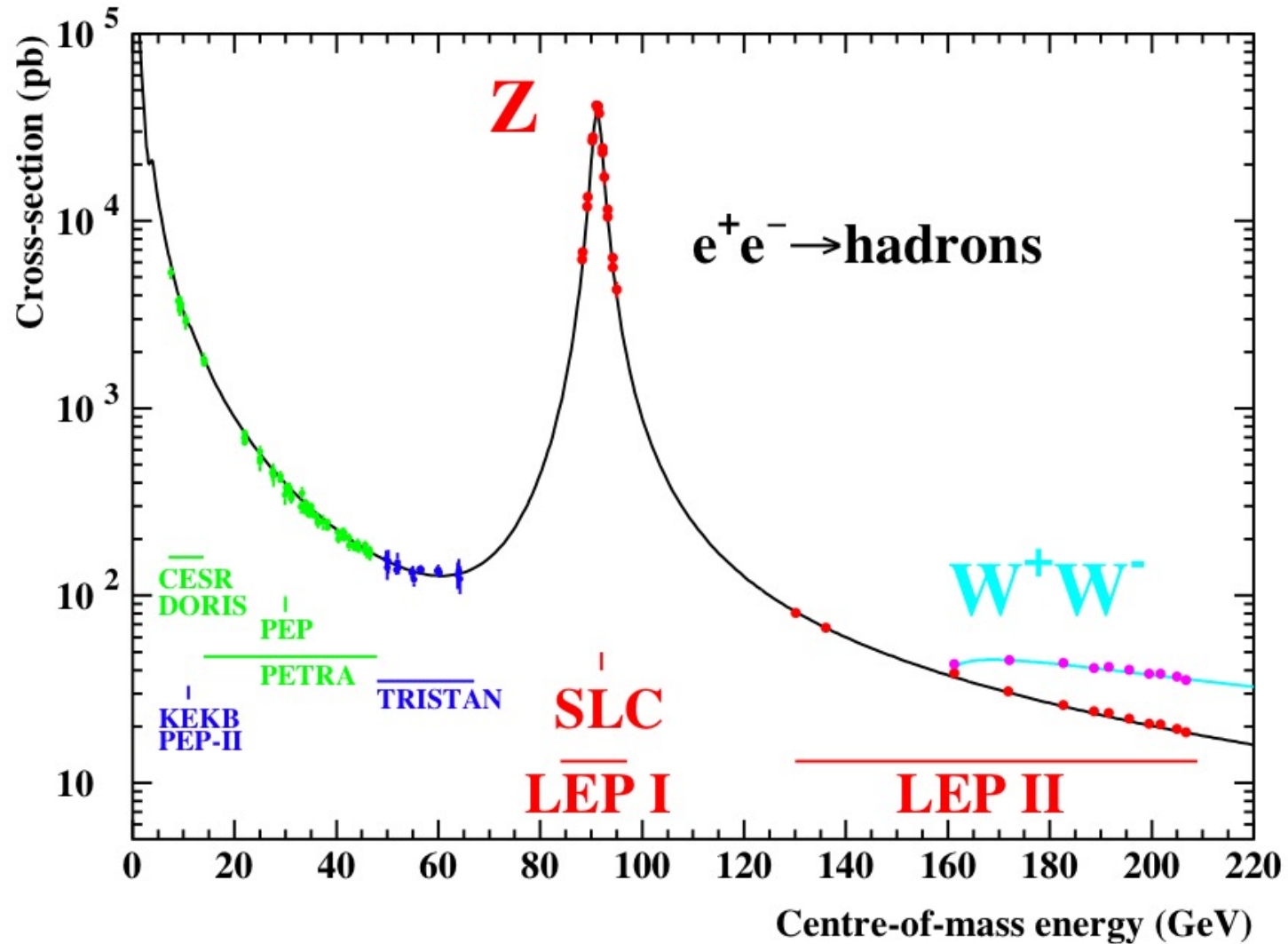
- And the cross section is proportional to:

$$\sigma \propto \frac{1}{(s - m_Z^2)^2 + m_Z^2\Gamma_Z^2}$$

Breit-Wigner resonance



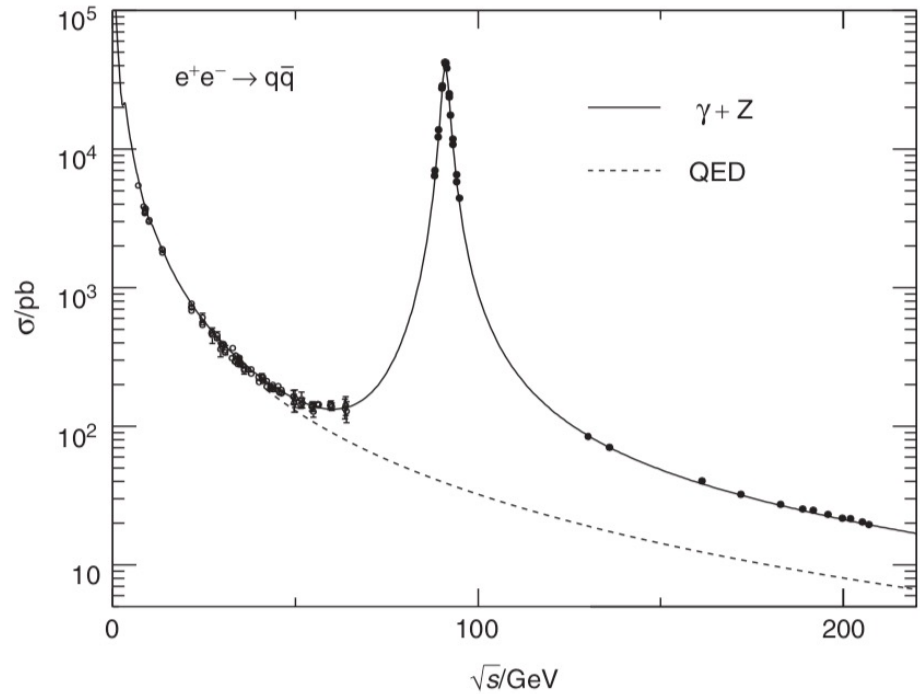
Z resonance





Z resonance

- Below 40 GeV: QED process dominates
- Between 50 and 80 GeV: contributions from both processes
- Around the resonance: Z boson process dominates
- Away from the resonance: both processes of the same order of magnitude (EW unification)

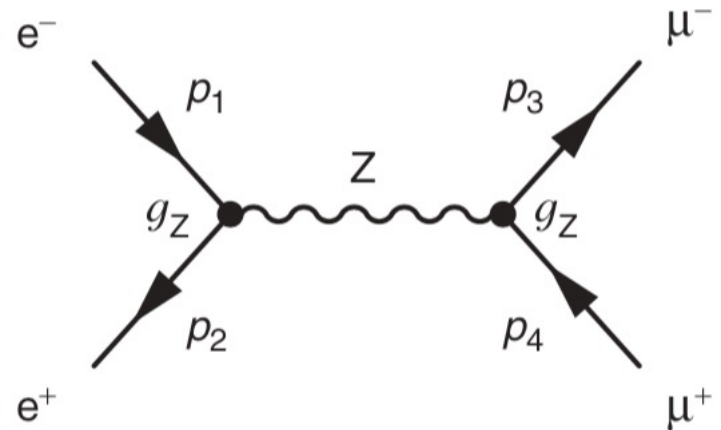




Z production cross section

- The cross section calculation around the resonance would only need the Z boson contribution

$$\mathcal{M}_{fi} = -\frac{g_Z^2}{(s - m_Z^2 + im_Z\Gamma_Z)} g_{\mu\nu} \left[\bar{v}(p_2) \gamma^\mu \frac{1}{2} (c_V^e - c_A^e \gamma^5) u(p_1) \right] \times \left[\bar{u}(p_3) \gamma^\nu \frac{1}{2} (c_V^\mu - c_A^\mu \gamma^5) v(p_4) \right],$$



with the corresponding vector and axial couplings to the Z boson

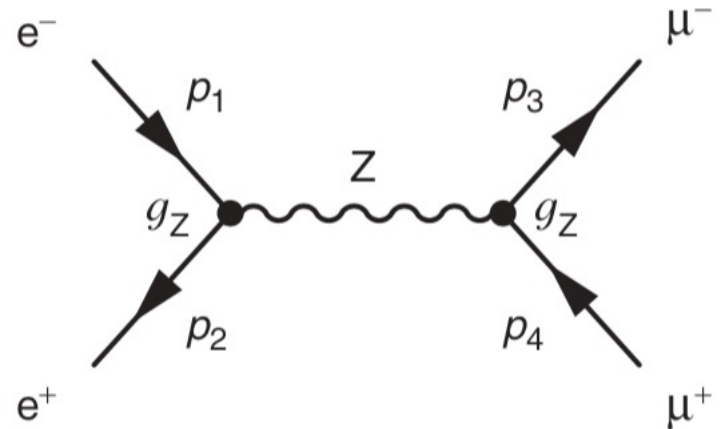
- The rest of the procedure is similar to the QED calculation
- Around the resonance, the masses of the leptons can be ignored
- In this scenario, the helicity and chiral states are the same



Z production cross section

- The cross section calculation around the resonance would only need the Z boson contribution

$$\mathcal{M}_{fi} = -\frac{g_Z^2}{(s - m_Z^2 + im_Z\Gamma_Z)} g_{\mu\nu} \left[\bar{v}(p_2) \gamma^\mu \frac{1}{2} (c_V^e - c_A^e \gamma^5) u(p_1) \right] \times \left[\bar{u}(p_3) \gamma^\nu \frac{1}{2} (c_V^\mu - c_A^\mu \gamma^5) v(p_4) \right],$$



with

$$c_V = (c_L + c_R)$$

$$c_A = (c_L - c_R)$$

$$\mathcal{M}_{fi} = -\frac{g_Z^2}{(s - m_Z^2 + im_Z\Gamma_Z)} g_{\mu\nu} \left[c_L^e \bar{v}(p_2) \gamma^\mu P_L u(p_1) + c_R^e \bar{v}(p_2) \gamma^\mu P_R u(p_1) \right] \times \left[c_L^\mu \bar{u}(p_3) \gamma^\nu P_L v(p_4) + c_R^\mu \bar{u}(p_3) \gamma^\nu P_R v(p_4) \right]$$



Z production cross section

- The cross section calculation around the resonance would only need the Z boson contribution

$$\mathcal{M}_{fi} = -\frac{g_Z^2}{(s - m_Z^2 + im_Z\Gamma_Z)} g_{\mu\nu} \left[c_L^e \bar{v}(p_2) \gamma^\mu P_L u(p_1) + c_R^e \bar{v}(p_2) \gamma^\mu P_R u(p_1) \right] \times \left[c_L^\mu \bar{u}(p_3) \gamma^\nu P_L v(p_4) + c_R^\mu \bar{u}(p_3) \gamma^\nu P_R v(p_4) \right]$$

here $m_Z \gg m_\mu$, so:

$$P_L u = u_\downarrow \quad P_R u = u_\uparrow \quad P_L v = v_\uparrow \quad P_R v = v_\downarrow$$

and only 4 contributions to the matrix element are non zero:

$$\mathcal{M}_{RR} = -P_Z(s) g_Z^2 c_R^e c_R^\mu g_{\mu\nu} [\bar{v}_\downarrow(p_2) \gamma^\mu u_\uparrow(p_1)] [\bar{u}_\uparrow(p_3) \gamma^\nu v_\downarrow(p_4)]$$

$$\mathcal{M}_{RL} = -P_Z(s) g_Z^2 c_R^e c_L^\mu g_{\mu\nu} [\bar{v}_\downarrow(p_2) \gamma^\mu u_\uparrow(p_1)] [\bar{u}_\downarrow(p_3) \gamma^\nu v_\uparrow(p_4)]$$

$$\mathcal{M}_{LR} = -P_Z(s) g_Z^2 c_L^e c_R^\mu g_{\mu\nu} [\bar{v}_\uparrow(p_2) \gamma^\mu u_\downarrow(p_1)] [\bar{u}_\uparrow(p_3) \gamma^\nu v_\downarrow(p_4)]$$

$$\mathcal{M}_{LL} = -P_Z(s) g_Z^2 c_L^e c_L^\mu g_{\mu\nu} [\bar{v}_\uparrow(p_2) \gamma^\mu u_\downarrow(p_1)] [\bar{u}_\downarrow(p_3) \gamma^\nu v_\uparrow(p_4)]$$



Z production cross section

- The cross section calculation around the resonance would only need the Z boson contribution

$$\mathcal{M}_{RR} = -P_Z(s) g_Z^2 c_R^e c_R^\mu g_{\mu\nu} [\bar{v}_\downarrow(p_2) \gamma^\mu u_\uparrow(p_1)] [\bar{u}_\uparrow(p_3) \gamma^\nu v_\downarrow(p_4)]$$

$$\mathcal{M}_{RL} = -P_Z(s) g_Z^2 c_R^e c_L^\mu g_{\mu\nu} [\bar{v}_\downarrow(p_2) \gamma^\mu u_\uparrow(p_1)] [\bar{u}_\downarrow(p_3) \gamma^\nu v_\uparrow(p_4)]$$

$$\mathcal{M}_{LR} = -P_Z(s) g_Z^2 c_L^e c_R^\mu g_{\mu\nu} [\bar{v}_\uparrow(p_2) \gamma^\mu u_\downarrow(p_1)] [\bar{u}_\uparrow(p_3) \gamma^\nu v_\downarrow(p_4)]$$

$$\mathcal{M}_{LL} = -P_Z(s) g_Z^2 c_L^e c_L^\mu g_{\mu\nu} [\bar{v}_\uparrow(p_2) \gamma^\mu u_\downarrow(p_1)] [\bar{u}_\downarrow(p_3) \gamma^\nu v_\uparrow(p_4)]$$

with $P_Z(s) = 1/(s - m_Z^2 + im_Z\Gamma_Z)$

- The combinations are identical to the ones derived for QED:

$$|\mathcal{M}_{RR}|^2 = |P_Z(s)|^2 g_Z^4 s^2 (c_R^e)^2 (c_R^\mu)^2 (1 + \cos \theta)^2$$

$$|\mathcal{M}_{RL}|^2 = |P_Z(s)|^2 g_Z^4 s^2 (c_R^e)^2 (c_L^\mu)^2 (1 - \cos \theta)^2$$

$$|\mathcal{M}_{LR}|^2 = |P_Z(s)|^2 g_Z^4 s^2 (c_L^e)^2 (c_R^\mu)^2 (1 - \cos \theta)^2$$

$$|\mathcal{M}_{LL}|^2 = |P_Z(s)|^2 g_Z^4 s^2 (c_L^e)^2 (c_L^\mu)^2 (1 + \cos \theta)^2$$



Z production cross section

- Analogous to the QED calculation, averaging over the initial state spin configurations:

$$\langle |\mathcal{M}|^2 \rangle = \frac{1}{4} (|\mathcal{M}_{RR}|^2 + |\mathcal{M}_{LL}|^2 + |\mathcal{M}_{LR}|^2 + |\mathcal{M}_{RL}|^2)$$

$$\langle |\mathcal{M}|^2 \rangle = \frac{1}{4} \frac{g_Z^4 s^2}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \times \left\{ \left[(c_R^e)^2 (c_R^\mu)^2 + (c_L^e)^2 (c_L^\mu)^2 \right] (1 + \cos \theta)^2 + \left[(c_R^e)^2 (c_L^\mu)^2 + (c_L^e)^2 (c_R^\mu)^2 \right] (1 - \cos \theta)^2 \right\}$$

- Going back to the vector and axial couplings, the differential cross section:

$$\frac{d\sigma}{d\Omega} = \frac{1}{256\pi^2 s} \cdot \frac{g_Z^4 s^2}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \times \left\{ \frac{1}{4} \left[(c_V^e)^2 + (c_A^e)^2 \right] \left[(c_V^\mu)^2 + (c_A^\mu)^2 \right] (1 + \cos^2 \theta) + 2c_V^e c_A^e c_V^\mu c_A^\mu \cos \theta \right\}$$



Z production cross section

- And the total cross section:

$$\sigma(e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-) = \frac{1}{192\pi} \frac{g_Z^4 s}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \left[(c_V^e)^2 + (c_A^e)^2 \right] \left[(c_V^\mu)^2 + (c_A^\mu)^2 \right]$$

- This can be expressed in terms of the partial decay rates of the Z boson:

$$\Gamma_{ee} = \frac{g_Z^2 m_Z}{48\pi} \left[(c_V^e)^2 + (c_A^e)^2 \right] \quad \Gamma_{\mu\mu} = \frac{g_Z^2 m_Z}{48\pi} \left[(c_V^\mu)^2 + (c_A^\mu)^2 \right]$$

$$\sigma(e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-) = \frac{12\pi s}{m_Z^2} \frac{\Gamma_{ee} \Gamma_{\mu\mu}}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2}$$



Z production cross section

- The cross section calculation around the resonance would only need the Z boson contribution

$$\sigma(e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-) = \frac{12\pi s}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{\mu\mu}}{(s - m_Z^2)^2 + m_Z^2\Gamma_Z^2}$$

- The cross section for other final state fermions can be obtained replacing the partial decay rates into muons with the corresponding partial decay rates: Γ_{ff}
- The maximum value for the cross section:

$$\sigma_{ff}^0 = \frac{12\pi}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}$$



Z production cross section

- The cross section calculation around the resonance would only need the Z boson contribution

$$\sigma(e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-) = \frac{12\pi s}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{\mu\mu}}{(s - m_Z^2)^2 + m_Z^2\Gamma_Z^2}$$

- The maximum value for the cross section:

$$\sigma_{\text{ff}}^0 = \frac{12\pi}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{\text{ff}}}{\Gamma_Z^2}$$

- The cross section falls to half the maximum value at:

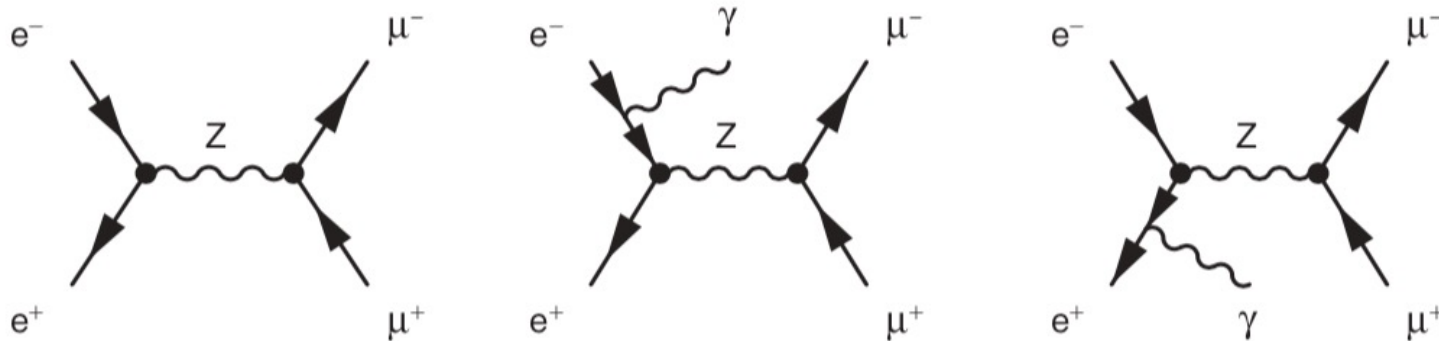
$$\sqrt{s} = m_Z \pm \Gamma_Z/2$$

- So the total decay rate is the width of the cross-section distribution



Z mass measurement

- In practice, initial state radiation diagrams should be included
- This reduces the centre of mass energy of the collision and smears out the resonance



- At LEP, the mass of the Z boson and its decay width were measured by measuring the cross section for $e^+e^- \rightarrow Z \rightarrow q\bar{q}$ at different centre-of-mass energies



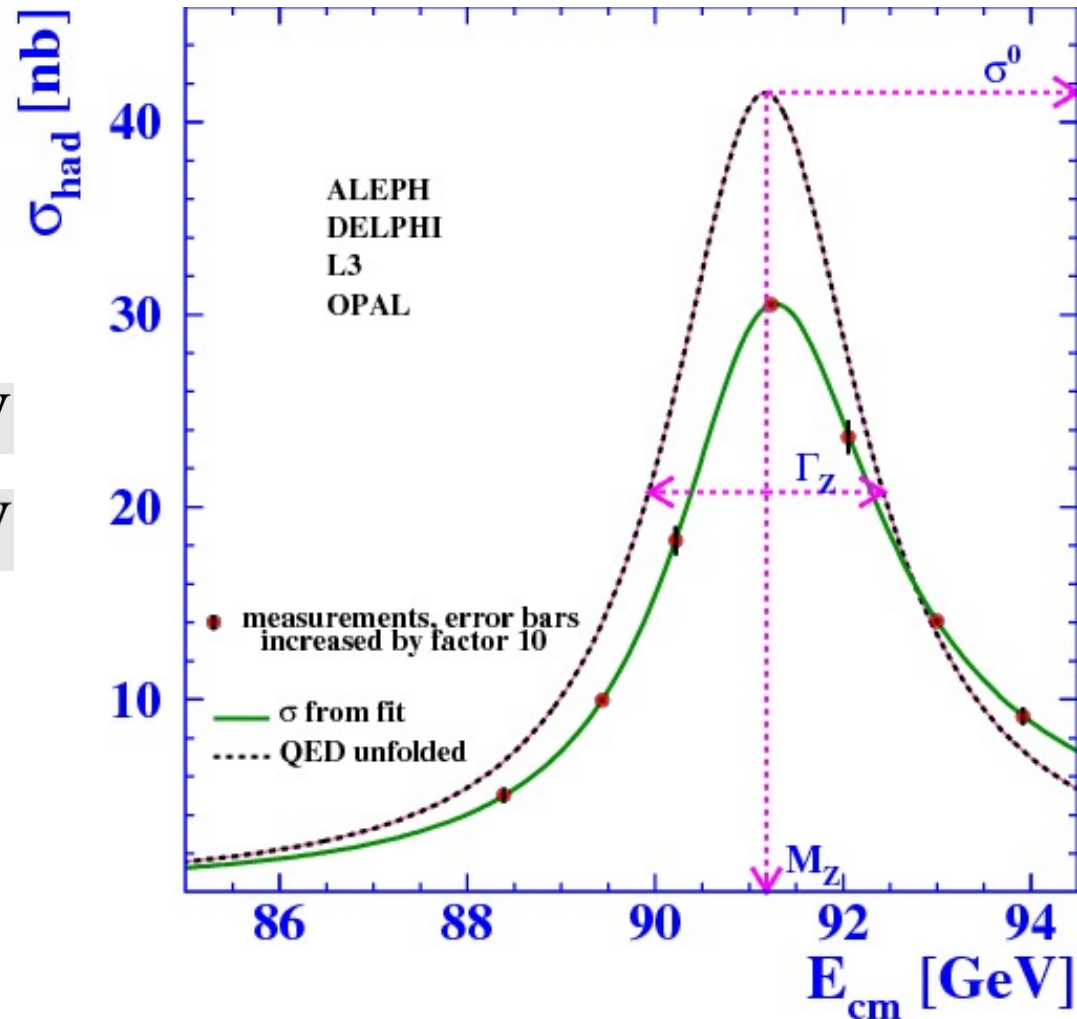
Z mass measurement

- Mass of the Z boson
- Total decay width
- Peak cross-section

$$m_Z = 91.1875 \pm 0.0021 \text{ GeV}$$

$$\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$$

- Measured with high level precision





Number of generations

- The Z boson couples to all fermions
- The total decay width has contributions from all fermions

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\text{hadrons}} + \Gamma_{\nu_e\nu_e} + \Gamma_{\nu_\mu\nu_\mu} + \Gamma_{\nu_\tau\nu_\tau}$$

- If there were more generations, the decay width of the Z boson would be affected by it



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$$\Gamma_Z = 3\Gamma_{\ell\ell} + \Gamma_{\text{hadrons}} + N_\nu\Gamma_{\nu\nu}$$

$$N_\nu = \frac{(\Gamma_Z - 3\Gamma_{\ell\ell} - \Gamma_{\text{hadrons}})}{\Gamma_{\nu\nu}^{\text{SM}}}$$



Number of generations

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$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\text{hadrons}} + \Gamma_{\nu_e\nu_e} + \Gamma_{\nu_\mu\nu_\mu} + \Gamma_{\nu_\tau\nu_\tau}$$

- If there were more generations, the decay width of the Z boson would be affected by it
- The decay width to fermions can be calculated using the vertex coupling:

$$\Gamma(Z \rightarrow f\bar{f}) = \frac{g_Z^2 m_Z}{48\pi} (c_V^2 + c_A^2)$$

- In particular for neutrinos:

$$\Gamma(Z \rightarrow \nu_e \bar{\nu}_e) = \frac{g_Z^2 m_Z}{48\pi} \left(\frac{1}{4} + \frac{1}{4} \right)$$



Number of generations

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- The total decay width has contributions from all fermions

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\text{hadrons}} + \Gamma_{\nu_e\nu_e} + \Gamma_{\nu_\mu\nu_\mu} + \Gamma_{\nu_\tau\nu_\tau}$$

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measured calculated



Number of generations

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$$N_\nu = \frac{(\Gamma_Z - 3\Gamma_{\ell\ell} - \Gamma_{\text{hadrons}})}{\Gamma_{\nu\nu}^{\text{SM}}}$$

- And from the cross section peak of the Z resonance:

$$\sigma^0(e^+e^- \rightarrow Z \rightarrow f\bar{f}) = \frac{12\pi}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}$$



Number of generations

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- The total decay width has contributions from all fermions

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Number of generations

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- The total decay width has contributions from all fermions

$$N_\nu = \frac{(\Gamma_Z - 3\Gamma_{\ell\ell} - \Gamma_{\text{hadrons}})}{\Gamma_{\nu\nu}^{\text{SM}}}$$

$$\Gamma(Z \rightarrow \nu_e \bar{\nu}_e) = \frac{g_Z^2 m_Z}{48\pi} \left(\frac{1}{4} + \frac{1}{4} \right) = 167 \text{ MeV}$$

$$N_\nu = 2.9840 \pm 0.0082$$

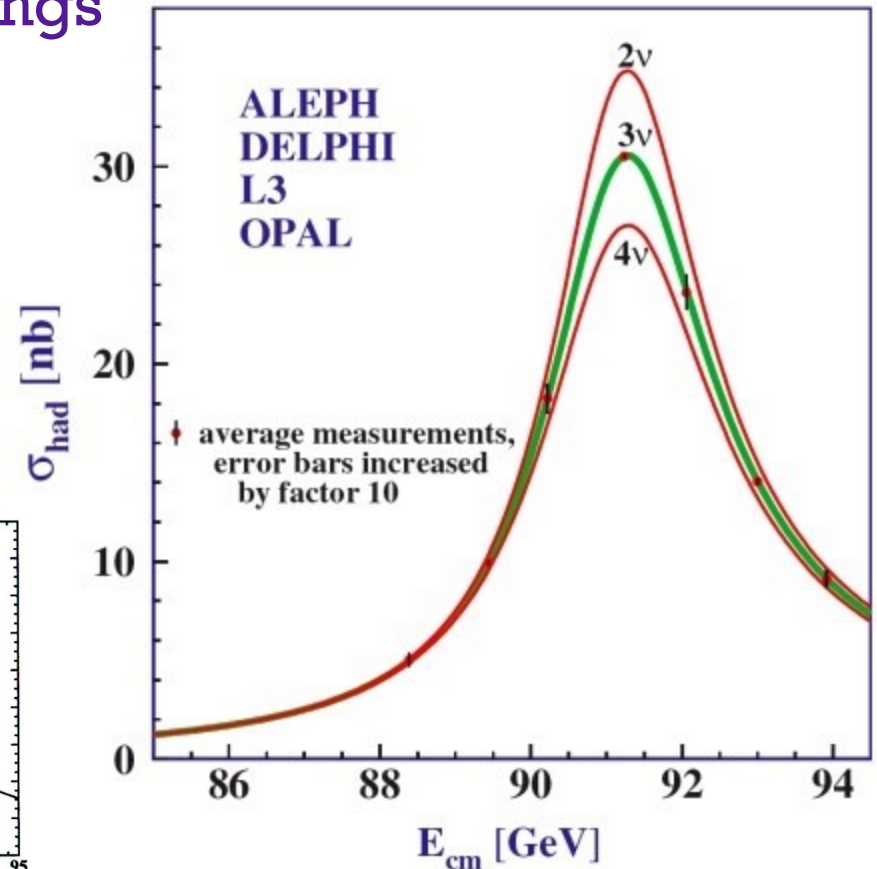
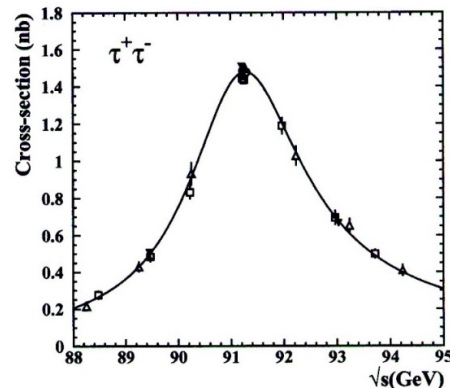
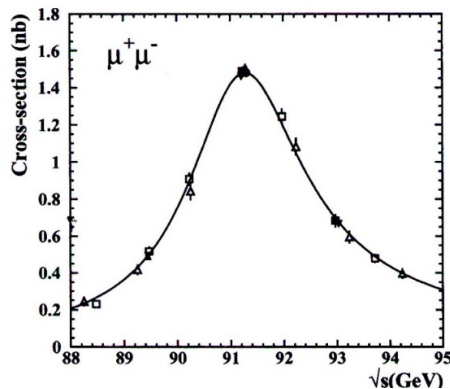
Γ_Z	2495.2 ± 2.3 MeV
Γ_{ee}	83.91 ± 0.12 MeV
$\Gamma_{\mu\mu}$	83.99 ± 0.18 MeV
$\Gamma_{\tau\tau}$	84.08 ± 0.22 MeV
Γ_{qq}	1744.4 ± 2.0 MeV
$N_\nu \Gamma_{\nu\nu}$	499.0 ± 1.5 MeV



Number of generations

- Most likely, only 3 generations of fermions
- Universality of lepton couplings
- Calculated cross section assumes 3 colours

$$N_v = 2.9840 \pm 0.0082$$





Weak mixing angle

- The weak mixing angle relates all couplings in the EW model and is therefore a fundamental parameter
- It can be measured through the ratio between vector and axial couplings:

$$\frac{c_V}{c_A} = 1 - \frac{2Q \sin^2 \theta_W}{I_W^{(3)}}$$

For charged leptons:

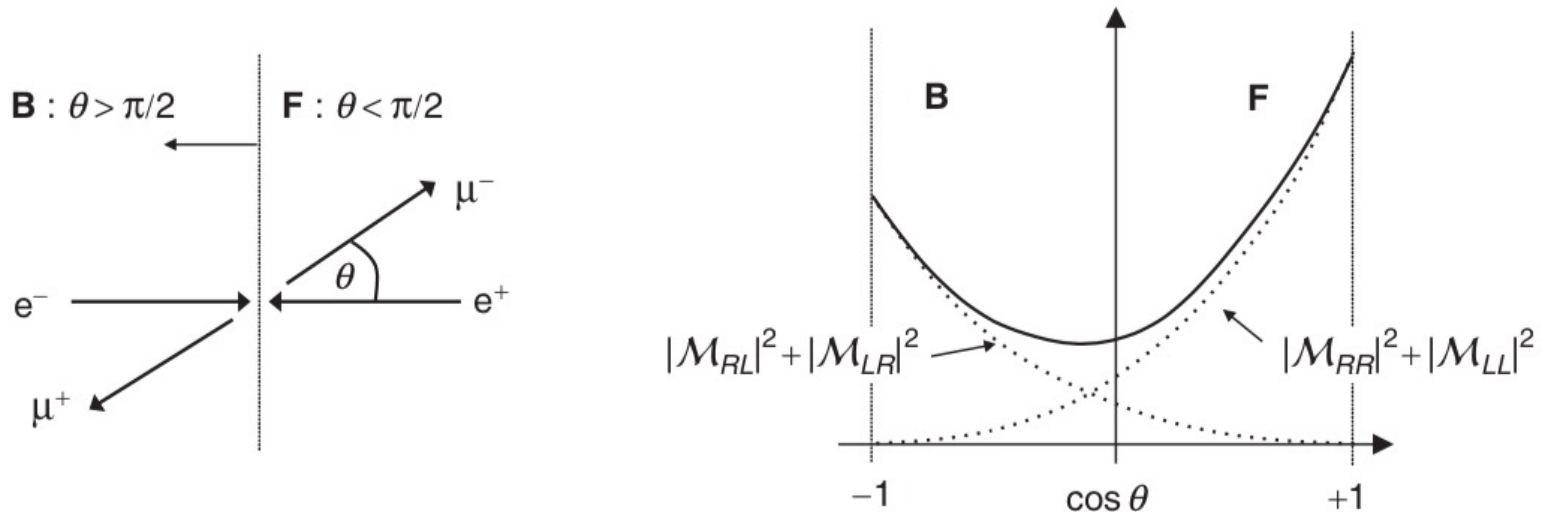
$$\frac{c_V^\ell}{c_A^\ell} = 1 - 4 \sin^2 \theta_W$$

- At LEP this could be measured by measuring the forward-backward asymmetry of leptons produced



Weak mixing angle

- At LEP this could be measured by measuring the forward-backward asymmetry of leptons produced



- The cross section has the form:

$$\frac{d\sigma}{d\Omega} \propto a(1 + \cos^2 \theta) + 2b \cos \theta$$

if the couplings were the same, b would be zero and the angular distribution would have the same form as in QED



Weak mixing angle

- At LEP this could be measured by measuring the forward-backward asymmetry of leptons produced

$$A_{\text{FB}}^{\ell} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

where the forward and backward cross sections can be obtained integrating in different hemispheres:

$$\sigma_F \equiv 2\pi \int_0^1 \frac{d\sigma}{d\Omega} d(\cos \theta) \quad \sigma_B \equiv 2\pi \int_{-1}^0 \frac{d\sigma}{d\Omega} d(\cos \theta)$$

$$\sigma_F \propto \int_0^1 [a(1 + \cos^2 \theta) + 2b \cos \theta] d(\cos \theta) = \int_0^1 [a(1 + x^2) + 2bx] dx = \left(\frac{4}{3}a + b\right)$$

$$\sigma_B \propto \int_{-1}^0 [a(1 + \cos^2 \theta) + 2b \cos \theta] d(\cos \theta) = \int_{-1}^0 [a(1 + x^2) + 2bx] dx = \left(\frac{4}{3}a - b\right)$$



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where the forward and backward cross sections can be obtained integrating in different hemispheres:

$$A_{\text{FB}} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3b}{4a}$$

$$a = [(c_L^e)^2 + (c_R^e)^2][(c_L^{\mu})^2 + (c_R^{\mu})^2] \quad b = [(c_L^e)^2 - (c_R^e)^2][(c_L^{\mu})^2 - (c_R^{\mu})^2]$$

$$A_{\text{FB}} = \frac{3}{4} \left[\frac{(c_L^e)^2 - (c_R^e)^2}{(c_L^e)^2 + (c_R^e)^2} \right] \cdot \left[\frac{(c_L^{\mu})^2 - (c_R^{\mu})^2}{(c_L^{\mu})^2 + (c_R^{\mu})^2} \right]$$



Weak mixing angle

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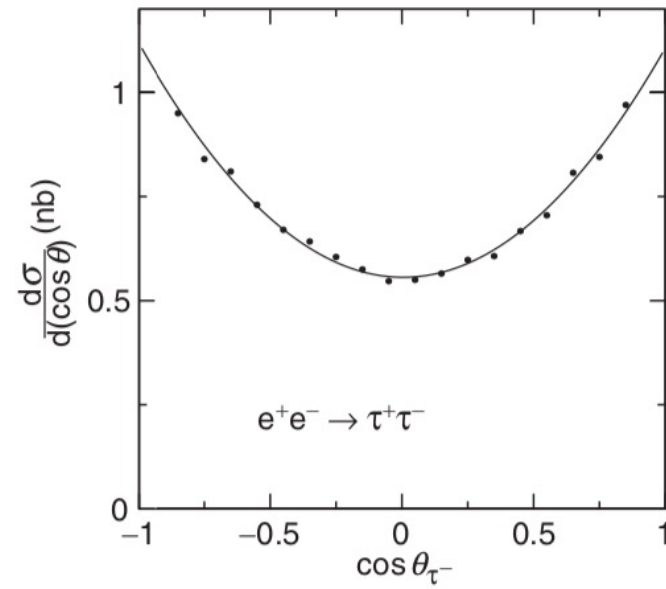
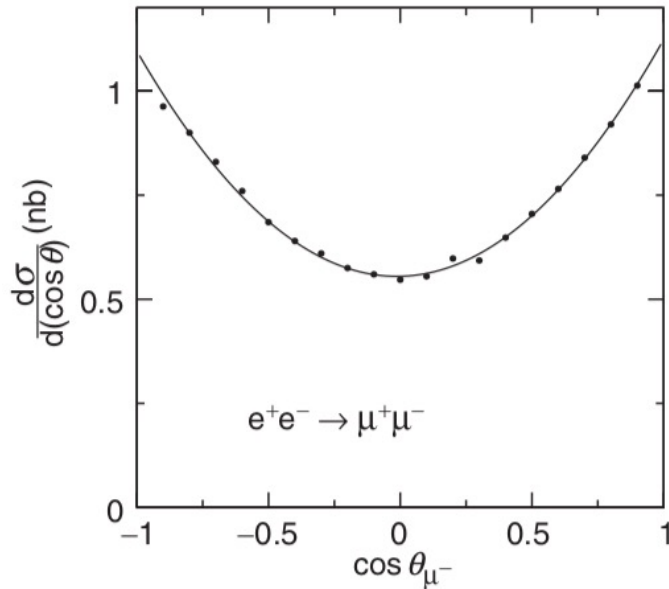
- In terms of the coupling constants: $A_{\text{FB}} = \frac{3}{4} \mathcal{A}_f \mathcal{A}_{f'}$

$$\mathcal{A}_f = \frac{2c_V^f c_A^f}{(c_V^f)^2 + (c_A^f)^2}$$



Weak mixing angle

- At LEP the cleanest way of measuring A_{FB} is in lepton final states



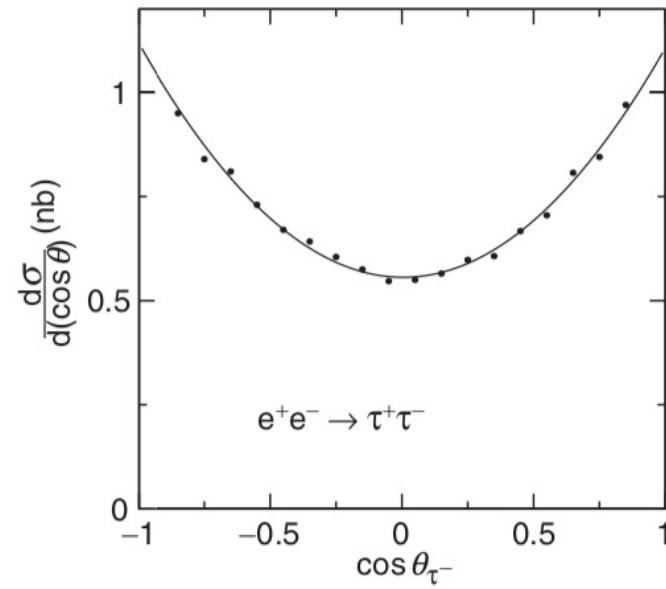
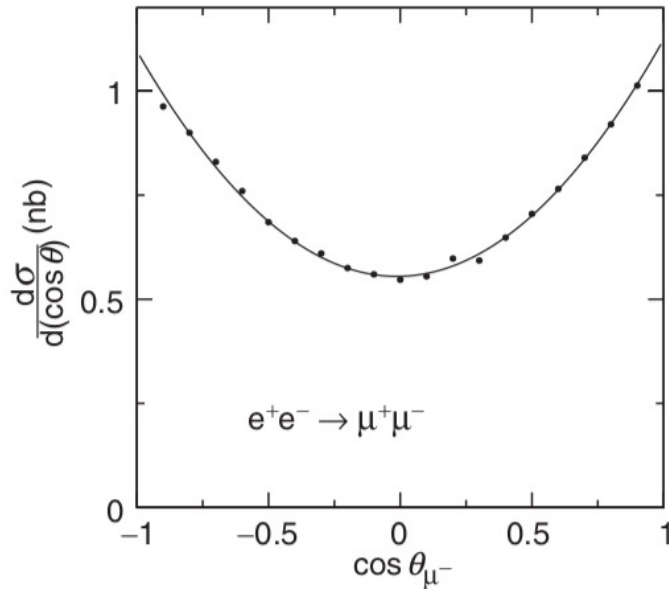
$$A_{FB}^e = 0.0145 \pm 0.0025 \quad A_{FB}^\mu = 0.0169 \pm 0.0013 \quad A_{FB}^\tau = 0.0188 \pm 0.0017$$

$$A_{FB}^e = \frac{3}{4} \mathcal{A}_e^2 \quad A_{FB}^\mu = \frac{3}{4} \mathcal{A}_e \mathcal{A}_\mu \quad A_{FB}^\tau = \frac{3}{4} \mathcal{A}_e \mathcal{A}_\tau$$



Weak mixing angle

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$$A_{FB}^e = 0.0145 \pm 0.0025 \quad A_{FB}^\mu = 0.0169 \pm 0.0013 \quad A_{FB}^\tau = 0.0188 \pm 0.0017$$

$$\mathcal{A}_e = 0.1514 \pm 0.0019 \quad \mathcal{A}_\mu = 0.1456 \pm 0.0091 \quad \mathcal{A}_\tau = 0.1449 \pm 0.0040$$



Weak mixing angle

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$$A_{FB}^e = 0.0145 \pm 0.0025 \quad A_{FB}^\mu = 0.0169 \pm 0.0013 \quad A_{FB}^\tau = 0.0188 \pm 0.0017$$

$$\mathcal{A}_e = 0.1514 \pm 0.0019 \quad \mathcal{A}_\mu = 0.1456 \pm 0.0091 \quad \mathcal{A}_\tau = 0.1449 \pm 0.0040$$

$$\mathcal{A} = \frac{2c_V/c_A}{1 + (c_V/c_A)^2}$$

$$\frac{c_V}{c_A} = 1 - 4 \sin^2 \theta_W$$

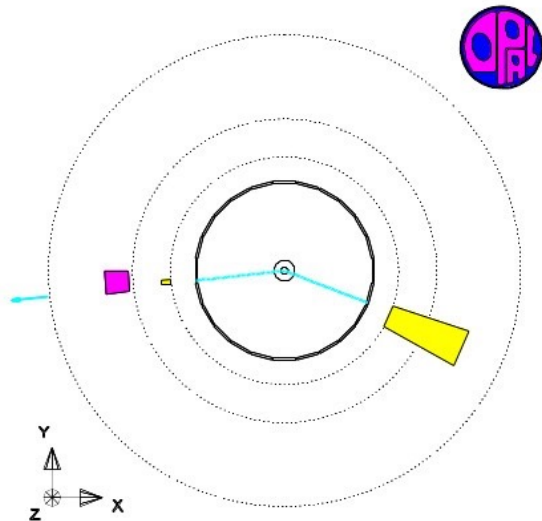
- Combining all measurements:

$$\sin^2 \theta_W = 0.23146 \pm 0.00012$$



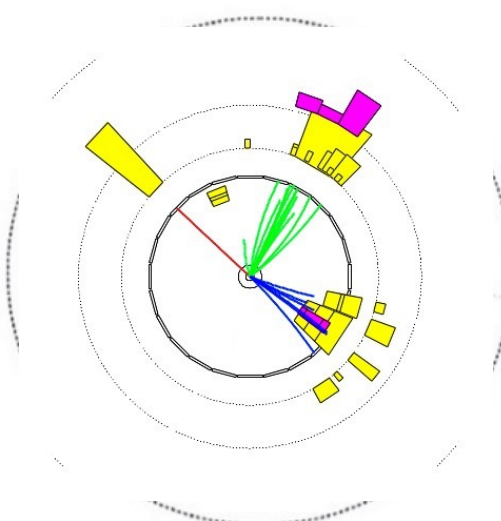
W bosons at LEP

- W bosons were produced in pairs at LEP



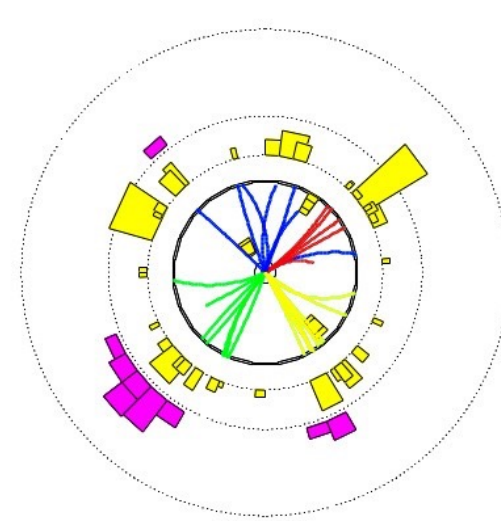
$$W^+W^- \rightarrow e^-\bar{\nu}_e\mu^+\nu_\mu$$

leptonic



$$W^+W^- \rightarrow e^-\bar{\nu}_eq_1\bar{q}_2$$

semi-leptonic



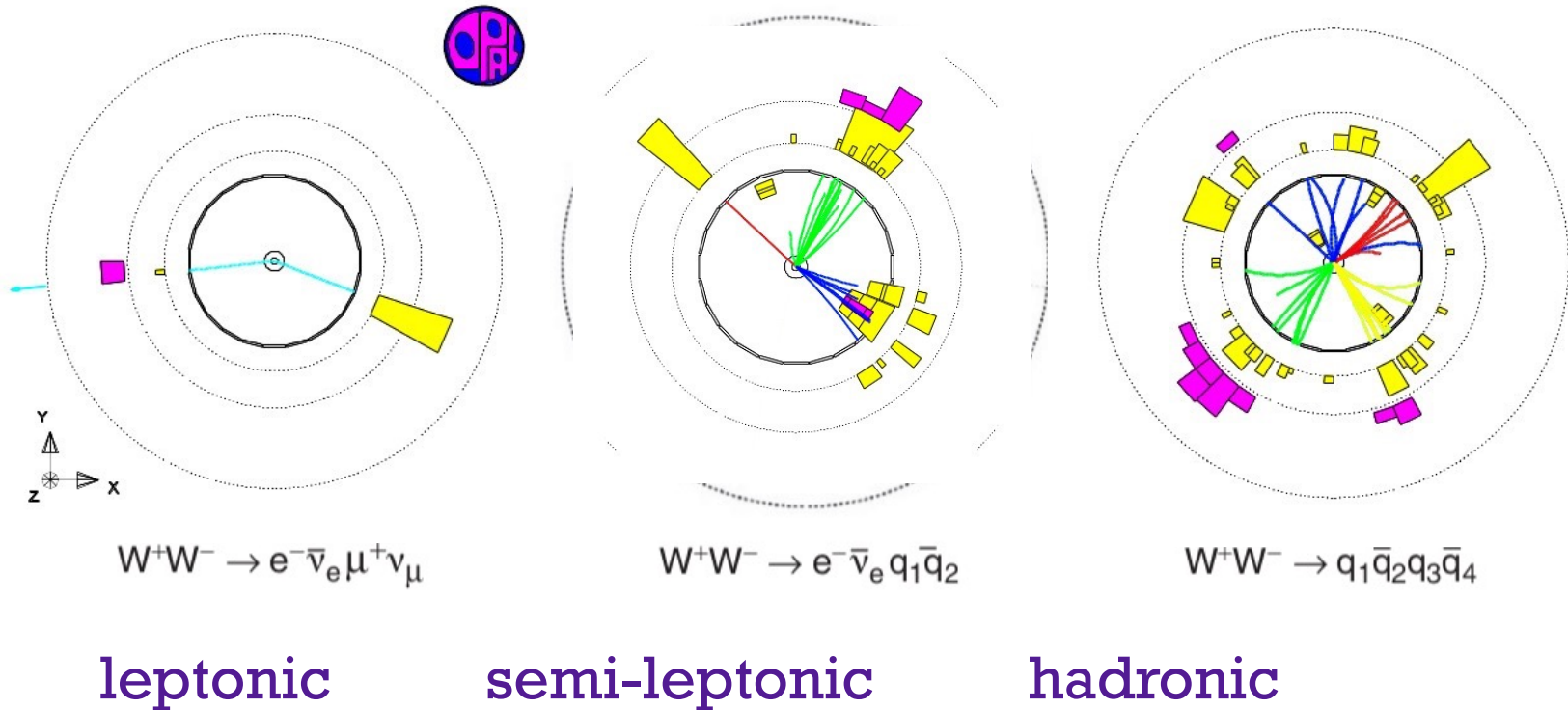
$$W^+W^- \rightarrow q_1\bar{q}_2q_3\bar{q}_4$$

hadronic



W bosons at LEP

- W bosons were produced in pairs at LEP



- We know the cross section is proportional to the decay rates, so we can measure the branching fractions



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$$N_{qqqq} \propto [BR(W \rightarrow q\bar{q}')]^2 \quad N_{\ell\nu\ell\nu} \propto [1 - BR(W \rightarrow q\bar{q}')]^2$$

$$BR(W \rightarrow q\bar{q}') = 67.41 \pm 0.27\%$$



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$$BR(W \rightarrow q\bar{q}') = 67.41 \pm 0.27\%$$

- From the EW Model:

$$\Gamma(W^- \rightarrow e^- \bar{\nu}_e) = \frac{g_W^2 m_W}{48\pi}$$

$$\Gamma(W^- \rightarrow e^- \bar{\nu}_e) = \Gamma(W^- \rightarrow \mu^- \bar{\nu}_\mu) = \Gamma(W^- \rightarrow \tau^- \bar{\nu}_\tau)$$

$$\Gamma(W^- \rightarrow d\bar{u}) = 3|V_{ud}|^2 \Gamma_{ev}$$

$$\Gamma(W^- \rightarrow d\bar{c}) = 3|V_{cd}|^2 \Gamma_{ev}$$

$$\Gamma(W^- \rightarrow s\bar{u}) = 3|V_{us}|^2 \Gamma_{ev}$$

$$\Gamma(W^- \rightarrow s\bar{c}) = 3|V_{cs}|^2 \Gamma_{ev}$$

$$\Gamma(W^- \rightarrow b\bar{u}) = 3|V_{ub}|^2 \Gamma_{ev}$$

$$\Gamma(W^- \rightarrow b\bar{c}) = 3|V_{cb}|^2 \Gamma_{ev}$$



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$$BR(W \rightarrow q\bar{q}') = 67.41 \pm 0.27\%$$

- From the EW Model:

$$\Gamma(W^- \rightarrow q\bar{q}') = 6 \Gamma(W^- \rightarrow e^- \bar{\nu}_e)$$

$$\kappa_{QCD} = \left[1 + \frac{\alpha_S(m_W)}{\pi} \right] \approx 1.038$$

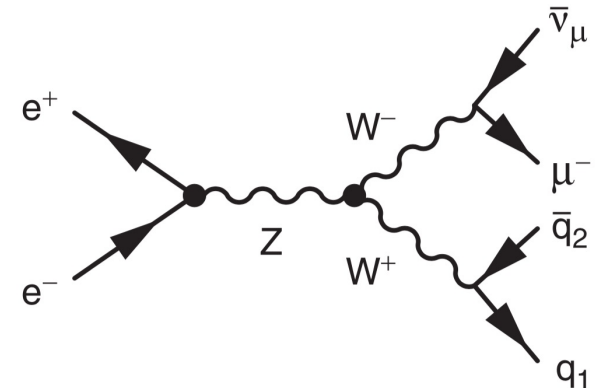
$$\Gamma_W = (3 + 6 \kappa_{QCD}) \Gamma(W^- \rightarrow e^- \bar{\nu}_e) \approx 9.2 \times \frac{g_W^2 m_W}{48\pi} = 2.1 \text{ GeV}$$

$$BR(W \rightarrow q\bar{q}') = \frac{6 \kappa_{QCD}}{3 + 6 \kappa_{QCD}} = 67.5\%$$



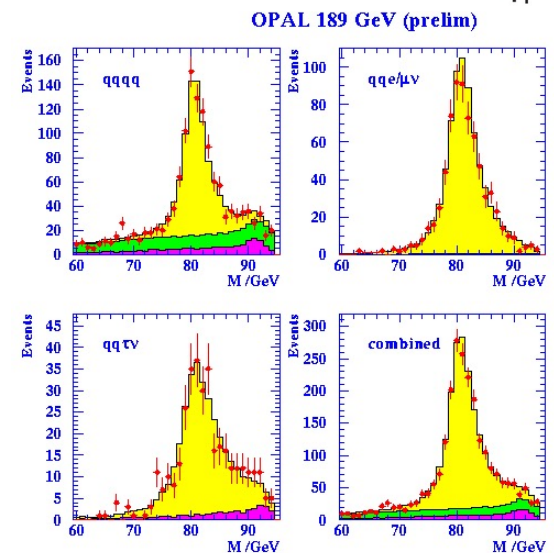
W boson mass and width

- The W-pair production at LEP is not a resonant process, like the Z boson production
- The mass and width can be obtained through direct reconstruction of the invariant masses of the W decays



$$m_W = 80.376 \pm 0.033 \text{ GeV}$$

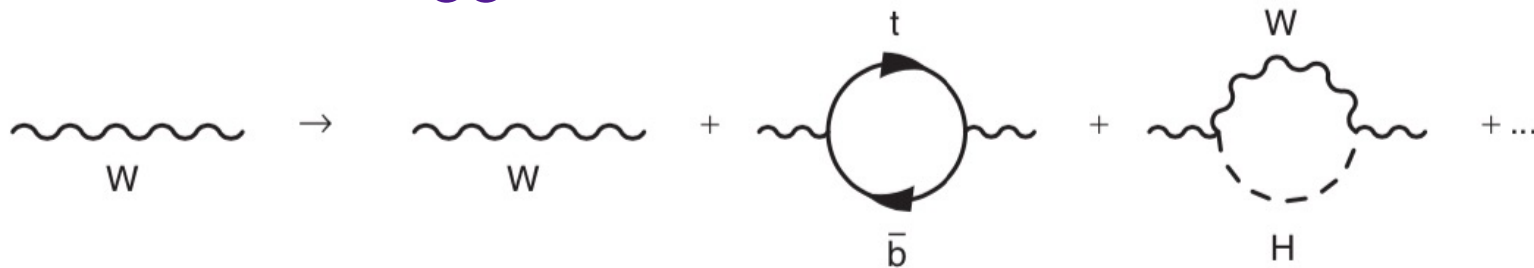
$$\Gamma_W = 2.195 \pm 0.083 \text{ GeV}$$





W mass loop corrections

- When comparing to the precise measurements from LEP, higher order corrections must be taken into account
- For example, the W mass has corrections related to the top quark and the Higgs boson



$$m_W = m_W^0 + a m_t^2 + b \ln \left(\frac{m_H}{m_W} \right) + \dots$$

- The measurements from LEP for the EW parameters, together with the quantum loop effects, predict a top quark mass of 175 ± 11 GeV...



The Top quark

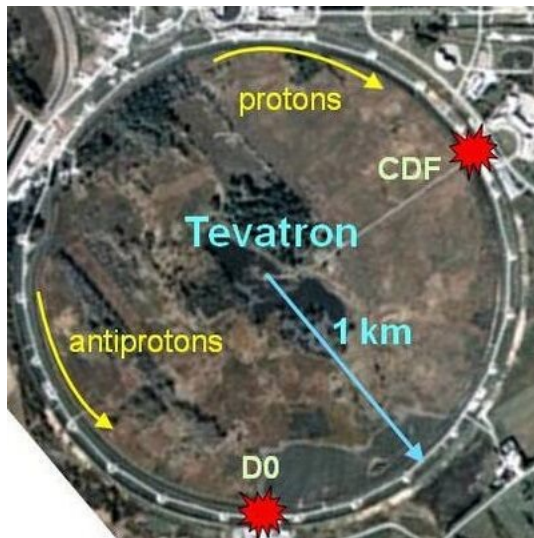
- The top quark is the “heaviest” fundamental particle we know
- It could not be observed at LEP, it was discovered at the Tevatron in 1994



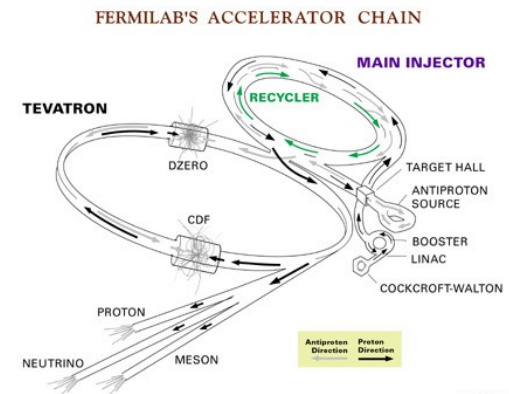
The TeVatron

- It is possible to achieve higher centre of mass energies with hadron colliders than with e^+e^- colliders
- They are central in the production of new heavy particles
- Underlying process: parton-parton scattering

TeVatron (1987-2010)



- Located at Fermilab, Chicago, USA
- $p\bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV
- Two main experiments: CDF and D0

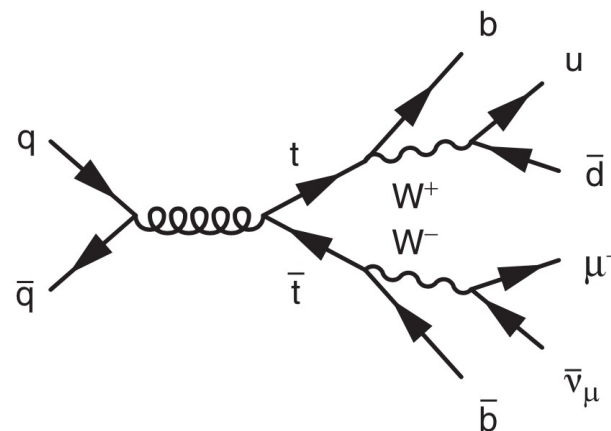




The Top quark

- The top quark is the “heaviest” fundamental particle we know
- It could not be observed at LEP, it was discovered at the Tevatron in 1994
- It has a short lifetime and decays before hadronisation
- It decays almost exclusively into a W boson and a b quark
- At hadron colliders it is easier to look for the semi-leptonic channel:

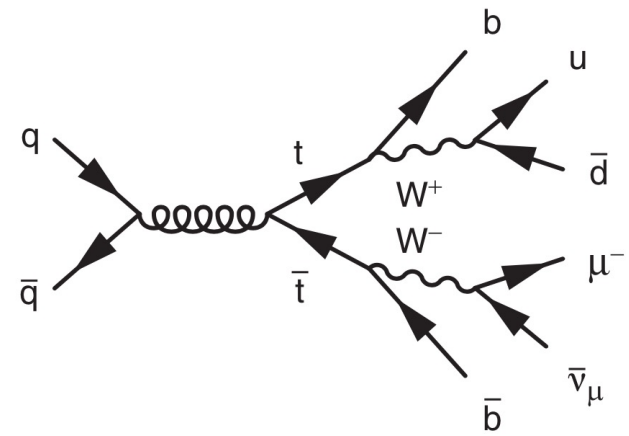
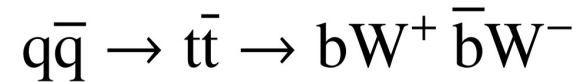
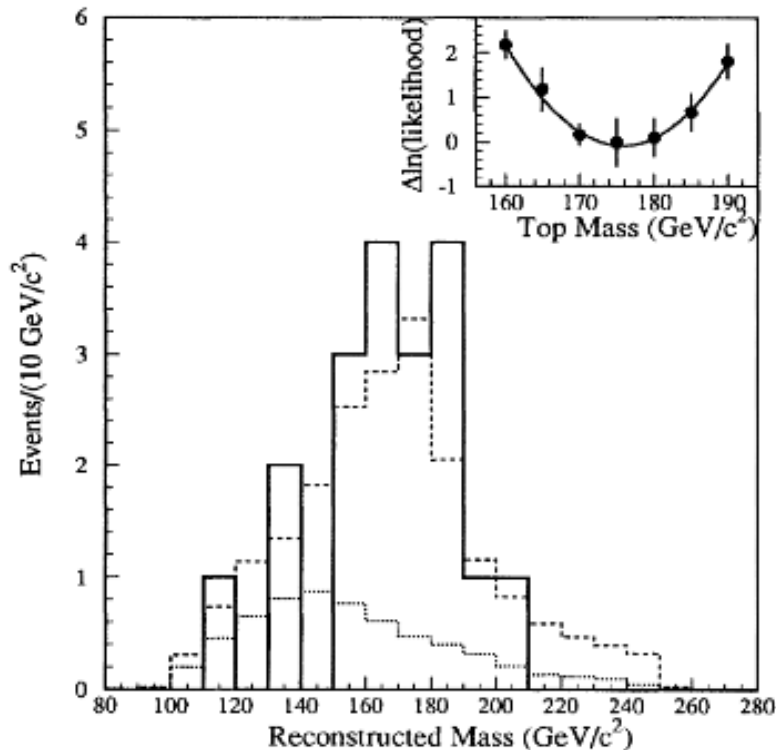
$$q\bar{q} \rightarrow t\bar{t} \rightarrow bW^+ \bar{b}W^-$$





The Top quark

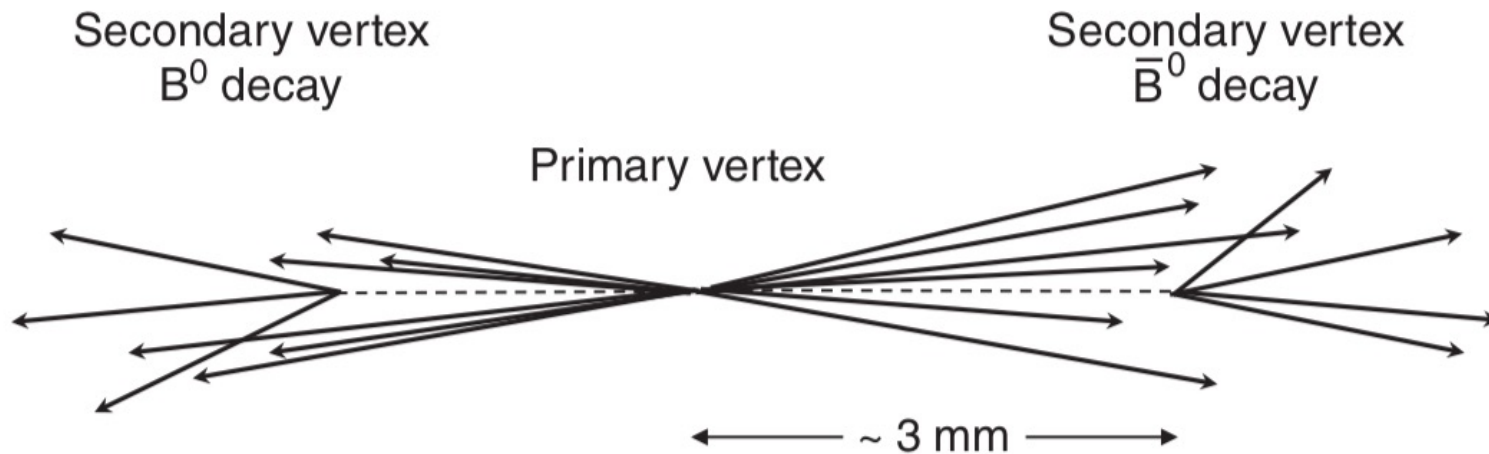
- At hadron colliders it is easier to look for the semi-leptonic channel
- First observation of top quark (CDF)





The Top quark

- To measure the top quark mass we reconstruct the invariant mass of its decay products (as with the W)
- Need to identify the b-jet: b-tagging
- b quarks have a longer lifetime than the other quarks
- The b quark travels some distance from the interaction point before decaying: secondary vertex





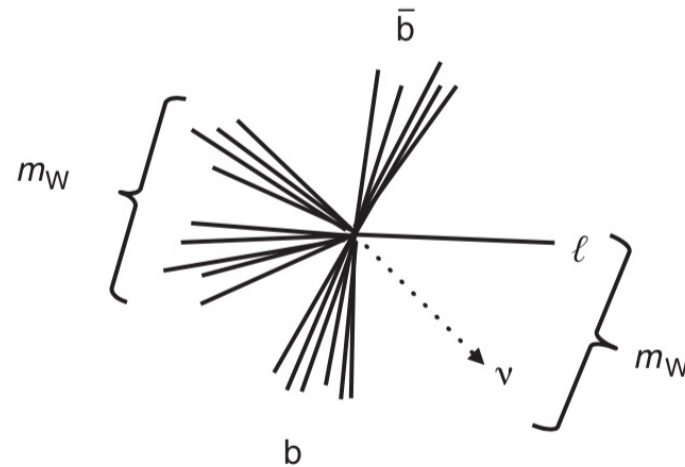
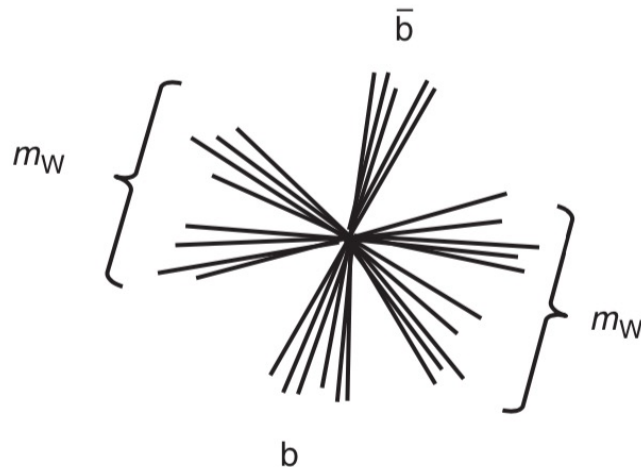
The Top quark

- To measure the top quark mass we reconstruct the invariant mass of its decay products (as with the W)

$$t\bar{t} \rightarrow (bW^+)(\bar{b}W^-) \rightarrow (b q_1 \bar{q}_2) (\bar{b} q_3 \bar{q}_4) \rightarrow 6 \text{ jets},$$

$$t\bar{t} \rightarrow (bW^+)(\bar{b}W^-) \rightarrow (b q_1 \bar{q}_2) (\bar{b} \ell^- \bar{\nu}_\ell) \rightarrow 4 \text{ jets} + 1, \text{ charged lepton} + 1 \nu$$

$$t\bar{t} \rightarrow (bW^+)(\bar{b}W^-) \rightarrow (b \ell^+ \nu_\ell) (\bar{b} \ell'^- \bar{\nu}_{\ell'}) \rightarrow 2 \text{ jets} + 2 \text{ charged leptons} + 2 \nu$$



Tevatron average result:

$$m_t = 173.5 \pm 1.0 \text{ GeV}$$



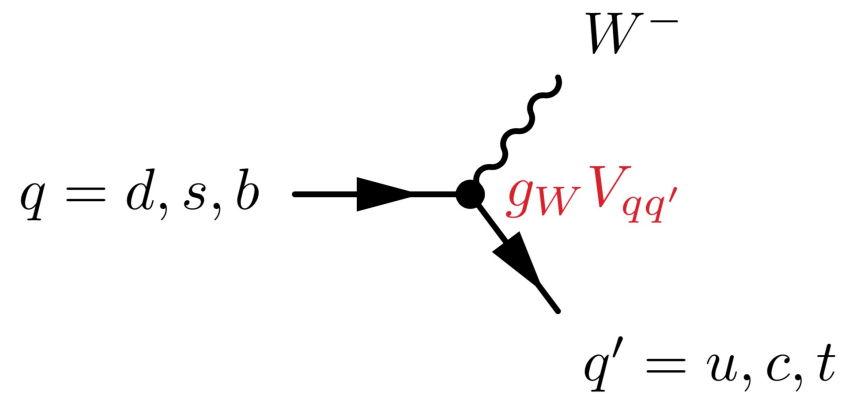
Summary

- The weak current has a V-A structure
- W boson coupling to leptons constant (lepton universality)

$$\alpha_W = \frac{g_W^2}{4\pi} = \frac{8m_W^2 G_F}{4\sqrt{2}\pi} \approx \frac{1}{30}$$

$$G_F = 1.16638 \times 10^{-5} \text{ GeV}^{-2}$$

$$\frac{-ig_W}{\sqrt{2}} \frac{1}{2} \gamma^\mu (1 - \gamma^5)$$



- W boson coupling to quarks depends on CKM matrix
- The W boson always changes quark flavour
- Mixing between different families can occur but less likely
- Parity and Charge conjugation is violated by the charged weak current: only couples to LH particles (RH anti-particles)



Summary

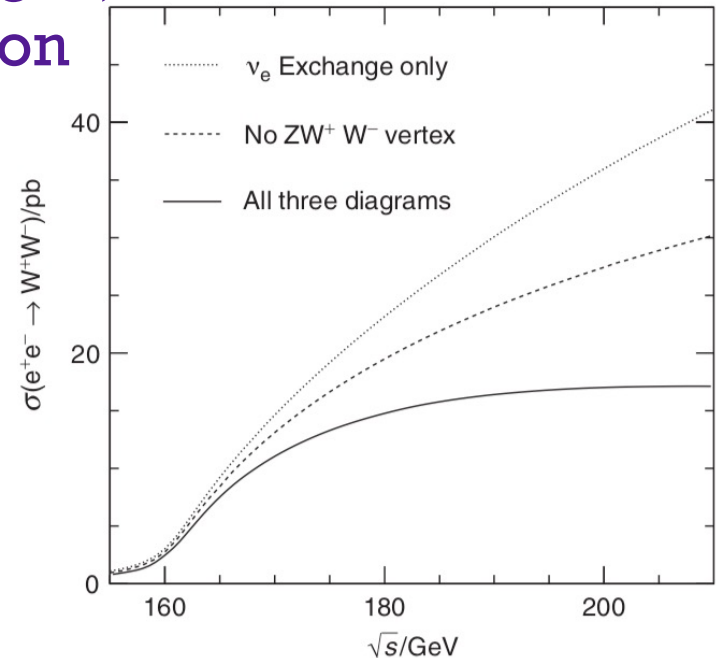
- The charged weak interaction alone does not fully explain W -pair production (cross section diverges)
- Neutral gauge boson needed: Z boson
- Unified $SU(2)_L \times U(1)_Y$ gauge theory
- One new parameter relates all couplings:

$$\sin^2 \theta_W = 0.23146 \pm 0.00012$$

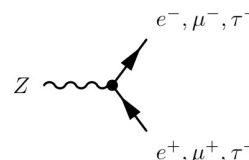
$$\frac{\alpha}{\alpha_W} = \frac{e^2}{g_W^2} = \sin^2 \theta_W \sim 0.23$$

$$g_Z = \frac{g_W}{\cos \theta_W} \equiv \frac{e}{\sin \theta_W \cos \theta_W}$$

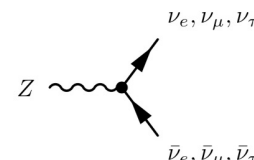
$$-i \frac{1}{2} g_Z \gamma^\mu [c_V - c_A \gamma^5]$$



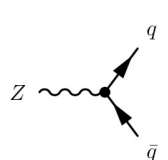
$Z \rightarrow e^+e^-, \mu^+\mu^-, \tau^+\tau^-$



$Z \rightarrow \nu_e \bar{\nu}_e, \nu_\mu \bar{\nu}_\mu, \nu_\tau \bar{\nu}_\tau$



$Z \rightarrow q\bar{q}$





- LEP made some impressive precise tests of the EW Model:

$$m_Z = 91.1875 \pm 0.0021 \text{ GeV}$$

$$m_W = 80.385 \pm 0.015 \text{ GeV}$$

$$\alpha(m_Z^2) = \frac{1}{128.91 \pm 0.02}$$

$$G_F = 1.166\,378\,7(6) \times 10^{-5} \text{ GeV}^{-2}$$

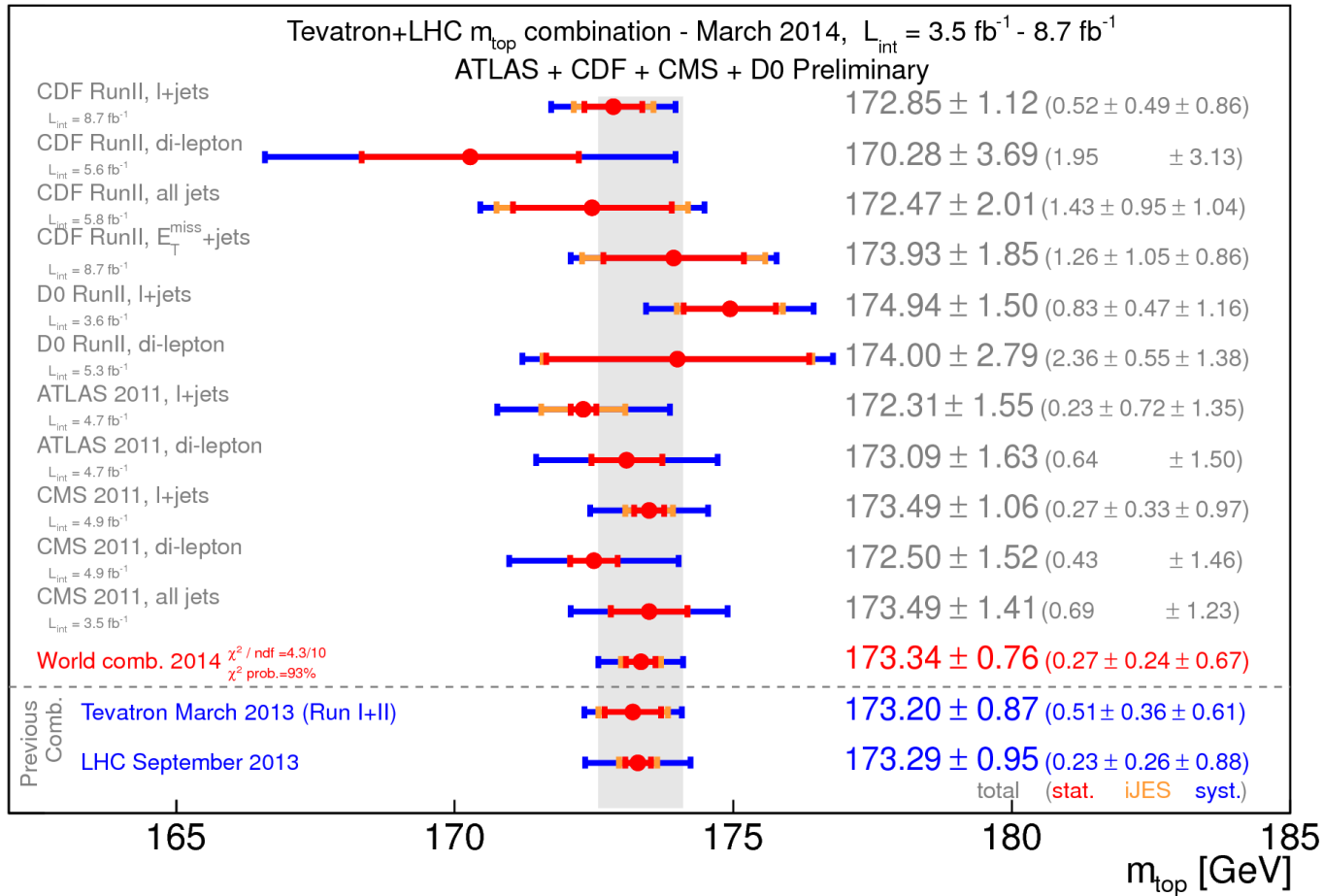
$$\sin^2 \theta_W = 0.23146 \pm 0.00012$$

$$N_\nu = 2.9840 \pm 0.0082$$

- These measurements are consistent with the relations between constants established in the EW model
- The model works! (we are missing a way of giving mass to the W and Z)
- The top quark discovered at the Tevatron completes the spectrum of quarks: $m_t = 173.5 \pm 1.0 \text{ GeV}$
- This value is as predicted by the quantum loop corrections of the EW model – impressive!

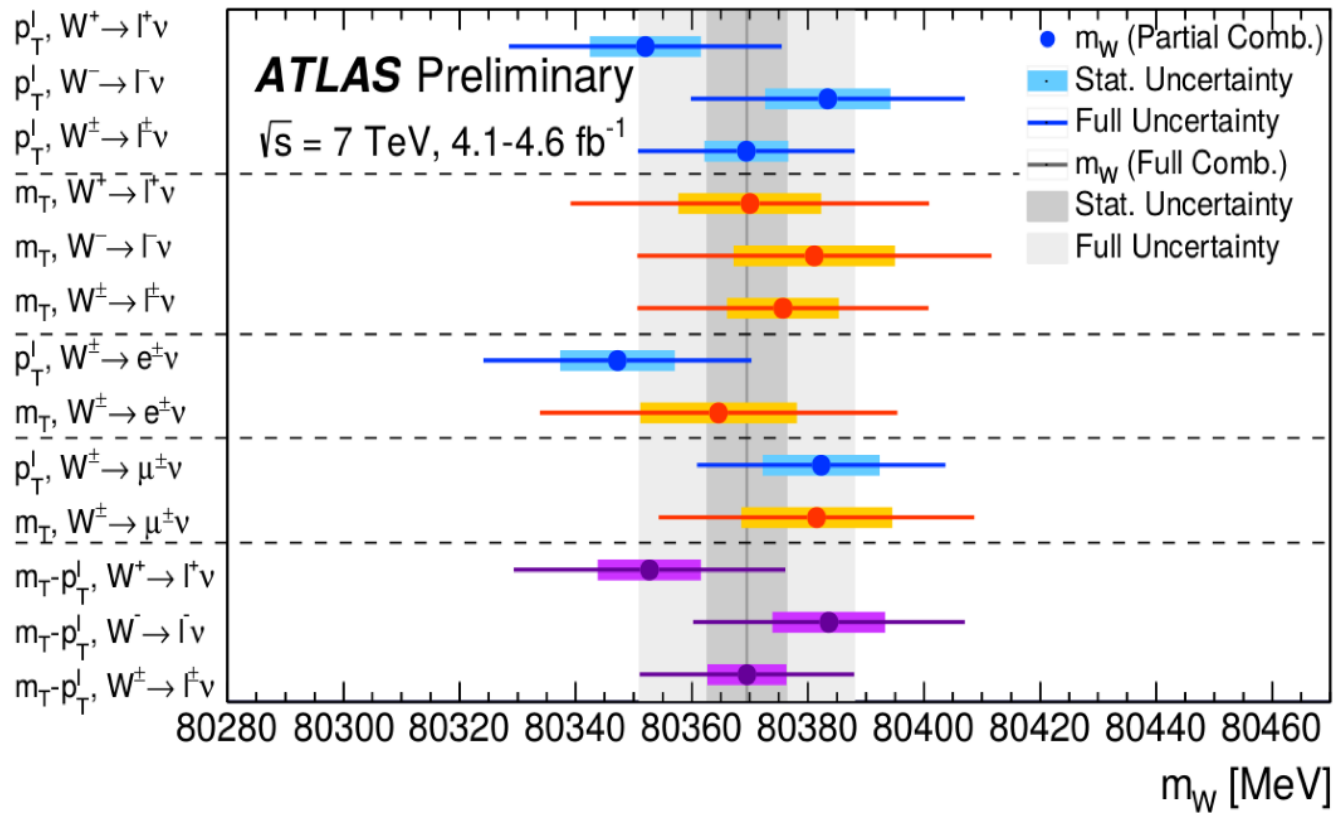


• Top mass world combination



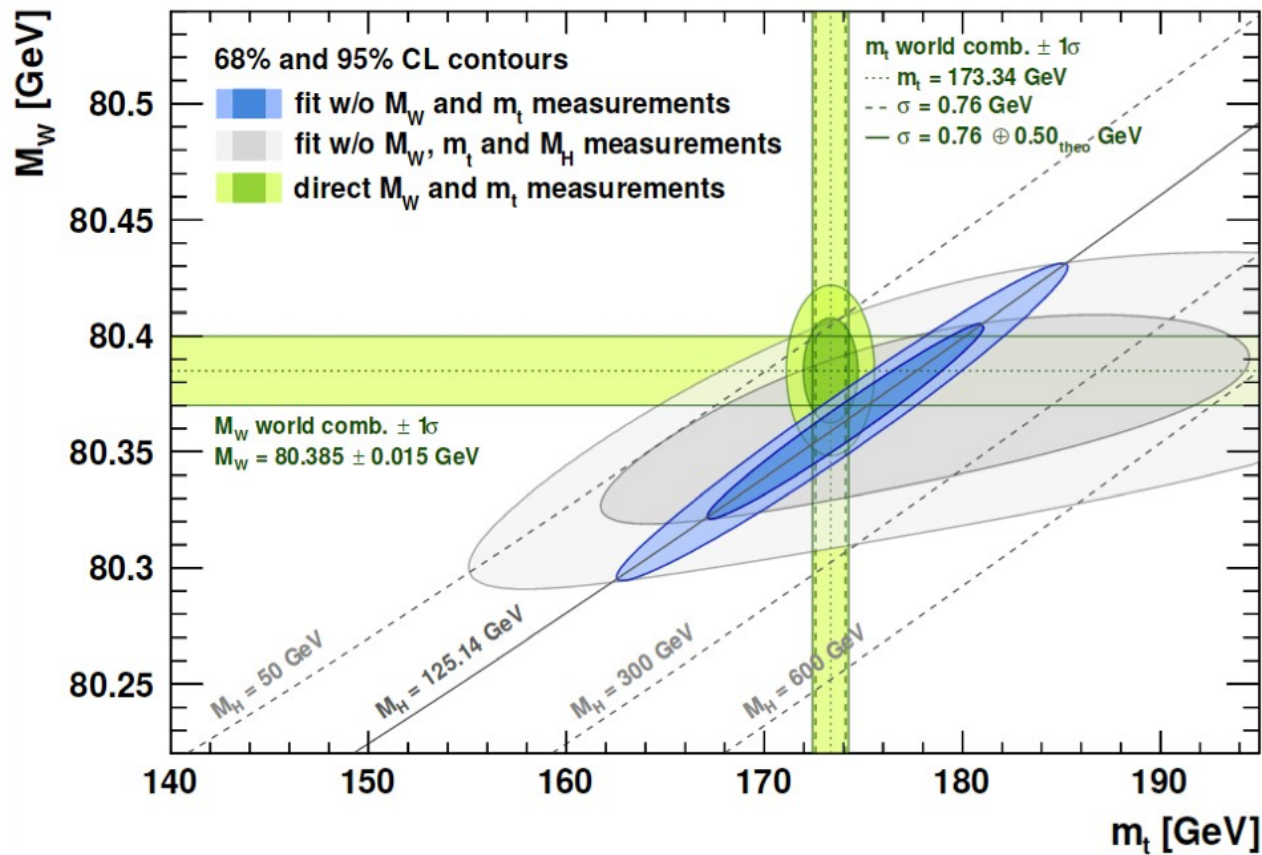


- W boson mass





- Top mass vs. W boson mass



* arXiv:1407.3792