

Módulo de Teoría

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Latin American alliance for
Capacity building in Advanced physics
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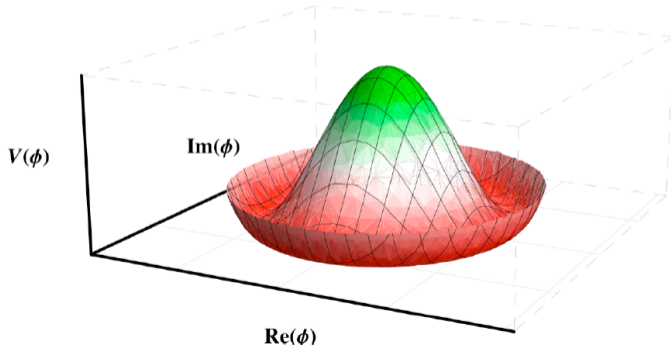


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Ruptura espontánea de simetría

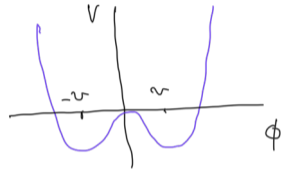
Mecanismo Higgs



Recap Clase 14

* ruptura espontánea de simetría: \mathcal{L} invariante pero el estado de mínima energía no.

Ej. $\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi)$, $V(\phi) = -\frac{1}{2} \mu^2 \phi^2 + \frac{\lambda}{4!} \phi^4$, $\mu^2 > 0$



simetría \mathbb{Z}_2 : $\phi' = -\phi$, $\mathcal{L}' = \mathcal{L}$ rota por $\langle \phi \rangle = v = \sqrt{\frac{6}{\lambda}} \mu$

$\langle \phi \rangle$: valor de expectación de vacío (vev) de ϕ

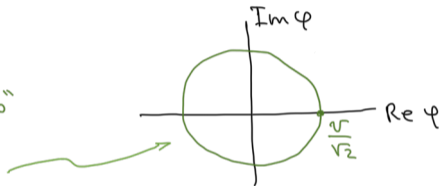
$\phi(x) = \sigma(x) + v$, $\langle \sigma \rangle = 0$

ejercicio $m_\sigma = \sqrt{2} \mu$

* ruptura de simetría global continua \Rightarrow bosones de Goldstone
campos escalares de masa nula

Ej. $\mathcal{L} = \partial_\mu \varphi^* \partial^\mu \varphi - V(\varphi^*, \varphi)$, simetría $U(1)$ global: $\varphi' = e^{i\alpha} \varphi$

$V(\varphi^*, \varphi) = -\mu^2 \varphi^* \varphi + \lambda (\varphi^* \varphi)^2$, $\mu^2 > 0$
potencial del "sombrero mexicano"
mínimo en $|\varphi|^2 = \frac{\mu^2}{2\lambda} = \frac{v^2}{2}$



$$\varphi = e^{i\xi/v} \left(\frac{\eta + v}{\sqrt{2}} \right)$$

$\langle \varphi \rangle = \frac{v}{\sqrt{2}}$ rompe la simetría

$$\langle \xi \rangle = 0, \quad \langle \eta \rangle = 0$$

ejercicio

$$m_\eta = \sqrt{2} \mu, \quad m_\xi = 0$$

ξ es el bosón de Goldstone

* Teo. de Goldstone : por cada simetría interna global continua rota existe un bosón de Goldstone

* simetría local $U(1)$

$$\psi' = e^{ig\chi(x)} \psi$$

derivada covariante $D_\mu \psi = \partial_\mu \psi + ig B_\mu \psi$

$$(D_\mu \psi)' = e^{ig\chi(x)} D_\mu \psi$$

$$B'_\mu = B_\mu - \partial_\mu \chi$$

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu = F'_{\mu\nu} \quad , \quad \mathcal{L}_B = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

(Grupos de simetría en el Modelo Estándar)

$$U(1) = \{ e^{i\alpha} \} \quad (e^{i\alpha})^\dagger e^{i\alpha} = 1$$

$$\text{grupo Abeliano: } e^{i\alpha} e^{i\beta} = e^{i\beta} e^{i\alpha}$$

1 parámetro

1 vector de calibre B_μ

$$SU(2) = \{ \text{matrices } U_{2 \times 2}, U^\dagger U = \mathbb{1}, \det U = 1 \}$$

$$U = \begin{pmatrix} c_0 + ic_3 & c_2 + ic_1 \\ -c_2 + ic_1 & c_0 - ic_3 \end{pmatrix}, \quad \begin{aligned} c_0^2 + c_1^2 + c_2^2 + c_3^2 &= 1 \\ c_0, c_1, c_2, c_3 &\in \mathbb{R} \end{aligned}$$

no Abeliano

3 parámetros

3 vectores de calibre

$$W_\mu^1, W_\mu^2, W_\mu^3$$

$$SU(3) = \{ \text{matrices } U_{3 \times 3}, U^\dagger U = \mathbb{1}, \det U = 1 \}$$

no Abeliano

8 parámetros

8 vectores de calibre

$$G_\mu^1, G_\mu^2, \dots, G_\mu^8$$

ruptura de simetría local $SU(2) \times U(1)$ en el Modelo Estándar

$W_\mu^1, W_\mu^2, W_\mu^3, B_\mu \rightarrow W_\mu^\pm, Z_\mu$ masivos, A_μ masa nula

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp i W_\mu^2)$$

$$Z_\mu = \cos \theta_w W_\mu^3 - \sin \theta_w B_\mu$$

$$A_\mu = \sin \theta_w W_\mu^3 + \cos \theta_w B_\mu$$

SU(2)

$$U = e^{i\alpha_a T_a} = e^{i(\alpha_1 T_1 + \alpha_2 T_2 + \alpha_3 T_3)},$$

$$= \cos \frac{|\vec{\alpha}|}{2} \sigma_0 + i \sin \frac{|\vec{\alpha}|}{2} \hat{n} \cdot \vec{\sigma}$$

$$\vec{\alpha} = (\alpha_1, \alpha_2, \alpha_3) = |\vec{\alpha}| \hat{n}$$

$$[T_1, T_2] = iT_3, \quad [T_2, T_3] = iT_1, \quad [T_3, T_1] = iT_2$$

$T_a = \frac{\sigma_a}{2}$ son los generadores en la representación doblete ó 2

Existen otras representaciones
Ej. rep. adjunta $(T_a)_{bc} = -i\epsilon_{abc}$

$$T_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad T_2 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \quad T_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

matrices de Pauli
 $T_a = \frac{\sigma_a}{2}$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

T_a : generadores

$$[T_a, T_b] = i\epsilon_{abc} T_c$$

transformaciones SU(2) locales

$$\bar{\Phi} = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \text{ doblete}, \quad \bar{\Phi}' = e^{ig\chi_a(x) T_a} \bar{\Phi}, \quad T_a = \frac{\sigma_a}{2}$$

$$D_\mu \bar{\Phi} = \partial_\mu \bar{\Phi} + ig T_a W_\mu^a \bar{\Phi} = \begin{pmatrix} \partial_\mu \psi_1 \\ \partial_\mu \psi_2 \end{pmatrix} + ig \begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & W_\mu^3 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$(D_\mu \bar{\Phi})' = e^{ig\chi_a T_a} D_\mu \bar{\Phi}$$

$$W_\mu^{\prime a} = W_\mu^a - \partial_\mu \chi^a + g \epsilon_{abc} W_\mu^b \chi^c, \quad \chi^c \text{ infinitesimal}$$

$$T_a W_\mu^{\prime a} = U \left(T_b W_\mu^b + \frac{i}{g} U^{-1} \partial_\mu U \right) U^{-1}, \quad U = e^{ig\chi^a T_a}$$

* para cualquier rep. con $[T_a, T_b] = i f_{abc} T_c$, no solo $T_a = \frac{\sigma_a}{2}$

$$\Phi' = e^{ig\chi^a T_a} \Phi, \quad D_\mu \Phi = \partial_\mu \Phi + ig W_\mu^a T_a \Phi$$

$$(D_\mu \Phi)' = e^{ig\chi^a T_a} D_\mu \Phi, \quad W_\mu'^a = W_\mu^a - \partial_\mu \chi^a + g f_{abc} W_\mu^b \chi^c$$

χ^c infinitesimal

* tensor de campo $F_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g f_{abc} W_\mu^b W_\nu^c$

Lagrangiano invariante: $\mathcal{L}_W = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a}$ Yang-Mills SU(2)

* se generaliza a SU(3), etc, $f_{abc} \rightarrow f_{abc}$
 $[T_a, T_b] = i f_{abc} T_c$

Mecanismo Higgs

P.W. Higgs (1964)

F. Englert, R. Brout (1964)

G.S. Guralnik, C.R. Hagen, T.W. Kibble (1964)

Mecanismo Higgs $U(1)$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \varphi)^* (D^\mu \varphi) + \mu^2 (\varphi^* \varphi) - \lambda (\varphi^* \varphi)^2$$

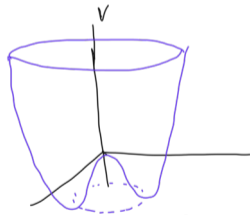
$$D_\mu \varphi = \partial_\mu \varphi + i g B_\mu \varphi, \quad F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

\mathcal{L} es invariante bajo transformaciones $U(1)$ locales

$$\varphi' = e^{i g \chi(x)} \varphi, \quad B'_\mu = B_\mu - \partial_\mu \chi$$

$$V = -\mu^2 \varphi^* \varphi + \lambda (\varphi^* \varphi)^2, \quad \mu^2 > 0$$

$$\langle \varphi \rangle = \frac{v}{\sqrt{2}}, \quad \varphi = e^{i \xi/v} \frac{(v + \eta)}{\sqrt{2}}$$



$$\text{mínimo en } |\varphi|^2 = \frac{v^2}{2} = \frac{\mu^2}{2\lambda}$$

$$V = \lambda v^2 \eta^2 + \lambda v \eta^3 + \frac{\lambda}{4} \eta^4 + \text{const}$$

$$\underbrace{\frac{1}{2} m_\eta^2 \eta^2}_{\frac{1}{2} m_\eta^2 \eta^2} \quad m_\eta = \sqrt{2} \lambda v = \sqrt{2} \mu$$

$$D_\mu \varphi = \frac{1}{\sqrt{2}} e^{i\xi/v} \left[\partial_\mu \eta + i \left(g B_\mu + \frac{1}{v} \partial_\mu \xi \right) (\eta + v) \right]$$

$$(D_\mu \varphi)^\dagger D^\mu \varphi = \frac{1}{2} g^2 (\eta + v)^2 \underbrace{\left(B_\mu + \frac{1}{g v} \partial_\mu \xi \right) \left(B^\mu + \frac{1}{g v} \partial^\mu \xi \right)}_{\text{incluye } B_\mu \partial^\mu \xi} + \frac{1}{2} \partial_\mu \eta \partial^\mu \eta$$

incluye $B_\mu \partial^\mu \xi$ 

! pero se puede hacer una transformaci3n de calibre!

$$B'_\mu = B_\mu - \partial_\mu \chi = B_\mu + \frac{1}{g v} \partial_\mu \xi, \quad \text{escogiendo } \chi = -\frac{1}{g v} \xi$$

$$F'_{\mu\nu} = F_{\mu\nu}$$

$$\begin{aligned}
 (D_\mu \varphi)^* D^\mu \varphi &= \frac{1}{2} g^2 (\eta + v)^2 B'_\mu B'^\mu + \frac{1}{2} \partial_\mu \eta \partial^\mu \eta \\
 &= \frac{1}{2} g^2 v^2 B'_\mu B'^\mu + \dots
 \end{aligned}$$

B'_μ es masivo, tiene $m_B = gv$

$$\begin{aligned}
 * \quad \mathcal{L} &= -\frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} + \frac{1}{2} g^2 v^2 B'_\mu B'^\mu + \frac{1}{2} \partial_\mu \eta \partial^\mu \eta - \frac{1}{2} (2\lambda v^2) \eta^2 \\
 &+ v g^2 \eta B'_\mu B'^\mu + \frac{1}{2} g^2 \eta^2 B'_\mu B'^\mu - \lambda v \eta^3 - \frac{1}{4} \lambda \eta^4 + \text{const.}
 \end{aligned}$$

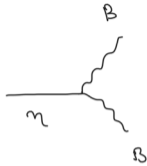
* bosón de Goldstone ξ desaparece de \mathcal{L} , en vez aparece B'_μ masivo

B'_μ se "come" a ξ .

ξ proporciona la polarización longitudinal de B'_μ

$$B'_\mu = B_\mu + \frac{1}{gv} \partial_\mu \xi$$

* interacciones



* Al hacer la transformación $B'_\mu = B_\mu - \partial_\mu \chi$, $\chi = -\frac{1}{g\nu} \xi$.

Se escoge el llamado calibre unitario, en el cual los campos en \mathcal{L} corresponden a partículas físicas. En calibre unitario

$$\varphi' = e^{ig\left(-\frac{1}{g\nu}\xi\right)} \varphi = e^{-\frac{i\xi}{\nu}} e^{\frac{i\xi}{\nu}} \frac{(\eta + \nu)}{\sqrt{2}}$$

$$\varphi' = \frac{(\eta + \nu)}{\sqrt{2}}$$

Mecanismo Higgs en el Modelo Estándar

doblete Higgs $\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$, φ^+, φ^0 campos escalares complejos

Φ transforma bajo $SU(2) \times U(1)_Y$ Y : hipercarga

$$\Phi' = e^{i\frac{g}{2}\chi^a \sigma_a + i\frac{g'}{2}\chi} \Phi, \quad (D_\mu \Phi)' = e^{i\frac{g}{2}\chi^a \sigma_a + i\frac{g'}{2}\chi} D_\mu \Phi$$

$$D_\mu \Phi = \partial_\mu \Phi + ig \frac{\sigma_a}{2} W_\mu^a \Phi + i\frac{g'}{2} B_\mu \Phi$$

en las convenciones usadas, Φ tiene $Y=1$

$SU(2)$: constante de acoplamiento g , campos de calibre $W_\mu^1, W_\mu^2, W_\mu^3$

$U(1)_Y$: constante de acoplamiento g' , campo de calibre B_μ

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi^\dagger, \Phi)$$

$$V(\Phi^\dagger, \Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2, \quad \mu^2 > 0$$

ejercicio mínimo en $\Phi^\dagger \Phi = |\varphi^+|^2 + |\varphi^0|^2 = \frac{v^2}{2}$, $v^2 = \frac{\mu^2}{\lambda}$

$$\langle \Phi \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \quad \text{rompe la simetría}$$

calibre unitario

$$\Phi(x) = \begin{pmatrix} 0 \\ \frac{v + h(x)}{\sqrt{2}} \end{pmatrix}$$

$h(x)$ campo escalar real

$$\Phi^\dagger \Phi = \frac{1}{2} (v+h)^2 \Rightarrow V = \lambda v^2 h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4 + \text{const.}$$

$$D_\mu \Phi = \partial_\mu \Phi + ig \frac{\sigma_a}{2} W_\mu^a \Phi + i \frac{g'}{2} B_\mu \Phi$$

$$D_\mu \Phi = \begin{pmatrix} \partial_\mu + i \frac{g'}{2} B_\mu + i \frac{g}{2} W_\mu^3 & i \frac{g}{2} (W_\mu^1 - i W_\mu^2) \\ i \frac{g}{2} (W_\mu^1 + i W_\mu^2) & \partial_\mu + i \frac{g'}{2} B_\mu - i \frac{g}{2} W_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$D_\mu \Phi = \left(\begin{array}{c} \frac{ig}{2\sqrt{2}} (W_\mu^1 - iW_\mu^2)(v+h) \\ \frac{1}{\sqrt{2}} \partial_\mu h + \frac{i}{2\sqrt{2}} (g' B_\mu - g W_\mu^3)(v+h) \end{array} \right)$$

$$\begin{aligned} (D_\mu \Phi)^\dagger (D_\mu \Phi) &= \frac{1}{2} \partial_\mu h \partial^\mu h + \frac{1}{8} g^2 (v+h)^2 (W_\mu^1 + iW_\mu^2)(W^{1\mu} - iW^{2\mu}) \\ &\quad + \frac{1}{8} (v+h)^2 (g W_\mu^3 - g' B_\mu)(g W^{3\mu} - g' B^\mu) \end{aligned}$$

$$\mathcal{L}_{\text{Higgs}} = \frac{1}{2} \partial_\mu h \partial^\mu h - \underbrace{\lambda v^2 h^2}_{-\frac{1}{2} m_h^2 h^2} - \lambda v h^3 - \frac{\lambda}{4} h^4$$

$$+ \frac{1}{8} g^2 (v+h)^2 (W_\mu^1 + i W_\mu^2)(W^{1\mu} - i W^{2\mu})$$

$$+ \frac{1}{8} (v+h)^2 (g W_\mu^3 - g' B_\mu)(g W^{3\mu} - g' B^\mu)$$

$$\boxed{m_h = \sqrt{2\lambda} v}$$

$h(x)$ es el bosón Higgs

Masas para vectores de calibre

$$\begin{aligned} \mathcal{L}_{\text{Higgs}} &\supset \frac{1}{8} g^2 v^2 (W_\mu^1 + iW_\mu^2)(W^{1\mu} - iW^{2\mu}) \\ &= \frac{1}{2} m_W^2 (W_\mu^1 W^{1\mu} + W_\mu^2 W^{2\mu}) = m_W^2 W_\mu^+ W^{-\mu} \end{aligned}$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp iW_\mu^2)$$

$$m_W = \frac{1}{2} g v$$

$$\mathcal{L}_{\text{Higgs}} \supset \frac{1}{8} v^2 (g W_\mu^3 - g' B_\mu)(g W^{3\mu} - g' B^\mu)$$

$$= \frac{1}{8} \frac{v^2 g^2}{\cos^2 \theta_w} Z_\mu Z^\mu = \frac{1}{2} m_Z^2 Z_\mu Z^\mu$$

$$\frac{g'}{g} = \tan \theta_w$$

$$Z_\mu = \cos \theta_w W_\mu^3 - \sin \theta_w B_\mu$$

$$m_Z = \frac{v g}{2 \cos \theta_w}$$

$$A_\mu = \sin \theta_w W_\mu^3 + \cos \theta_w B_\mu$$

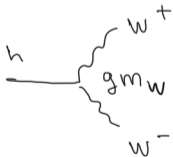
$$m_A = 0$$

A_μ es el fotón

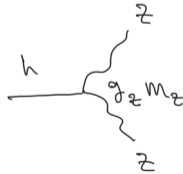
Acoplamiento h, W_μ^\pm, Z_μ

$$\mathcal{L}_{\text{Higgs}} \supset \frac{g^2}{8\cos^2\theta_w} (2vh + h^2) Z_\mu Z^\mu + \frac{1}{4} g^2 (2vh + h^2) W_\mu^+ W^{-\mu}$$

↳ acoplamiento cúbico proporcional a masas



$h \rightarrow W^+ W^-$



$h \rightarrow Z Z$

$$g_Z = \frac{g}{\cos\theta_w}$$

Derivada covariante redux

Con hipercarga Y genérica

$$D_\mu = \partial_\mu + i g \frac{\sigma_a}{2} W_\mu^a + i \frac{g'}{2} Y B_\mu$$

Ejercicio

$$D_\mu = \partial_\mu + \frac{i g}{\sqrt{2}} (T^+ W_\mu^+ + T^- W_\mu^-) + \frac{i g}{\cos \theta_w} (T_3 - \sin^2 \theta_w Q) Z_\mu + i e Q A_\mu$$

$$T^+ = \frac{1}{2} (\sigma_1 + i \sigma_2) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$T^- = \frac{1}{2} (\sigma_1 - i \sigma_2) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, T_3 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$$

$$Q = T_3 + \frac{Y}{2}$$

$$e = g \sin \theta_w$$

Resumen ¹

↙ grados de libertad

Antes	gdl	Después	gdl
φ^+, φ^-	2 + 2	h	1
$W_\mu^1, W_\mu^2, W_\mu^3$	2 + 2 + 2	W_μ^+, W_μ^-	3 + 3
B_μ	2	Z_μ	3
	<hr/>	A_μ	<hr/>
	12		12

$$SU(2) \times U(1)_Y$$

$$U(1)_{EM}$$

Resumen 2

$$m_W = \frac{1}{2} g v, \quad m_Z = \frac{g v}{2 \cos \theta_W}$$

$$m_h = \sqrt{2\lambda} v, \quad e = g \sin \theta_W$$

dependen de g, g', λ, μ en $\mathcal{L}_{\text{Higgs}}$

$$g' = g \tan \theta_W, \quad \mu^2 = \lambda v^2$$

W, Z descubiertos en 1983,

h descubierta en 2012

$$m_W = 80.379 \text{ GeV}$$

$$m_Z = 91.1876 \text{ GeV}$$

$$\frac{g^2}{m_W^2} = 4\sqrt{2} G_F, \quad G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$$

$$m_h = 125.10 \text{ GeV}$$

\Rightarrow

$$v = 246 \text{ GeV}$$

$$\lambda = 0.13$$

Jim

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