Buenas tardes!

Curso: Datos: MC + DM

La CONG1

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Funciones de variables aleatorias:

$f_X(x)$ ← valores que toma X

X → variable aleatoria X

Si $Z = g(X)$ cual es la dist. $f_Z(z)$?

→ $Z = aX + b$ Transformacion lineal

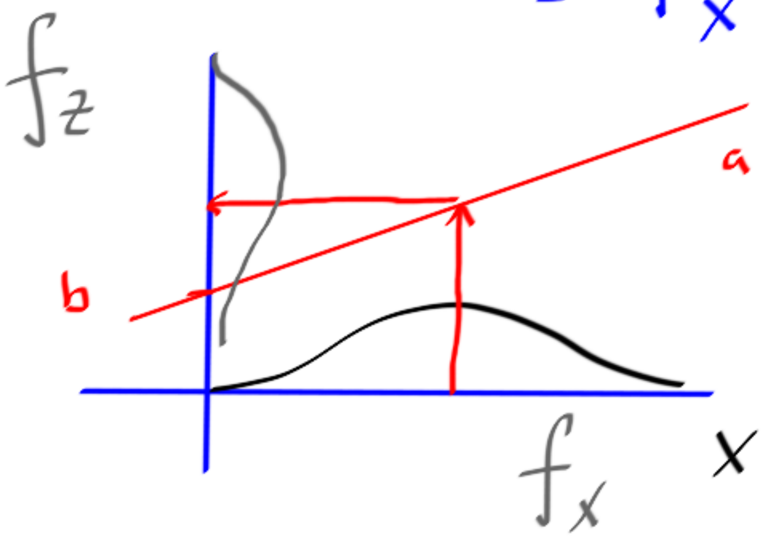
a, b escalares

$a \neq 0$

$$F_Z(z) = P(Z < z)$$ prob. acumulada

$$= P\left(X \le \frac{z-b}{a}\right)$$

$$= F_X\left((z-b)/a\right)$$



$$\frac{d}{dz} P(Z \le z) = \frac{d}{dz} P\left(X < \left(\frac{z-b}{a}\right)\right)$$

$$f_Z(z) = \frac{d}{dx} P\left(X < \left(\frac{z-b}{a}\right)\right) \left|\frac{dx}{dz}\right|$$

$$x = \frac{z-b}{a} \qquad dx/dz = \frac{1}{a}$$

$$f_z(z) = \frac{1}{|a|} f_X\left(\frac{z-b}{a}\right)$$

y

densidad de prob

- $z = g(X)$

$$F_Z(z) = P(Z \le z) = P(X < g^{-1}(z))$$

$$= F_X(g^{-1}(z))$$

$$f_Z(z) = \frac{d}{dz} P(Z<z)$$

$$= \frac{d}{dx} P(x < g^{-1}(z)) \frac{dx}{dz} = \underbrace{\frac{d}{dx} P(x < g^{-1}(z))} \left| \frac{d g^{-1}(z)}{dz} \right|$$

$$\int_{-\infty}^{\infty} \delta(x-a)\, dx = 1$$

$$= \frac{\frac{d}{dx} P(x < g^{-1}(z))}{\left| dg/dz \right|_{g^{-1}(z)}}$$

Demostrar (Tarea)

e.g.

$$\int_{-\infty}^{\infty} \delta(g(x))\, dx \Rightarrow \frac{\delta(x - x_0)}{|g'(x_0)|}$$

Ej.

$$z = x^2 = g(x)$$

$$\Rightarrow f_z(z) = f_x(\sqrt{z}) \frac{1}{|\sqrt{z}|} \quad \frac{d\sqrt{z}}{dz}$$

Ej:

f_X = Gaussiana

$$= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$$

(Tarea) demostrar

$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$$

$$Z = aX + b$$

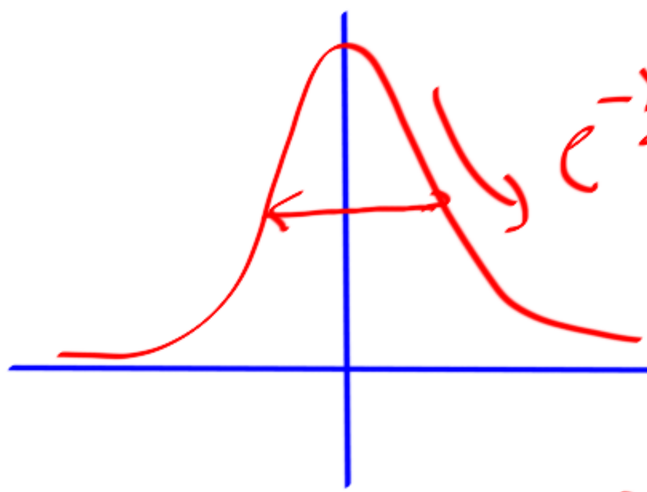
$$f_Z(z) = \frac{1}{|a|} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\left(\frac{z-b}{a}\right)^2 \frac{1}{2\sigma^2}}$$

demostrar que está normalizada

(Tarea)

(Tarea) $z = a e^{x}$ $a > 0$

hallar $f_z(z) = ?$ log-normal



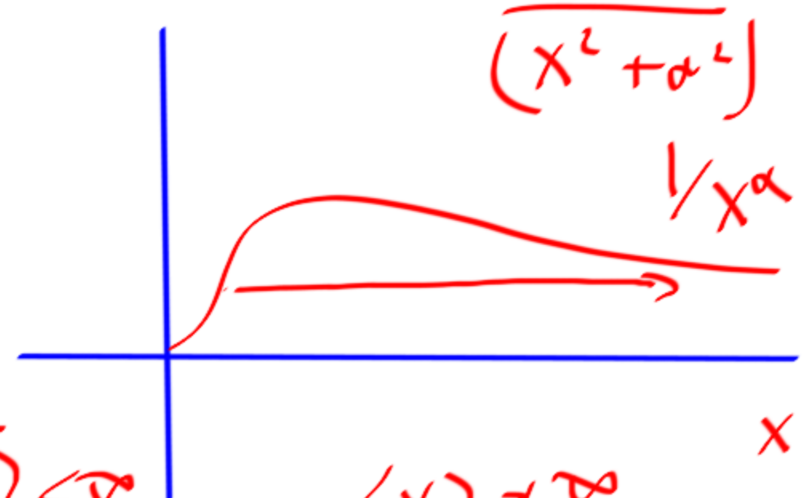
$e^{-x^2/2\sigma^2}$

$\langle x\rangle = E[x]$

$\exists$ $\langle x\rangle, \langle x^2\rangle < \infty$

concentral.

$\dfrac{1}{(x^2+a^2)}$



$1/x^{\alpha}$

x

$\langle x\rangle \neq \infty$

$\langle x^2\rangle - \langle x\rangle^2 = \infty$

Coulomb $\qquad\qquad$ Yukawa

$$\underline{\underline{\frac{1}{r^2}}} \longrightarrow \underline{\underline{\frac{e^{-x/\xi}}{r^2}}}$$

corto alcance

apantallamiento

$$\frac{1}{x^\alpha} \qquad \alpha \leq 2 \Rightarrow \cancel{\text{Gaussiana}}$$

$\Rightarrow$ <u>Levy</u>

<u>Phys. Rep.</u>

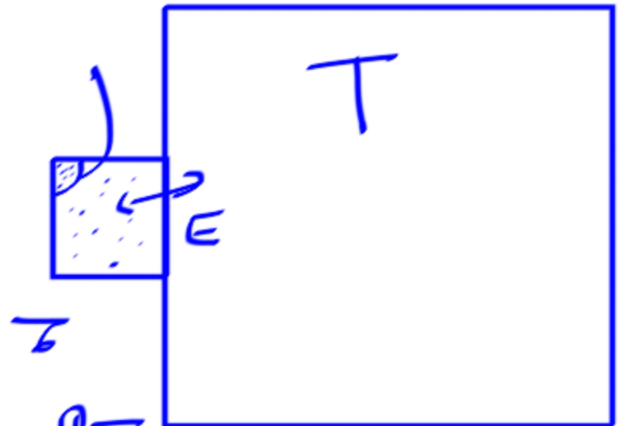
<u>J.P. Bouchard</u>

equil ⟺ reversible

Temporal

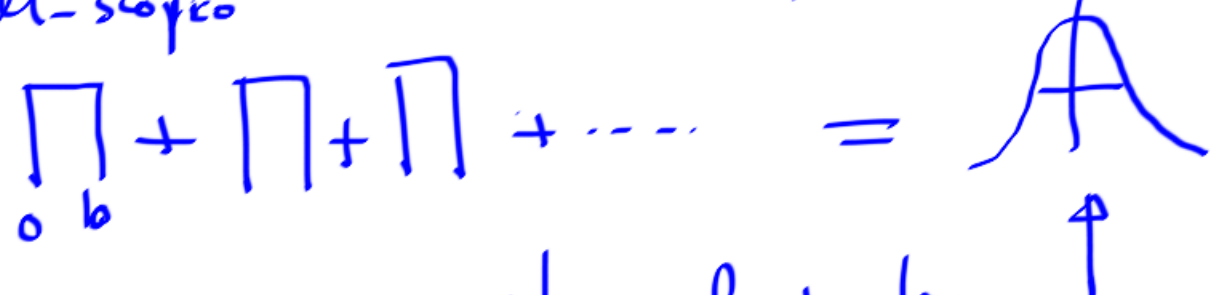
$t \to \infty$

$\infty \leftarrow t$



P, T, S . . . no depende de la historia

Microscópico



relación de muchos a uno

Irreversibilidad

Generalización de transformaciones lineales

$$\vec{z} = \vec{A}\,\vec{x}$$

matriz cuadrada

no

$$\vec{x} = (x_1 \cdots x_n)^T = \begin{pmatrix} \\ \\ \end{pmatrix}$$

$$\vec{x}^T \cdot \vec{x} = \begin{pmatrix} & & \end{pmatrix} \cdot \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

$=$ # escalar

$$(\vec{A})_{ij} =$$ # comps i $\neq$ # comps j

no necesario

Cuales son $\vec{\mu}_z$, $\overleftrightarrow{\Sigma}_z$ $(x_1, x_2, x_3 \ldots x_n)$ misma dist.

$$\vec{\mu}_z = E[\vec{z}] = E[\overleftrightarrow{A}\cdot\vec{x}]$$

$$\vec{\mu}_z = \overleftrightarrow{A}\cdot E[\vec{x}] = \overleftrightarrow{A}\cdot\vec{\mu}_x$$

$$\overleftrightarrow{\Sigma}_z = \overleftrightarrow{A}\cdot\overleftrightarrow{\Sigma}_x\cdot\overleftrightarrow{A}^T \quad \text{(Tarea)}$$

$z = g(x)$ transformación lineal

(Tarea) demostrar

$$f_{\vec{z}}(\vec{z}) = \frac{f_{\vec{x}}(\overleftrightarrow{A}^{-1}\vec{z})}{|A|}$$

$|A| \rightsquigarrow$ determinante de $\overleftrightarrow{A}$

$$f_{\vec{z}}(\vec{z}) = f_{\vec{x}}\left(g^{-1}(\vec{z})\right) \left|J_z(g^{-1})\right|$$

$$\vec{z} = \vec{g}(\vec{x})$$

$$J_x(g) = \begin{vmatrix} \partial g_1^{-1}/\partial x_1 & \cdots & \dfrac{\partial g_1^{-1}}{\partial x_n} \\ \vdots & & \\ \dfrac{\partial g_n^{-1}}{\partial x_1} & \cdots & \dfrac{\partial g_n^{-1}}{\partial x_n} \end{vmatrix}$$

Jacobiano de la transformación

Ej.

$$f_{\vec{X}}(\vec{x}) = \frac{1}{(2\pi)^{n/2}} e^{-\frac{1}{2}(x_1 \cdots x_n)\begin{pmatrix}x_1\\ \vdots \\ x_n\end{pmatrix}}$$

$$= \frac{1}{(2\pi)^{n/2}} e^{-\frac{1}{2} X^T X}$$

$$\rightarrow \quad \vec{Z} = \vec{\mu} + \overleftrightarrow{B}\vec{X} \qquad \text{Transformacion lineal}$$

$$\overleftrightarrow{\Sigma} = \overleftrightarrow{B}\,\overleftrightarrow{B}^T \quad //$$

$$\vec{Y} = \vec{Z} - \vec{\mu} \qquad \vec{Y} = \overleftrightarrow{B}\vec{X} \Rightarrow f_Y(\vec{y}) = \frac{f_X(\overleftrightarrow{B}^{-1}\vec{y})}{|\overleftrightarrow{B}|}$$

$$f_Y(y) = \frac{1}{|B|\,(2\pi)^{n/2}}\, e^{-\frac{1}{2}\underbrace{(B^{-1}y)^T(B^{-1}y)}_{y^T\underbrace{(B^{-1})^T B^{-1}}\, y}}$$

$$(B^{-1}y)^T = y^T (B^{-1})^T$$

$$(B^{-1})^T B^{-1} = \underbrace{(B^T)^{-1}}_{(\text{Tarea})} B^{-1} = (B^T B)^{-1} = (\Sigma)^{-1}$$

$$|B^T|\,|B| = |B^T B| = |\Sigma| \Rightarrow \underline{\underline{|B| = \sqrt{|\Sigma|}}}$$

$$|B^T| = |B|$$

$$f_Y(y) = \frac{1}{\sqrt{|\Sigma|}\,(2\pi)^{n/2}}\, e^{-\frac{1}{2} y^T (\Sigma)^{-1} y}$$

Gaussiana multidimensional

z

$$f_Z(z) = \frac{1}{\sqrt{|\Sigma|}\,(2\pi)^{n/2}}\, e^{-\frac{1}{2} (\vec{z} - \vec{\mu})^T (\Sigma)^{-1} (\vec{z} - \vec{\mu})}$$

gen n variables

Forma canónica de la Gaussiana

$$P(z) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(z-\mu)^2}{2\sigma^2}}$$

$$\langle \vec{z} \rangle = \vec{\mu}$$

$$\langle z^2 \rangle - \langle z \rangle^2 = \sigma^2$$

La Gaussiana tiene ∞ momentos

$\rightarrow \langle x \rangle, \langle x^2 \rangle, \langle x^3 \rangle \ldots\ldots \infty$

Estadísticos $\Rightarrow \langle x \rangle, \langle x^2 \rangle - \langle x \rangle^2 \ldots\ldots$ cumulantes de la dist.!

Gaussiana tiene solo 2 cumulantes

Teorema del límite Central

Dados $x_1 \ldots\ldots x_n$ $\quad x_i$ idénticamente dist. e independientes

$$S_n = X_1 + X_2 + X_3 + \dots + X_n$$

Variabili macroscopiche

$$E[S_n] = E\left[\sum_i^n X_i\right] = \sum_i^n E[X_i]$$

$$E[S_n] = n\,E[X]$$

!!

(Teorema) $\quad Var(S_n) = n\,Var[X]$

Es. $\quad y = x_1 + x_2 \qquad \langle x_1 \rangle = \langle x_2 \rangle$

$$Var(y) = \langle y^2 \rangle - \langle y \rangle^2 = \langle (x_1 + x_2)^2 \rangle - \langle x_1 + x_2 \rangle^2$$

$$= 2\left(\langle x^2 \rangle - \langle x \rangle^2\right) = 2\,Var(x)$$

$$\frac{Var\,[x_1+x_2+\dots+x_n]}{\left(E\,[x_1+x_2+\dots]\right)^2} = \frac{n\,Var[x]}{n^2\,E^2[x]} \sim \left(\begin{matrix}\text{dep}\\ \mu\end{matrix}\right)\frac{1}{n}$$

variable microscópica

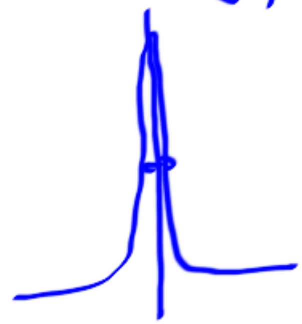
n pequeños

$P + \Delta P$

$P \sim \Delta P$

Sistemas pequeños

n grande



$P + \Delta P$

$P \gg \Delta P$

Termodinámica

Teorema del límite Central:

$$\lim_{n\to\infty} P\left(\frac{S_n - n\mu}{\sigma\sqrt{n}} \le x\right) = \Phi(x)$$

acumulada

Gaussiana acumulada!

$\mu, \sigma < \infty$

leer $\hookleftarrow$ Reif Mecanica Estadistica

(no Berkeley)

$$P(\overbrace{x_1+x_2}^{y}) = \int \underbrace{P(x_1,x_2)}_{P(x_1)P(x_2)} \delta(y-(x_1+x_2))\,dx_1\,dx_2$$

$$\sim \Pi$$