Buenas tardes!

Curso: La CoNGA

Mecánica Estadística Avanzada

(Dinámica)

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$$-\sum_i \frac{\partial H}{\partial q_i} \cdot v_i = 0 \qquad \frac{dH}{dt} = 0$$ ley de conservación ✓

$$-\quad \vec{\nabla} \cdot \vec{v} = 0$$

$-$ Cons. de prob. $\quad \dfrac{\partial P}{\partial t} = 0 \qquad [P, H]_P = 0$

$$P = P(H) \Rightarrow P \propto e^{-H/T}$$

M_c $\qquad\qquad$ $H \neq H(t)$

$$\frac{\partial P}{\partial t} = -\sum_i \frac{\partial J_i}{\partial q_i}$$

$$J_i = \underbrace{V_i \overbrace{P}^{e^{-H/T}}}_{\text{drift}} - \underbrace{\frac{\Gamma_i}{T}\frac{\partial H}{\partial q_i} P}_{\text{drag}} - \underbrace{D_i \frac{\partial P}{\partial q_i}}_{\text{diffusive}}$$

drift

convective

drag

diffusive

$$\sum_i \frac{\partial J_i}{\partial q_i} = \sum_i \left(\cancelto{0}{\frac{\partial V_i}{\partial q_i}} e^{-H/T} + \cancelto{0}{V_i \left(-\frac{1}{T}\right) \frac{\partial H}{\partial q_i}} e^{-H/T} + \dots \right.$$

$$= -D_i \frac{\partial^2}{\partial q_i^2} e^{-H/T} - \frac{\Gamma_i}{T} \frac{\partial}{\partial q_i}\left(\frac{\partial H}{\partial q_i} e^{-H/T}\right)$$

$$= \sum_i +\frac{D_i}{T}\left[\frac{\partial^2 H}{\partial q_i^2} e^{-H/T} - \frac{1}{T}\frac{\partial H}{\partial q_i}\frac{\partial H}{\partial q_i} e^{-H/T}\right]$$

$$-\frac{\Gamma_i}{T}\left[\frac{\partial^2 H}{\partial q_i^2} e^{-H/T} - \frac{1}{T}\left(\frac{\partial H}{\partial q_i}\right)^2 e^{-H/T}\right] = 0$$

Coefficiente de transporte $\longleftarrow D_i = \Gamma_i \Rightarrow$ coefficiente de fricción

Fluctuación disipación

relación de Einstein

Fluctuación disipación → Sistema cerca del equilibrio

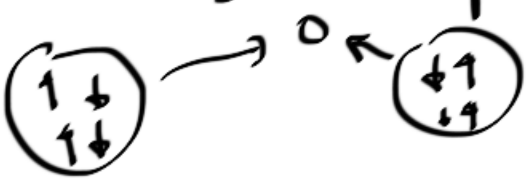
→ cumple con balance detallado

$P \sim e^{-H/T}$ es solución a la ec. de evolución

Grupo

— Renormalización

Semi Grupo ←



estático: Eliminamos de escalas μ-scópicas de longitud ⇒ comp. macro del sistema.

Sumar sobre

Conservamos Forma de H

Dinámico: Eliminamos de tiempos cortos

Conservamos forma ec. de evol.

'Renormalización' dinámica : Preservamos

$i = 1, 2$ modos

$$\frac{\partial q_i}{\partial t} = v_i - \frac{\Gamma_i}{T} \frac{\partial H}{\partial q_i} + \xi_i(t)$$

$$\langle \xi(t) \rangle = 0 \qquad \langle \xi_i(t)\, \xi_j(t') \rangle = 2 D_i \delta_{ij} \delta(t-t')$$

(HW) Si $v_1 = 0$ $\quad \dot{p} = 0$ $\quad$ desprecio $\dot{p}$

que ocurre con el decrecimiento de q_i ?

$$\frac{1}{\tau_+} = \frac{\Gamma_1}{T_m} \quad ; \quad \frac{1}{\tau_-} = \frac{KT}{\Gamma_1}$$

Suposición

$$\frac{\Gamma_1}{mT} \gg \left(K/m\right)^{1/2}$$

largos tiempos

$$\frac{1}{\tau_+} \gg \frac{1}{\tau_-} \Rightarrow \tau_+ \ll \tau_-$$

Modos lentos

cortos tiempos

Modos rapidos
rapidos

Podemos escribir ec de movimiento

$$\frac{\partial q_-}{\partial t} = -\frac{q_-}{\tau_-} + \tau_+ g_1 \qquad \text{(HW)}$$

$$q_\pm = \tau_\mp g_1 + g_2$$

$$\left\langle \frac{\partial q_-}{\partial t} \right\rangle = -\frac{\langle q_- \rangle}{\tau_-} + \cancelto{0}{\langle \tau_+ g_1 \rangle}$$

$$\langle q_- \rangle = C e^{-t/\tau_-} \qquad //$$

$$\boxed{\begin{aligned} \dot{q}_1 &= -K q_2 = v_1 \\ \dot{q}_2 &= \frac{q_1}{m} = v_2 \end{aligned}}$$

$\frac{1}{\tau_+}$ tiempo rápido

$$\frac{\partial q_1/m}{\partial t} = -\frac{K}{m} q_2 - \frac{\Gamma_1}{mT}\frac{q_1}{m} + \frac{S_1}{m}$$

inhomogéneo involucra evolución lenta del sistema

$$\frac{\partial q_1/m}{\partial t} + \frac{\Gamma_1}{mT}\frac{q_1}{m} = -\frac{K q_2}{m} + \frac{S_1}{m}$$

$1/\tau_+$

$$q_{1/m}(t) = -\frac{K}{m}\int_{-\infty}^{t} dt'\, e^{-\gamma(t-t')} q_2(t') + \int_{-\infty}^{t} dt'\, e^{-\gamma(t-t')}\frac{S_1(t')}{m}$$

$\tau_+ < t < \tau_-$ $\qquad q_2 \sim \text{const.}$

$$\frac{\partial q_2}{\partial t} = -\frac{K_1}{m}\left(\int_{-\infty}^{t} dt'\, e^{-\gamma(t-t')}\right) q_2 + \xi'(t)$$

$$= -\frac{\Gamma'}{T}\frac{\partial H}{\partial q_2} + \xi'(t) \longrightarrow \text{ruido renormalizado} \longrightarrow D'$$

Fricción renormalizada

relación de Einstein $\quad D' = \Gamma'$

$$H = \frac{q_1^2}{2m} + \frac{1}{2} k q_2^2 \qquad \frac{\partial H}{\partial q_2} = k q_2$$

$$\frac{\Gamma'}{T} = \frac{1}{m} \int_{-\infty}^{t} dt' \, e^{-\gamma(t-t')} \qquad (HW)$$

$$= \frac{1}{m\gamma} = \frac{1}{\cancel{m}\,\Gamma_1 / \cancel{m}\,T} = \frac{T}{\Gamma_1}$$

$$\frac{\partial q_2}{\partial t} = \underbrace{-\frac{\Gamma'}{T} \frac{\partial H}{\partial q_2}} + \zeta'(t)$$

$$\xi'(t) = \int_{-\infty}^{t} dt' \, e^{-\gamma(t-t')} \, \xi_1(t')/m$$

$$\langle \xi' \rangle = 0$$

$$\langle \xi_1(t)\,\xi_1(t') \rangle = 2D_1 \, \delta(t-t') \leftarrow$$

HW $$\langle \xi'(t)\,\xi'(t') \rangle = 2D_1 \left(\frac{I}{I_1}\right)^2 \frac{\gamma}{2} e^{-\gamma(t-t')} \approx \delta(t-t')$$

$$= 2D' \, \delta(t-t') \qquad \text{s:} \; t > \tau_r$$

$$\boxed{D' = 2D_1 \left(\frac{I}{I_1}\right)^2 \frac{\gamma}{2}}$$

$$\langle \varphi_z^2 \rangle \sim D' t \qquad t > \tau_+$$

Modelo anterior es un modelo de juguete (Gallina colérica) (dos escalas de tiempo)

- Teoría de Van - Hove

(Modelo Gaussiano en dinámica)

Siempre $\underline{\underline{T > T_c}}$ (instable para $T < T_c$)

V-Hove

$$\beta \mathcal{H} = \int dr^3 \left(\underset{\substack{\downarrow \\ \text{Temp de transición}}}{a_2}\, m^2 + c\, (\nabla m)^2 \right) \quad (\nabla m)^2 \rightarrow \vec{\nabla} m \cdot \vec{\nabla} m$$

$$m = \frac{1}{\sqrt{V}} \sum_k e^{i \vec{k} \cdot \vec{r}} \sigma_k$$

$$\nabla m = \frac{1}{\sqrt{V}} \sum_k i\vec{k}\, e^{+i \vec{k} \cdot \vec{r}} \sigma_k$$

(HW) . . . . $\Rightarrow \beta \mathcal{H} \xrightarrow{1/T} = \sum_{k < \Lambda = 1/a} (a_2 + c k^2) |\sigma_k|^2$

$k < \Lambda = 1/a \rightarrow$ parámetro de red

Ec. de mov. para comp de Fourier

$$\frac{\partial \sigma_k}{\partial t} = -\frac{\Gamma_k}{T} \frac{\partial H}{\partial \sigma_k} + S_k$$

$$\frac{\partial \sigma_k}{\partial t} = -\Gamma_k \, 2 \left(a_2 + c k^2 \right) \sigma_k + S_k$$

$$\langle S_k(t) \, S_{k'}(t') \rangle = 2 \, \Gamma_k \, \delta_{k', -k} \, \delta(t - t')$$

(HW) $$\frac{\partial m}{\partial t} = -\Gamma\, 2\left(a_2 m^2 - c\nabla^2 m\right) + \zeta$$

Comprobar

$$\langle \zeta(x,t)\, \zeta(x',t')\rangle = 2\Gamma\,\delta(x-x')\,\delta(t-t')$$

$$\Gamma_k = \Gamma \quad \text{indep de } k$$

suposición simplificación

relación de dispersión!

$$\omega = \frac{1}{\tau_k} = 2\Gamma\left(a_2 + Ck^2\right)$$

contínuo de vec de onda

un contínuo de tiempos

$k \to 0$ $\quad \lambda \to \infty$ escalas de long. muy grandes

$$\tau_k \to \frac{1}{2a_2}\,\frac{1}{\Gamma}$$

$$\to \tau_k \sim \left[2 a_2' (T-T_c)\right]^{-1} \Gamma^{-1}$$

Alentamiento crítico (slowing down)

$$T \to T_c \qquad \tau_k \to \infty$$

T.C.M $\qquad \xi = \left(\frac{a_2}{c}\right)^{-1/2} = \frac{a_2'}{c}(T-T_c)^{-1/2}$

$\nu = 1/2$

es la TCM

exacto para Modelo Gaussiano

$$\tau_k = \left[2\Gamma\,(a_2 + c k^2)\right]^{-1}$$

$$= \left[2\Gamma\left(\frac{c}{\xi^2} + c k^2\right)\right]^{-1}$$

$$\tau_k = \xi^2\, \underbrace{\left[2\Gamma c\right]^{-1} \left(1 + \xi^2 k^2\right)^{-1}}$$

$$\tau_k = \xi^z f(k\xi)$$

$$\boxed{z = 2}\ !$$

$$\tau_k = \left[2\,(a_2 + c k^2)\right]^{-1} \Gamma^{-1}$$

$G(k)$

Función de correlación espacio

Que ocorre quando Π_k

$\Pi_k = \Pi$

Model A $\quad z = 2$

Model B $\quad z = 4 \qquad \Pi_k = \gamma k^2$ //

Halperin – Hohenberg