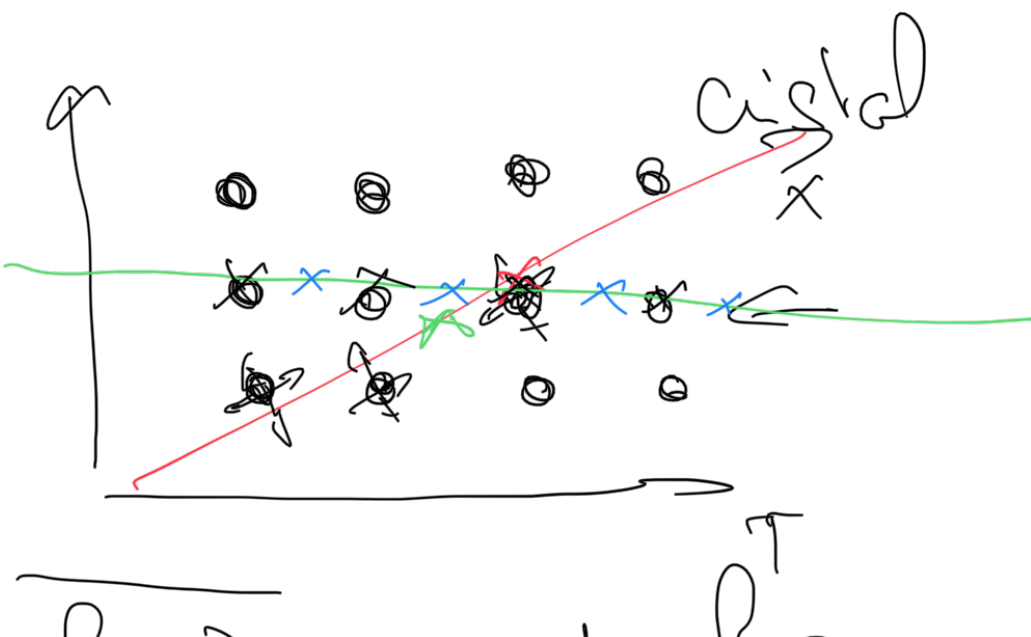
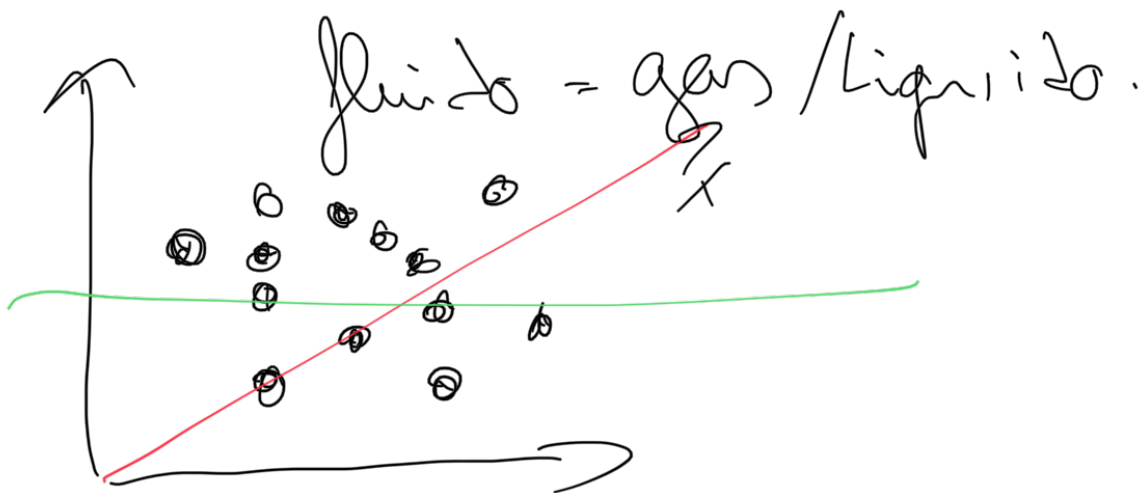
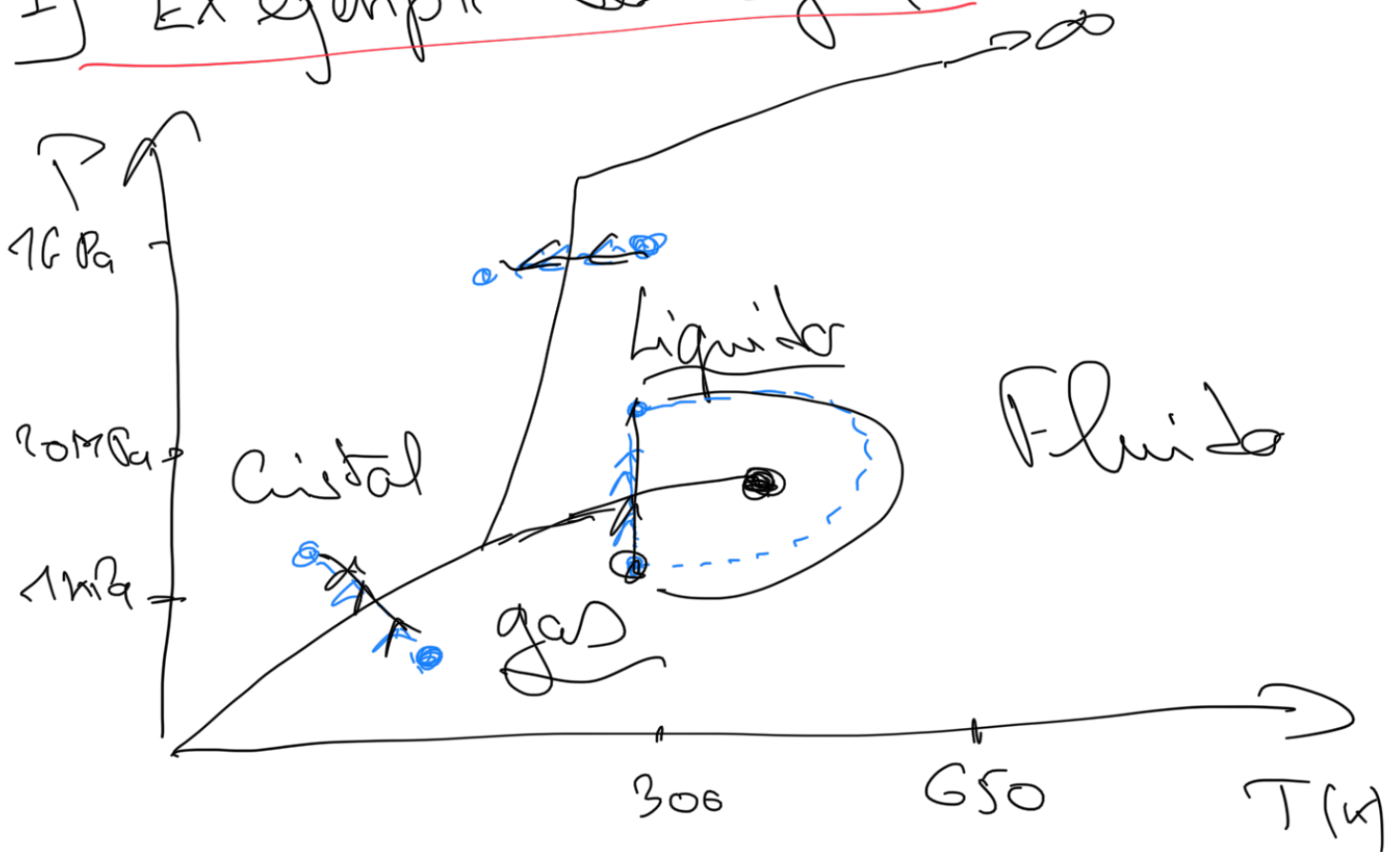
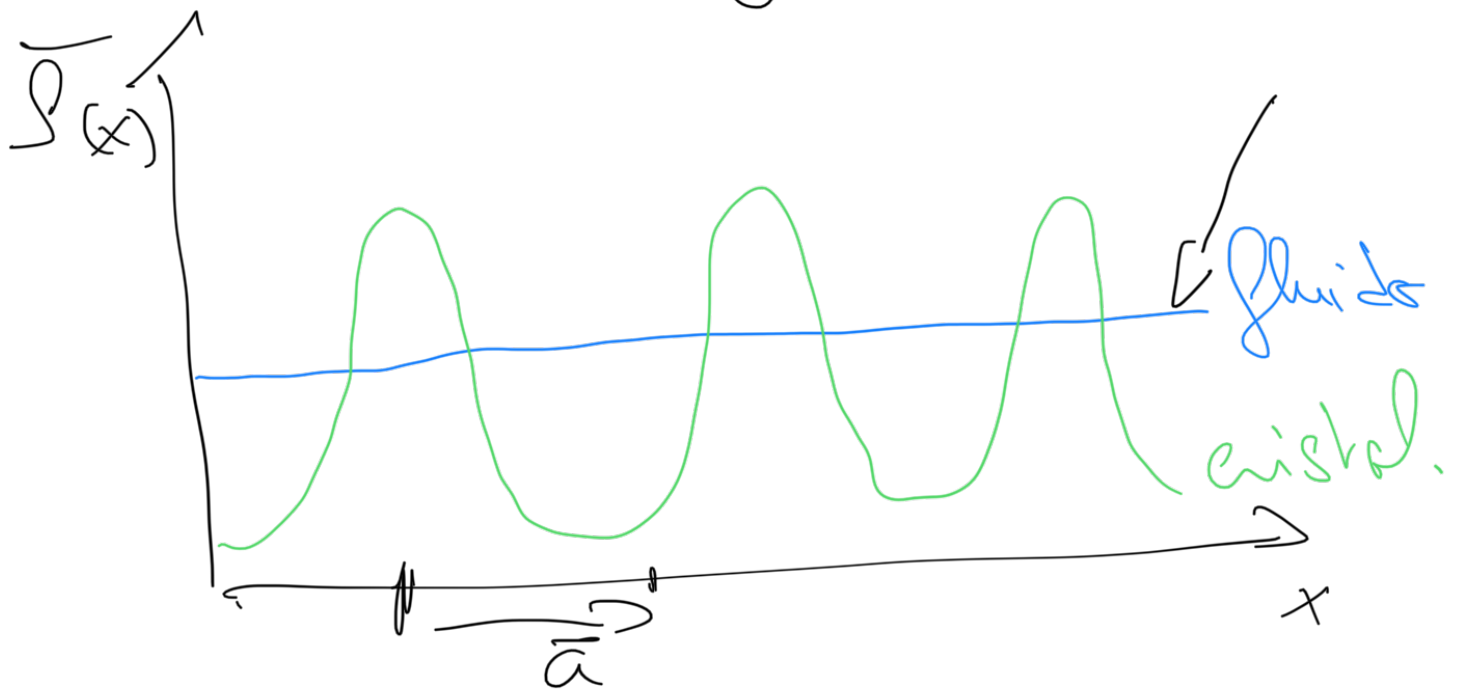


I Introducción a las transiciones de fase y la criticalidad

1) El ejemplo del agua



$$\rho(\vec{r}) = \frac{1}{T} \int_0^T \rho(\vec{r}, t) dt$$



$$H = \sum_i \frac{p_i^2}{2m} + \frac{1}{2} \sum_{i \neq j} V(\vec{r}_i - \vec{r}_j)$$

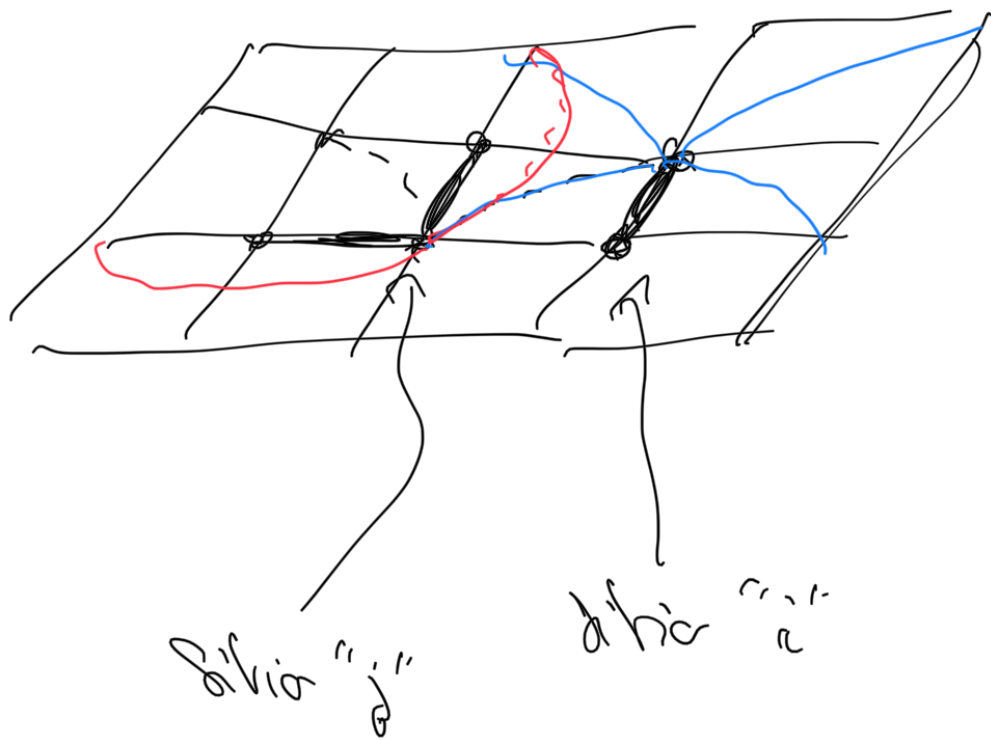
→ Simetría de traslación

$$\forall i \quad \vec{r}_i \rightarrow \vec{r}_i + \vec{a}$$

Cristal → ruptura espontánea de la simetría de traslación.

→ Clasificación de las fases por los ruptores de simetría.

2) El modelo de Ising



en cada sitio "i" \rightarrow variable σ_i

$$\sigma_i = \pm 1 \quad \sigma_i^2 = 1$$

para una partícula de Spin $1/2$

$$S^2 = \frac{\hbar}{2}$$

$$H = - \sum_{\langle i,j \rangle} J_{ij} \sigma_i \sigma_j$$

$$\underline{J_{ij}} \rightarrow 0 \text{ si } |i-j| \rightarrow \infty$$

\uparrow distancia entre "i" y "j"

Caso particular $J_{ij} \neq 0$ únicamente si "i" y "j" son primeros vecinos.



$$H = -J \sum_{\langle i,j \rangle} \sigma_i \sigma_j \quad \leftarrow \underline{J > 0}$$

↑
primeros vecinos.

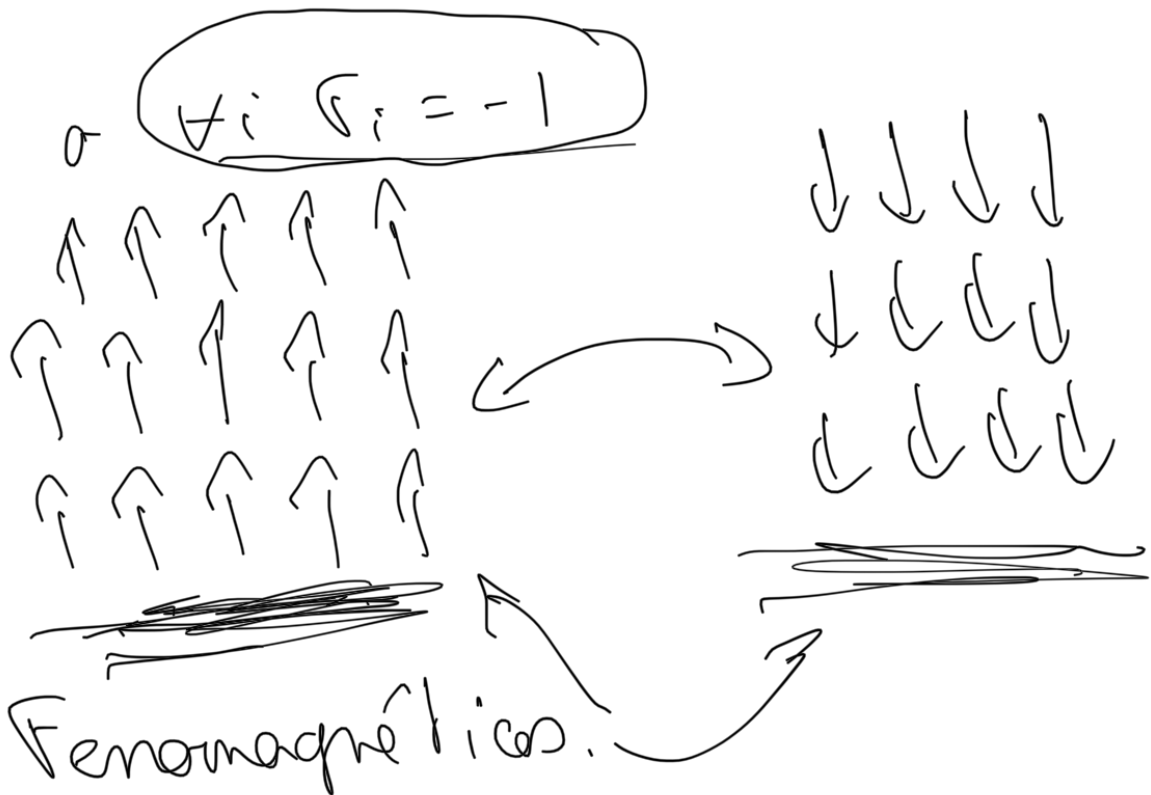
Obs 1: simetrías de H .

$$\left. \begin{array}{l} * \sigma_i \rightarrow \sigma_i \\ * \sigma_i \rightarrow -\sigma_i \end{array} \right\} \text{Grupo } \mathbb{Z}_2$$

Obs 2 $J_{ij} > 0 \quad \forall i,j$

Configuración de más baja energía?

→ $\forall i \sigma_i = +1$



$$H = -J \sum_{i,j} (\Delta_z S_i^z S_j^z + \Delta_x S_i^x S_j^x + \Delta_y S_i^y S_j^y)$$

$$\Delta_2 \gg \Delta_x, \Delta_y$$

$$Z = \sum_{\{\sigma_i\}} e^{-\beta H} \quad \beta = \frac{1}{k_B T}$$

$\uparrow \downarrow \downarrow \uparrow$
 N spins, 2^N possibilities

$$Z = e^{-\beta H}$$

\rightarrow "Curie magnetic"

$$H(h) = - \sum_{\langle ij \rangle} J_{ij} \sigma_i \sigma_j - h \sum_i \sigma_i$$

\downarrow
 \uparrow
 hores
 int. $Z/2$

$$m_i = \langle \sigma_i \rangle =$$

$$\langle \sigma_i \rangle = \frac{1}{Z} \left[\sum_{\{\sigma_j\}} e^{-\beta H} \sigma_i \right]$$

$$1 \rightarrow \left[\sum_{\{\sigma_j\}} e^{-\beta H} \right]$$

$$\langle \sigma_i \rangle = \frac{1}{Z} \sum_{\{\sigma_i\}} \sigma_i e^{-\beta H}$$

Properiedad:

$$F = \sum_i m_i = \frac{1}{Z} \left[\sum_{\{\sigma_i\}} e^{-\beta H} \sum_i \sigma_i \right]$$

ej: $F = \frac{1}{\beta} \frac{\partial}{\partial h} \ln Z(h) = - \frac{\partial}{\partial h} F(h)$

Susceptibilidad magnética

$$\chi(h) = \frac{\partial F(h)}{\partial h} = \frac{1}{\beta} \frac{\partial^2}{\partial h^2} \ln Z(h)$$

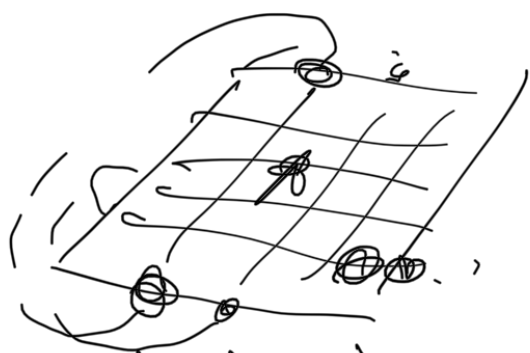
$$\frac{\beta}{Z(h)} \sum_{\{\sigma_i\}} e^{-\beta H} \sum_{i,j} \sigma_i \sigma_j - \beta \left[\frac{\sum_{\{\sigma_i\}} e^{-\beta H} \sum_i \sigma_i}{Z(h)} \right]^2$$

$$\chi(h) = \beta \sum_{i,j} \left[\langle \sigma_i \sigma_j \rangle - \langle \sigma_i \rangle \langle \sigma_j \rangle \right]$$

$$E = \frac{1}{Z} \sum_{\{\sigma_i\}} e^{-\beta H} H = - \frac{\partial}{\partial \beta} \ln Z$$

$$C = \frac{\partial E}{\partial T} = - k_B \beta^2 \frac{\partial E}{\partial \beta} = k_B \beta^2 \frac{\partial^2}{\partial \beta^2} \ln Z$$

obs



Condiciones de borde periódicas

→ Todos los sitios son equivalentes

$$\forall i \quad m_i = m$$

$$\mathcal{H} = N m \quad N = \# \text{ total de sitios}$$

$$\chi(n) = \beta \sum_{i,j} \langle \sigma_i \sigma_j \rangle - \beta N^2 m^2$$

Función de correlación:

$$\langle \sigma_i \sigma_j \rangle_c = \langle \sigma_i \sigma_j \rangle - \langle \sigma_i \rangle \langle \sigma_j \rangle$$

$$\approx \langle \sigma_i \sigma_j \rangle + m^2$$

A.P.B.P.

$$\langle \sigma_i \sigma_j \rangle_c \underset{|i-j| \rightarrow \infty}{\sim} e^{-\frac{|i-j|}{\xi}}$$

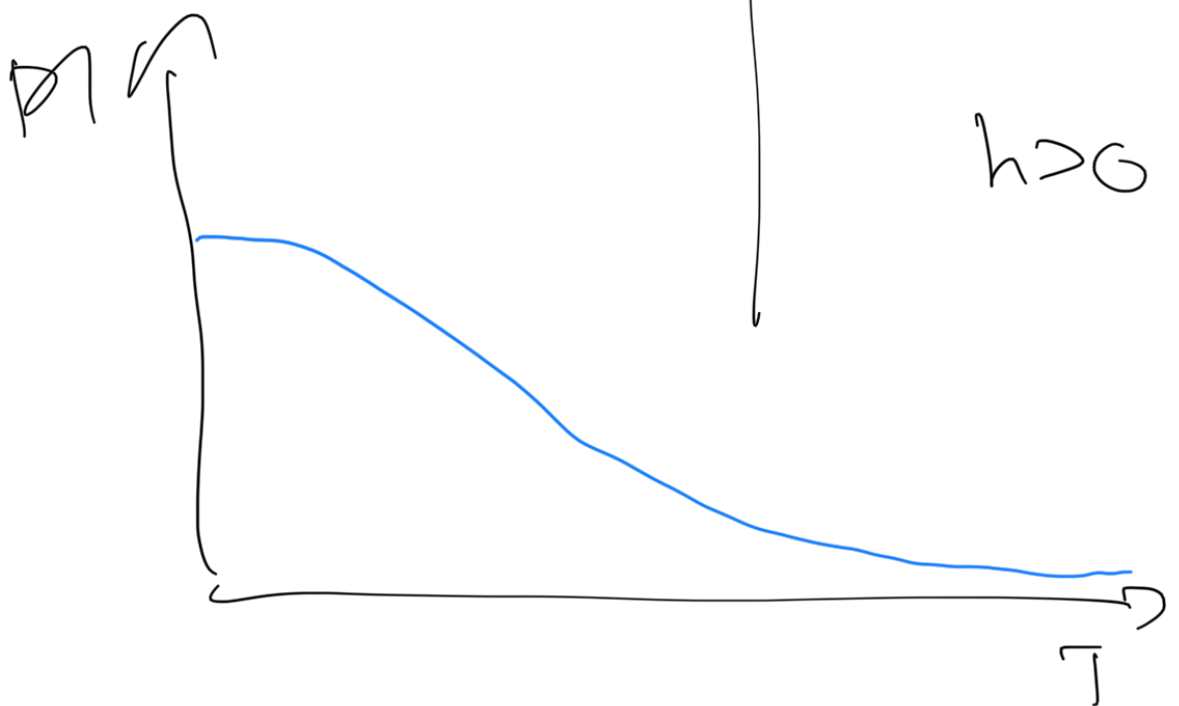
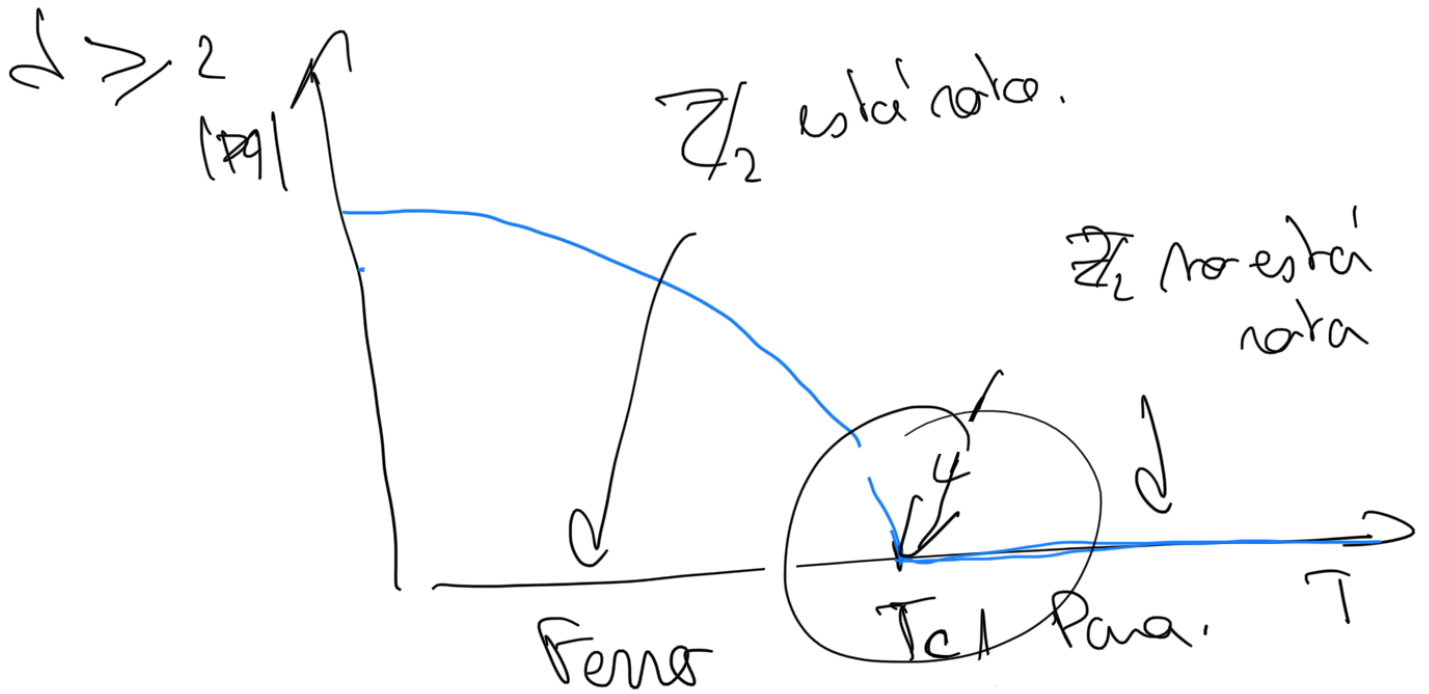
ξ = longitud de correlación.

Diagrama de fases

$$d=1$$

$$d \geq 2$$

$$h=0$$

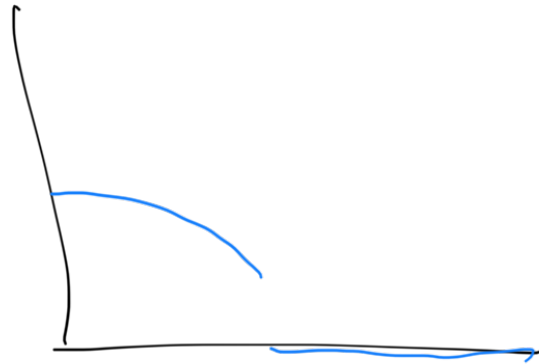


El punto de transición va a ser de una transición de segundo orden.

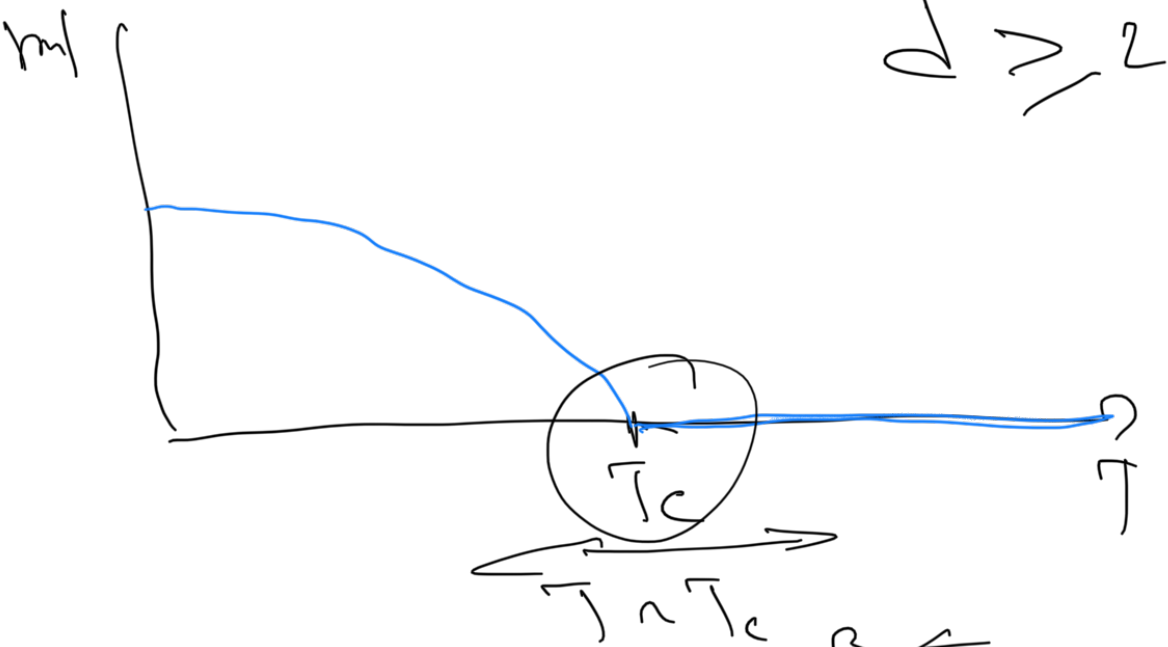
$P(T)$ en función de T es una función continua, pero $\frac{dP}{dT}$ es discontinua.

→ no transición de orden "n"
 es que $\frac{d^{n-1}}{dT^{n-1}} P(T)$ es discreta.

Sea ad



Exponentes críticos:



si $T \approx T_c$ $(m) \sim (T_c - T)^\beta \sim (m) \sim (t)^\beta$

$\left\{ \begin{array}{l} \xrightarrow{\text{a } T \rightarrow T_c} \Rightarrow \\ \sim |T - T_c| \sim |t|^\nu \end{array} \right.$

$t = T - T_c$ otros $t = \frac{T - T_c}{T_c}$

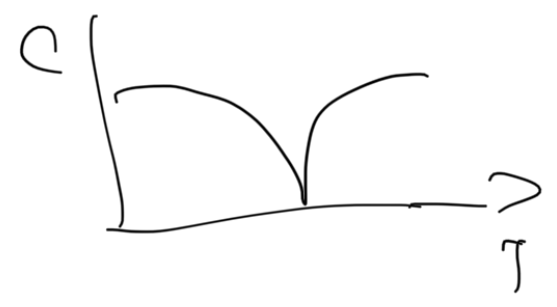
$\frac{\chi(t)}{t} \sim |t|^{-\gamma}$ $\chi \uparrow$

$$\frac{C}{N} \sim |t|^{-\alpha}$$



$$At T = T_c$$

$$\alpha < 0$$



$$\langle \sigma_i \sigma_j \rangle_c \sim$$

$$\frac{1}{|i-j|^{d-2+\eta}}$$

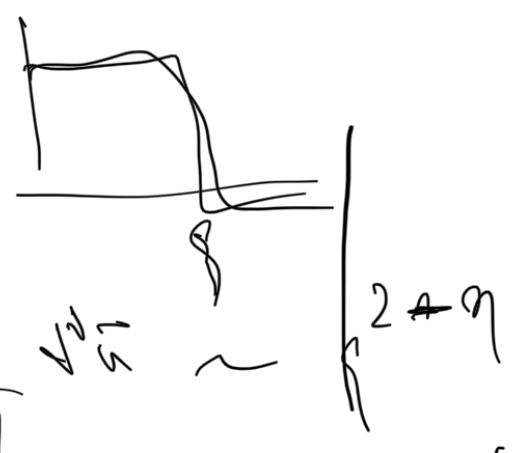
$$\nu d = 2 - \alpha = 2\beta + \gamma$$

$$2 - \eta = \frac{\gamma}{\nu}$$

hyper scaling.

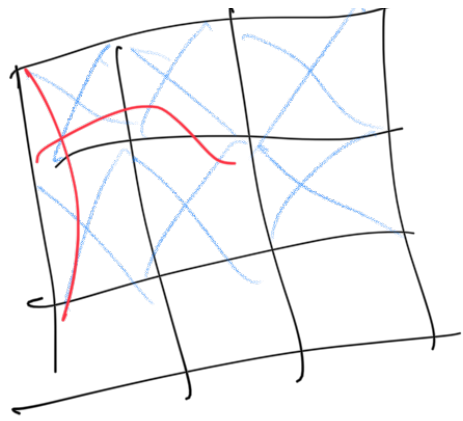
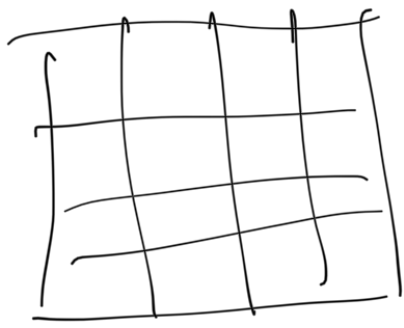
$$\frac{\chi}{N} \sim \frac{1}{T} \int \langle \sigma(\vec{0}) \sigma(\vec{r}) \rangle_c d^d r$$

$$\langle \sigma(\vec{0}) \sigma(\vec{r}) \rangle_c \sim \frac{1}{|\vec{r}|^{d-2+\eta}} f\left(\frac{|\vec{r}|}{\xi}\right)$$



$$\frac{\chi}{N} \sim \frac{1}{T} \int_{|\vec{r}| < \xi} \frac{1}{|\vec{r}|^{d-2+\eta}} d^d r \sim \frac{1}{T} \xi^{2+\eta}$$

$$\frac{\chi}{N} \sim |t|^{-\nu(2-\eta)}$$



$d = 1$
 $|m|$

