Modelo de Ising

$\sigma_i = \pm 1$

$\Rightarrow$ transformación de Hubbard-Stratonovich

$$\boxed{Z = \text{cte} \int \mathcal{D}[\phi]\, e^{-S}}$$

S acción de Ginzburg-Landau

$$\boxed{S[\phi] = \int d\vec{r} \left[\frac{c}{2} |\vec{\nabla}\phi|^2 + \frac{a_2}{2}\phi^2 + \frac{a_4}{4}\phi^4 + \dots \right]}$$

a_2 podía ser positivo o negativo

$a_2 \sim (T - T_c)$, $t = \frac{T - T_c}{T_c}$

$a_2 = a t$

$a_2 < 0$ si $t < 0$

$a_2 > 0$ si $t > 0$

y $a_4 > 0$

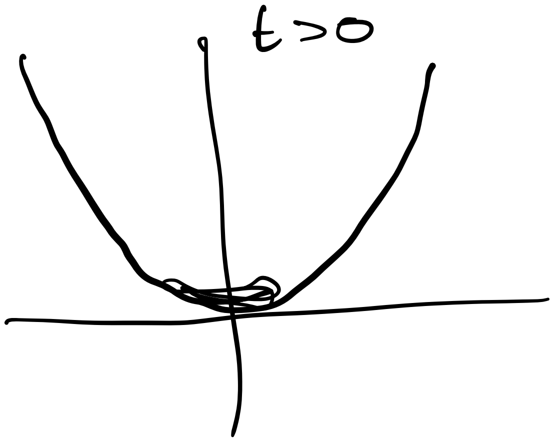
las config. mas relevantes son las que hacen que S sea mínima.

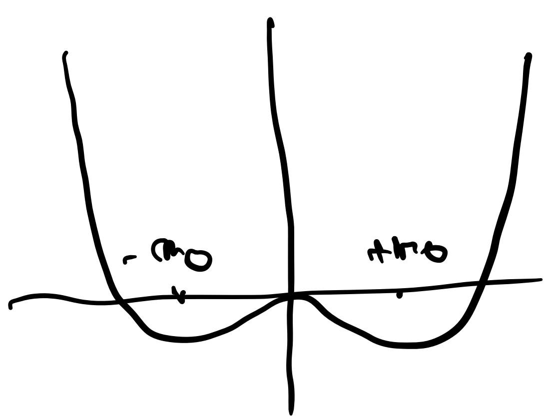
$\overline{\nabla \phi} = \overline{0}$ $\quad \phi = m$

$$S\{m\} = V\left[\frac{a_2}{2} m^2 + \frac{a_4}{4} m^4\right]$$

$a_4 > 0$

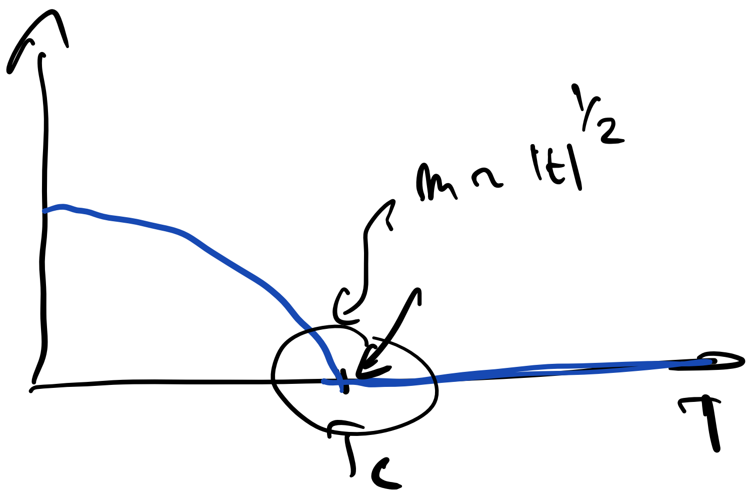
$a_2 = a t \begin{cases} > 0 \\ < 0 \end{cases}$





$$m_0 = \sqrt{\frac{-a_2}{a_4}} \Rightarrow m_0 \sim \sqrt{|t|} = |t|^{1/2}$$

$$\boxed{\beta = \frac{1}{2}}$$



$m \sim 0$

$T \gtrsim T_c \quad (t > 0)$

$m_0 = 0$

ϕ pequeño $\quad \phi^2 \gg \phi^4, \phi^6 \ldots$

$$S[\phi] = \int d\vec{r} \left[\frac{c}{2} |\vec{\nabla}\phi|^2 + \frac{a_2}{2} \phi^2 + \ldots \right]$$

$\Rightarrow$ aproximación gaussiana.

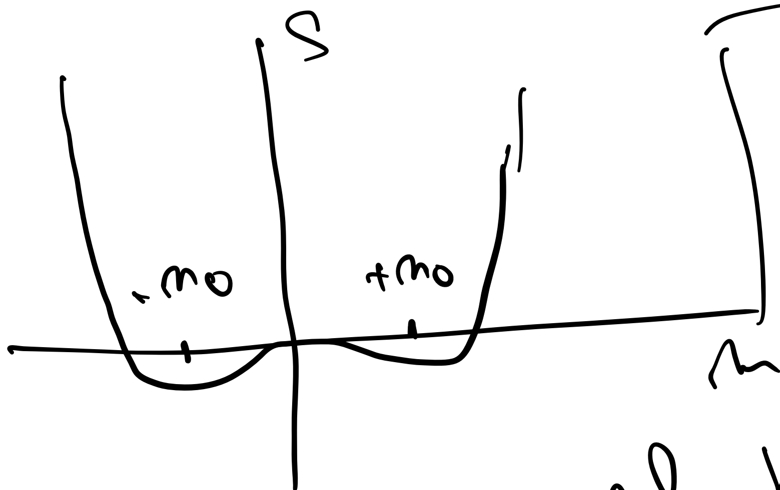
$\hat{\phi} = \sqrt{c}\, \phi$

$$S[\hat\phi] = \int d\vec{r} \left[\frac{1}{2}|\nabla\hat\phi|^2 + \frac{\hat a_2}{2}\hat\phi^2 + \cdots\right]$$

$[\hat a_2] = \frac{1}{L^2}$ $\quad \hat a_2 = \frac{1}{\xi^2} \sim t$

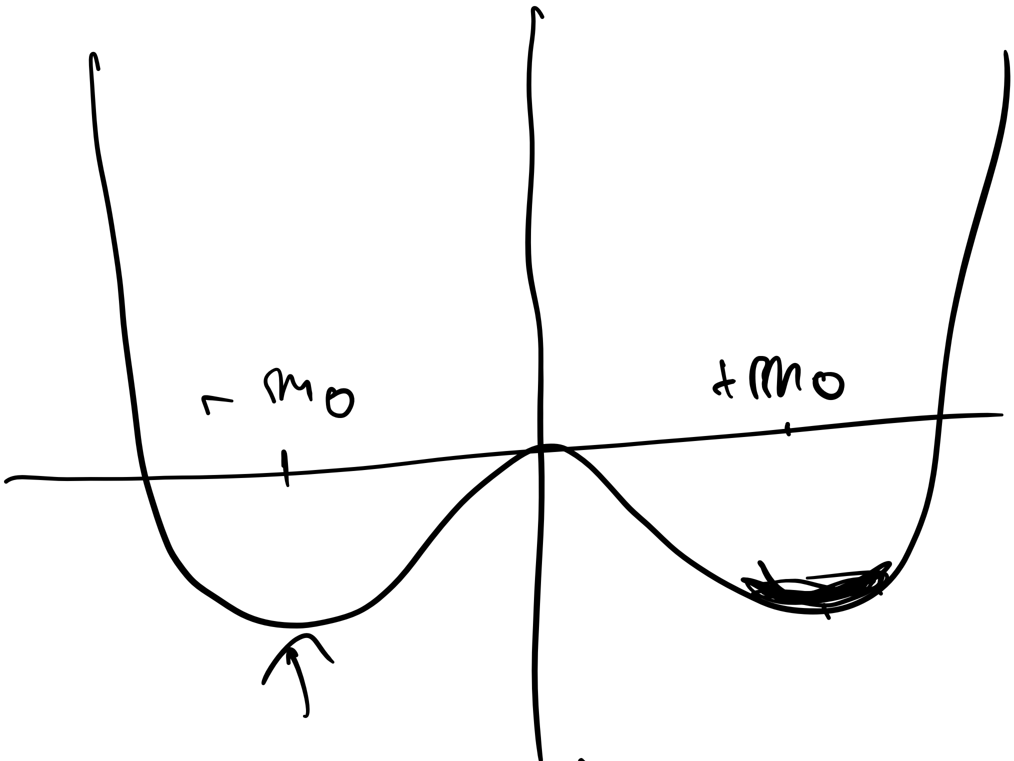
$\xi \sim t^{-1/2}$ $\quad \boxed{\nu = \frac{1}{2}}$

Ahora si $T<T_c$ $\quad t<0$



$$\boxed{m_0 = \sqrt{\frac{-a_2}{a_4}}}$$

m_0 tal de $\frac{d}{dm}\left[\frac{a_2}{2}m^2 + \frac{a_4}{4}m^4\right] = 0$



$$\phi(\vec{r}) = \underline{m_0} + \delta\phi(\vec{r})$$

$\delta\phi(\vec{r})$ pequeno

$\delta\phi^2 \gg \delta\phi^4$ etc...

$$S\{\delta\phi\} = \int d\vec{r} \left[\frac{c}{2} |\nabla \delta\phi|^2 + \left(\frac{a_2}{2} + \frac{3}{2} a_4 m_0^2 \right) \delta\phi^2 + \dots \right]$$

$$S\{\delta\phi\} = \int d\vec{r} \left[\frac{c}{2} |\vec{\nabla}\delta\phi|^2 - \underset{>0}{\underset{\uparrow}{a_2}} \delta\phi^2 + \dots \right.$$

$$-a_2 \sim \frac{1}{\xi^2} \qquad +a_2 \sim |t|$$

$$\Rightarrow \xi \sim |t|^{-1/2} \qquad \nu = \frac{1}{2}$$

$$|\vec{\nabla}\phi|^2 + \underline{|\vec{\nabla}\Delta\phi|^2} + \dots$$

→ los términos que aparecen.

→ invariantes por rotaciones en el espacio.

$$\vec{\nabla}\phi \cdot \vec{\nabla}\phi$$

para el caso $\phi(\vec{r}) = m_0$ cte.

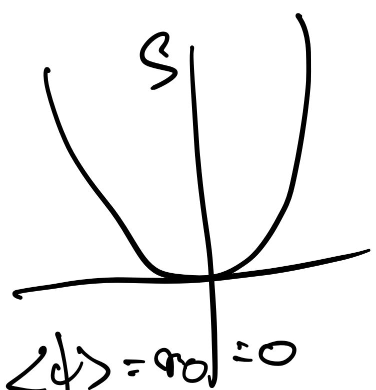
$$S\{\phi\} = \int d\vec{r}\left[\frac{c}{2}|\nabla\phi|^2 + \dots + \frac{a_2}{2}\phi^2 + \frac{a_4}{4}\phi^4 + \frac{a_6}{6}\phi^6 + \frac{a_8}{8}\phi^8 + \dots\right]$$

$a_4 > 0$

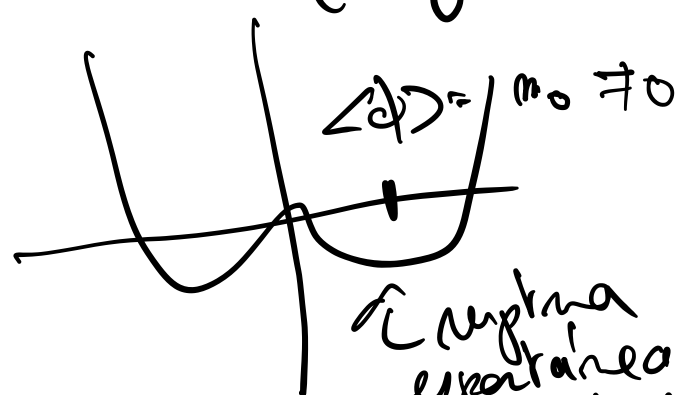
ϕ pequeño. $\beta = \frac{1}{2}$, $\gamma = \frac{1}{2}$

etc. . .

$t > 0$



$t < 0$



ruptura espontánea de la simetría $\mathbb{Z}_2$

¿Qué pasa si $a_4 < 0$?

$$\int_{-\infty}^{\infty} dx\, e^{-\alpha x^2 - \beta x^4 + \gamma x^6}$$

$\beta < 0$

$\gamma > 0$

si $a_4 < 0 \Rightarrow$ tomar en cuenta
el término $\frac{a_6}{6}\phi^6$; $a_6 > 0$

obs

$$S[\phi] = \int d\vec{r}\left[\frac{c}{2}|\nabla\phi|^2 + \ldots\right.$$

potencias pares de ϕ ?

$\ln[\operatorname{ch}\phi] \Leftarrow$

simetría $\phi \to -\phi$

grupo de simetría $\begin{cases}\phi \to \phi\\ \phi \to -\phi\end{cases}$

$\mathbb{Z}_2$

$\sigma_i \to \begin{cases}\sigma_i\\ -\sigma_i\end{cases} \forall i$

$\mathbb{Z}_2$

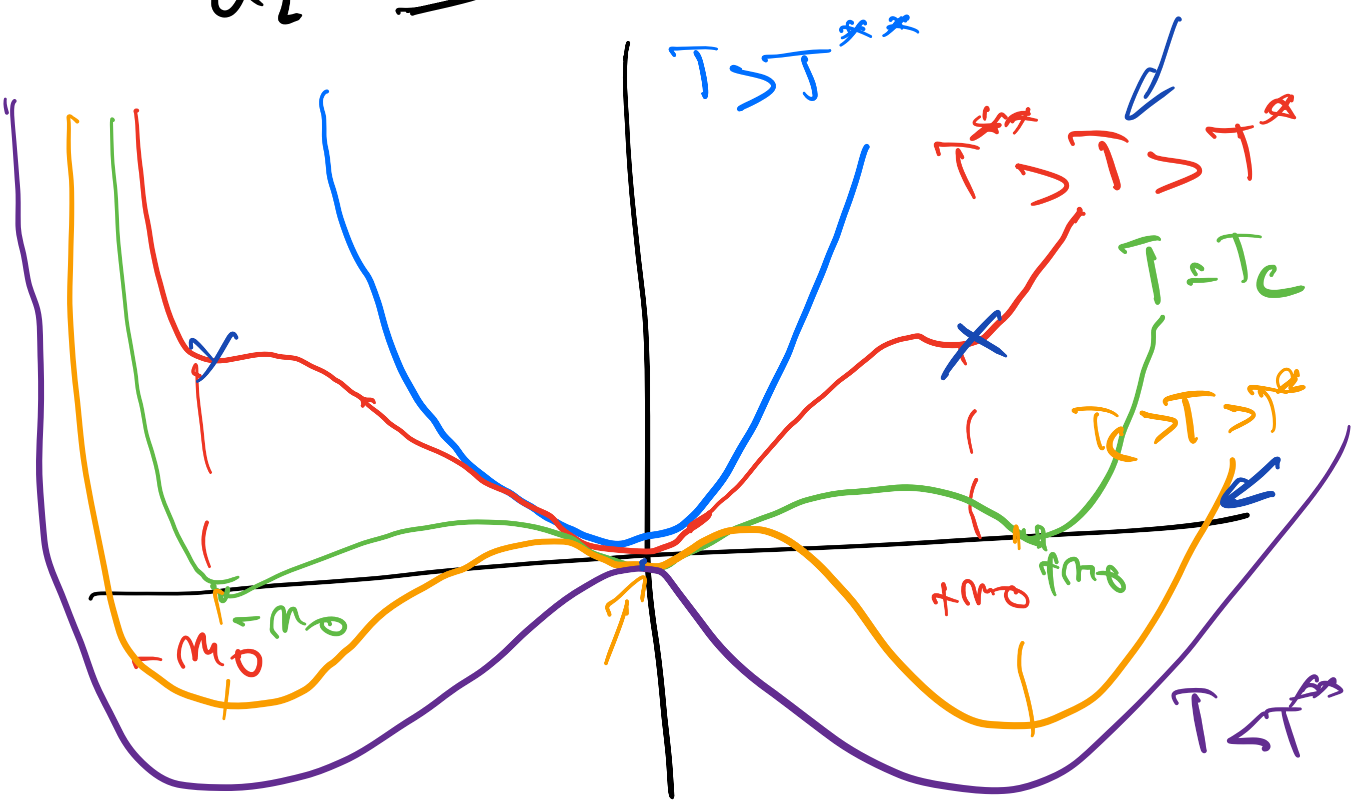
El caso $a_4<0$, $a_6>0$

$$S\{\phi\} = \int d\bar{r}\left[\frac{c}{2}|\bar{\nabla}\phi|^2 + \frac{a_2}{2}\phi^2 + \frac{a_4}{4}\phi^4 + \frac{a_6}{6}\phi^6\right]$$

$\phi = m$ indep. de $\bar{r} \Rightarrow \bar{\nabla}\phi = \bar{0}$

$$\boxed{S\{\phi\} = V\left[\frac{a_2}{2}m^2 + \frac{a_4}{4}m^4 + \frac{a_6}{6}m^6\right]}$$

$a_2 = \underline{a}\,t \qquad t = \dfrac{T-T^*}{T^*}$

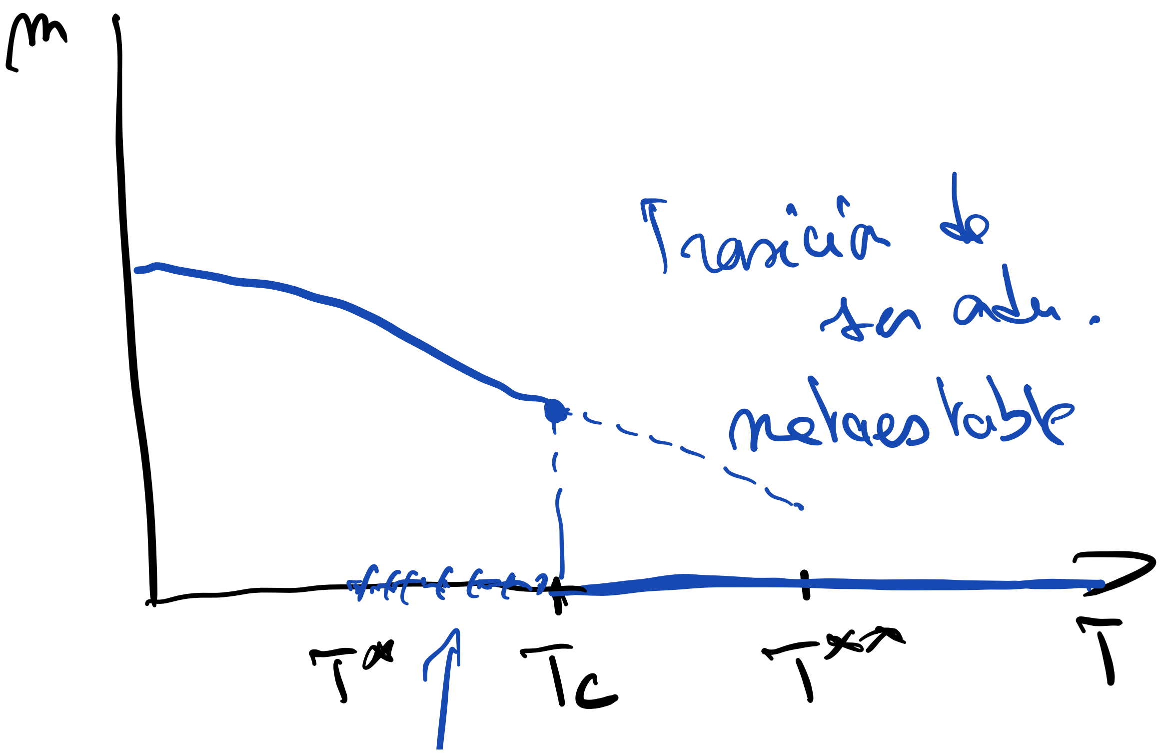


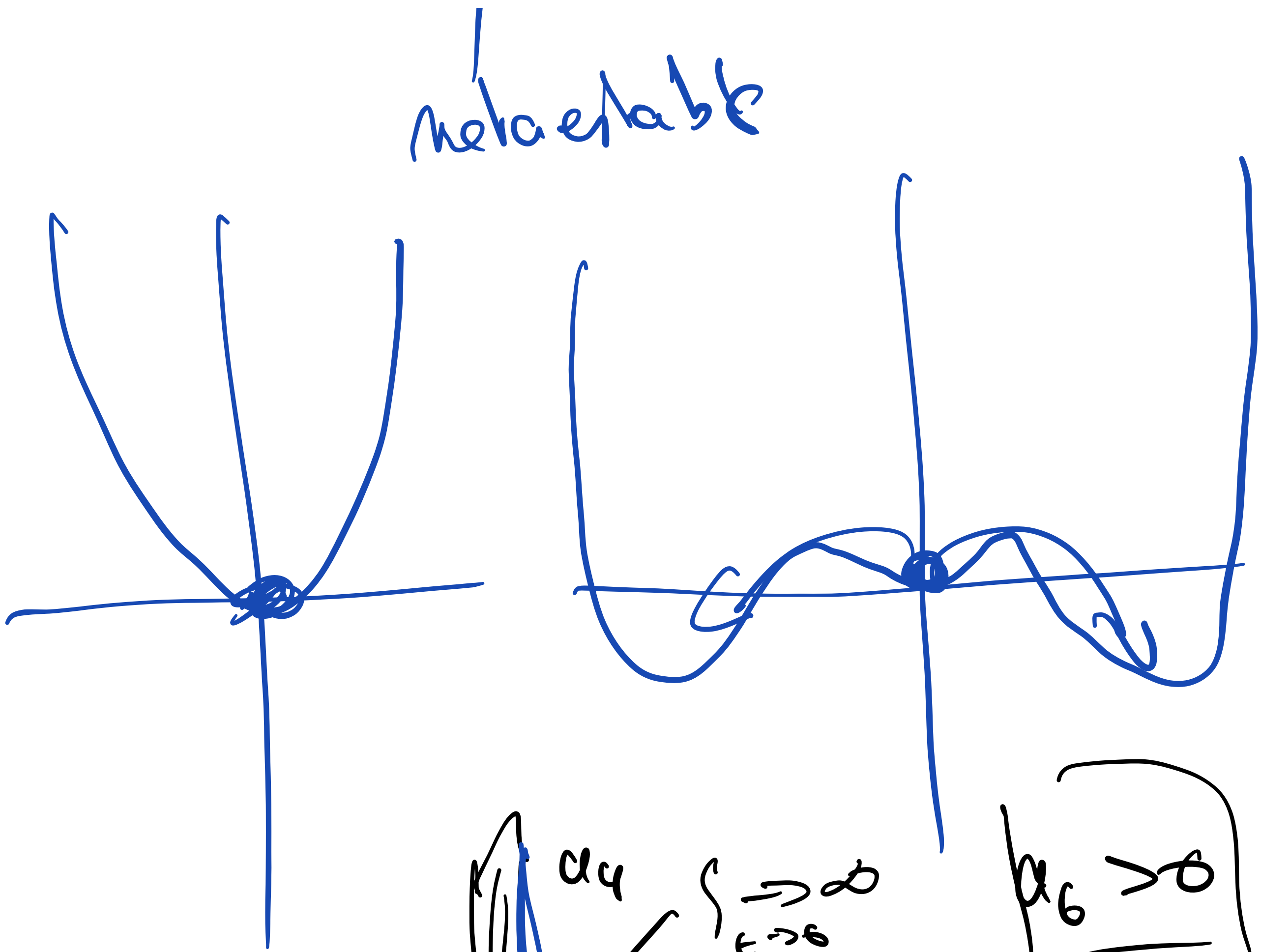
$$T^{**} = \frac{3a_4^2}{4 a a_6} > T^*$$

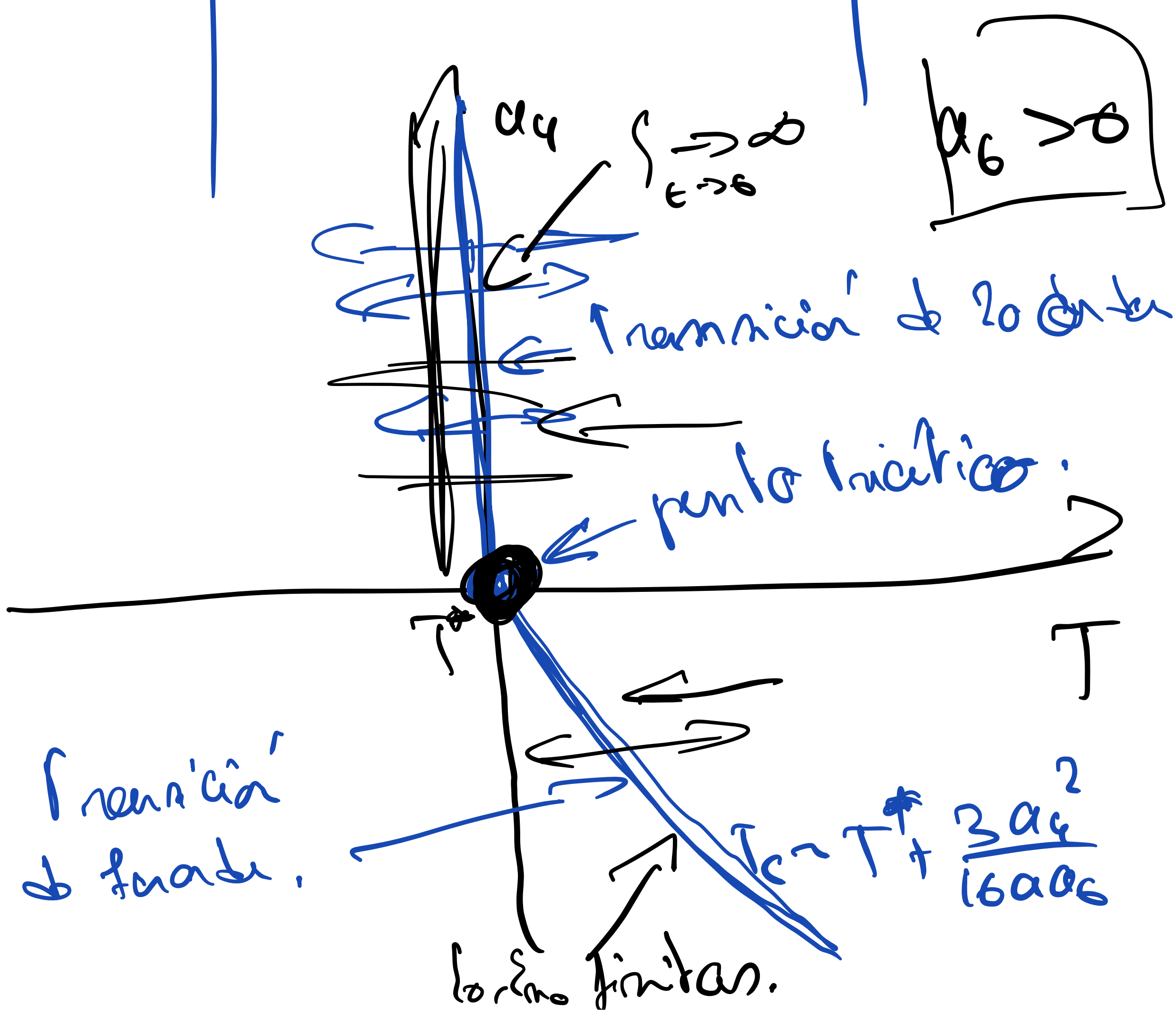
$$S(0) = S(\pm m_0)$$

$$m_0 = \left[\frac{2|a_4|}{a_6} + \frac{2|a_4|}{a_6}\sqrt{1 - \frac{4a(T-T^*)a_6}{a_4^2}}\right]$$

$$\boxed{T_c = T^* + \frac{3a_4^2}{16 a a_6}}$$

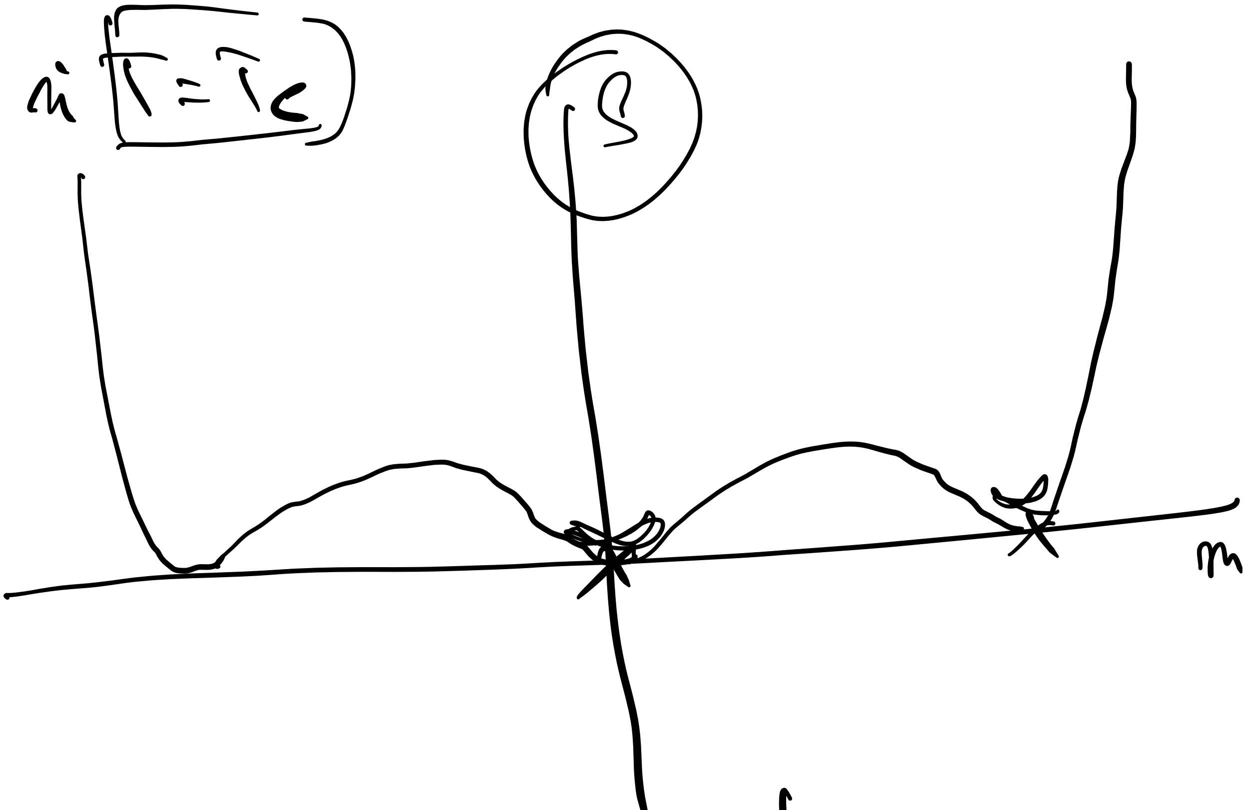



metaestable


a_4
$\xi \to \infty$
$\epsilon \to 0$
$a_6 > 0$
Transición de 2o orden
punto tricrítico.
T^*
T
Transición de fase.
$T_c \simeq T^* + \frac{3a_4^2}{16 a a_6}$
lo ξno finitas.

el caso $a_4 < 0$, $a_6 > 0$

si $\boxed{T = T_c}$



tres mínimos para ϕ

$\underline{\phi = 0}$ y $\underline{\phi = \pm m_0} = \pm \frac{3|a_4|}{4a_6}$

$\phi \overset{m=0}{=} \delta\phi$ $\quad \delta\phi$ pequeño

$S[\phi] = \int d\vec{r} \left[\frac{c}{2} |\nabla \delta\phi|^2 + \frac{m_0}{2} \delta\phi^2 + \dots \right]$

$$u_0 = \left.\frac{d^2 S}{dm^2}\right|_{m=0} > 0$$

$$\frac{1}{\xi_0^2} = u_0$$

alrededor de $m = m_0$

$$\phi = m_0 + \delta\phi$$

$$\boxed{S[\phi] = \int d\vec{r} \left[\frac{c}{2} |\nabla\delta\phi|^2 + \frac{u_{m_0}}{2} \delta\phi^2 + \ldots \right]}$$

$$u_{m_0} = \left.\frac{d^2 S}{dm^2}\right|_{m_0} > 0$$

$$\frac{1}{\xi_{m_0}^2} = u_{m_0}$$

→ coexistencia de dos fases

$m = 0$ y $m = m_0$

cada una con su longitud

de correlación ξ_0 y ξ_{m_0} ambos finitos!

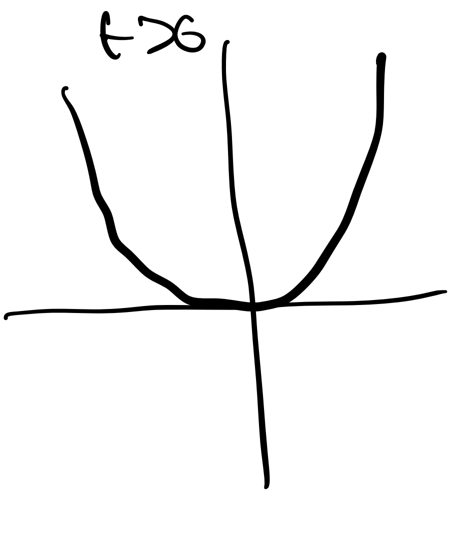
Por último:

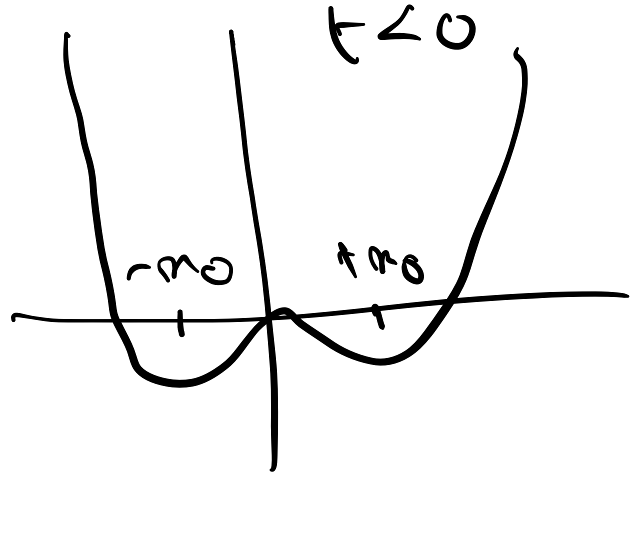
El punto tri-crítico:

$a_4 = 0 \quad , \quad a_6 > 0$

$$S[\phi] = \int d\vec{r} \left[\frac{c}{2} |\nabla \phi|^2 + \frac{a_2}{2}\phi^2 + \frac{a_6}{6}\phi^6 \right]$$

$a_2 = at \qquad a > 0 \quad t = T - T_c$





magnetización de $at + a_6 m^4 = 0$

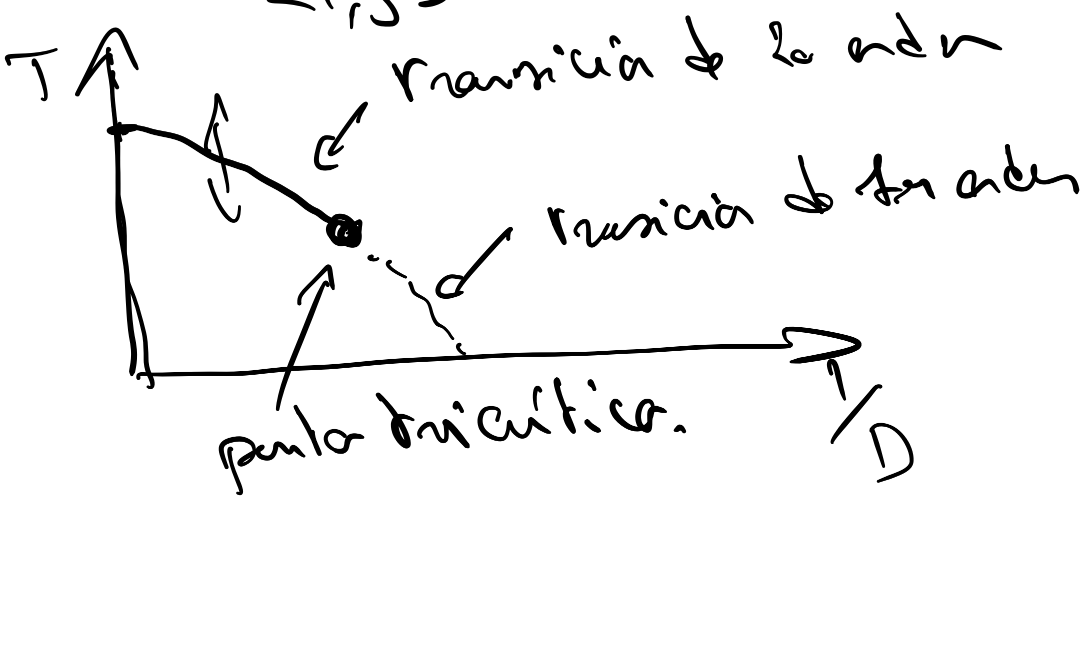
$$m_0 = \pm \left(\frac{-at}{a_6} \right)^{1/4} \quad t < 0$$

$$\Rightarrow m_0 \sim |t|^{1/4} \quad \beta = \frac{1}{4}$$

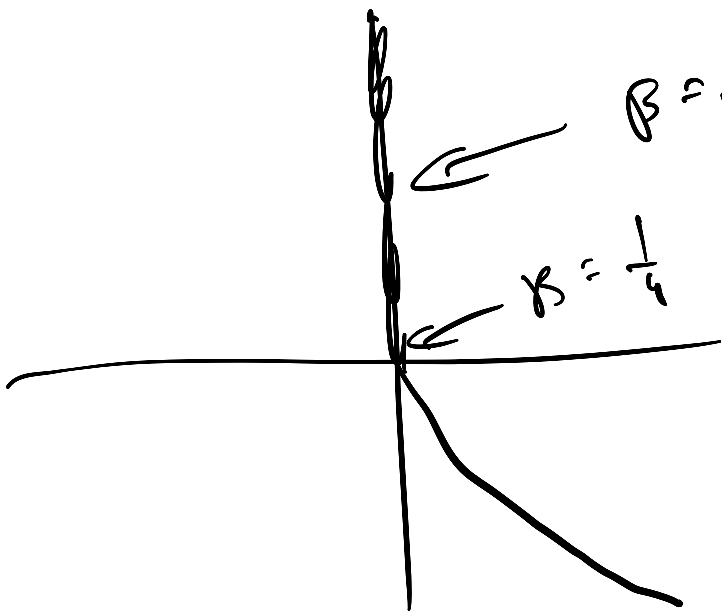
Modelo de Blume - Capel (1966)

$$\sigma_i = \pm 1, 0$$

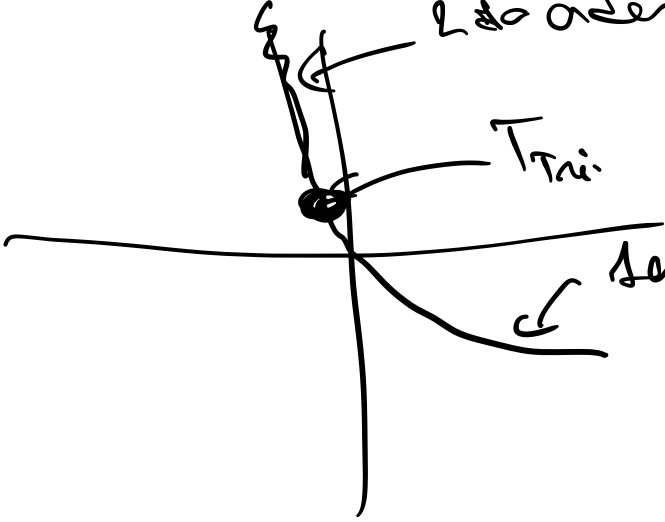
$$H = -J \sum_{\langle i,j \rangle} \sigma_i \sigma_j - D \sum_i \sigma_i^2$$



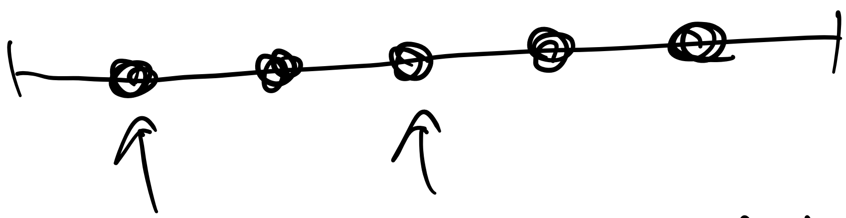
→ Equivalente a la aproximación de campo medio.



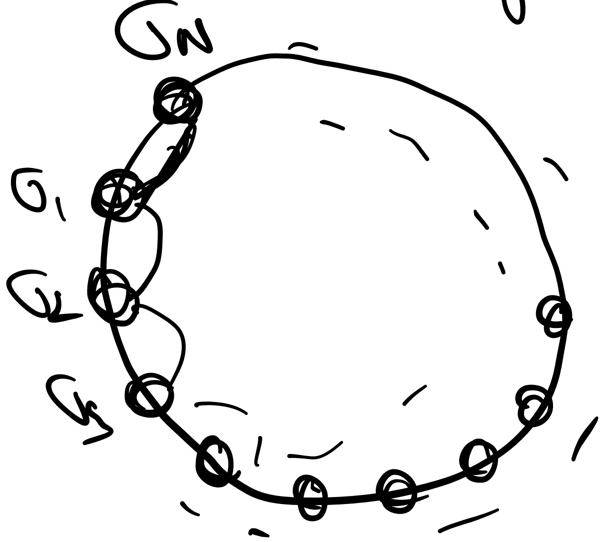
↓ Fluctuaciones



III Solución exacta de Ising en 1-D



→ Cond. de Bord. periódicas.



$$\sigma_{N+1} \equiv \sigma_1$$

$$\sigma_{i+N} = \sigma_i$$

$$H = -J \sum_{i=1}^{N} \sigma_i \sigma_{i+1} - h \sum_{i=1}^{N} \sigma_i$$

def $K = \frac{J}{k_B T}$ $\hat{h} = \frac{h}{k_B T}$

$$Z = \sum_{\{\sigma\}} e^{K \sum_{i=1}^{N} \sigma_i \sigma_{i+1} + \hat{h} \sum_i \sigma_i}$$

$$= \sum_{\substack{\sigma_1 = \pm 1 \\ \sigma_2 = \pm 1 \\ \vdots \\ \sigma_N = \pm 1}} \Big[\underline{e^{K\sigma_1\sigma_2}}\; e^{\frac{\hat{h}}{2}\sigma_1}\; \underline{e^{\frac{\hat{h}}{2}\sigma_2}}$$

$$\times\, e^{K\sigma_2\sigma_3}\; \underline{e^{\frac{\hat{h}}{2}\sigma_2}}\; e^{\frac{\hat{h}}{2}\sigma_3} \times \cdots$$

$$\times\, e^{K\sigma_N\sigma_1}\; e^{\frac{\hat{h}}{2}\sigma_1}\; e^{\frac{\hat{h}}{2}\sigma_N}$$

$$T(\sigma_i, \sigma_{i+1})_{11} = e^{K\sigma_i\sigma_{i+1} + \frac{h}{2}\sigma_i + \frac{h}{2}\sigma_{i+1}}\Big|_{\substack{\sigma_i = 1 \\ \sigma_{i+1} = 1}}$$

$$T_{21} = \text{---}\Big|_{\substack{\sigma_i = -1 \\ \sigma_{i+1} = +1}}$$

$$T_{12} = \text{--}\Big|_{\substack{\sigma_i = 1 \\ \sigma_{i+1} = -1}}$$

$$T_{22} = \text{--}\Big|_{\sigma_i = \sigma_{i+1} = -1}$$

$$T = \begin{pmatrix} e^{K+h} & e^{-K} \\ e^{-K} & e^{K-h} \end{pmatrix}$$

$$Z = \sum_{\substack{i_1 = 1,2 \\ i_2 = 1,2 \\ \vdots \\ i_N = 1,2}} T_{i_1, i_2} T_{i_2, i_3} \cdots T_{i_N, i_1}$$

$$\Rightarrow \boxed{Z = \mathrm{Tr}\{T^N\}}$$