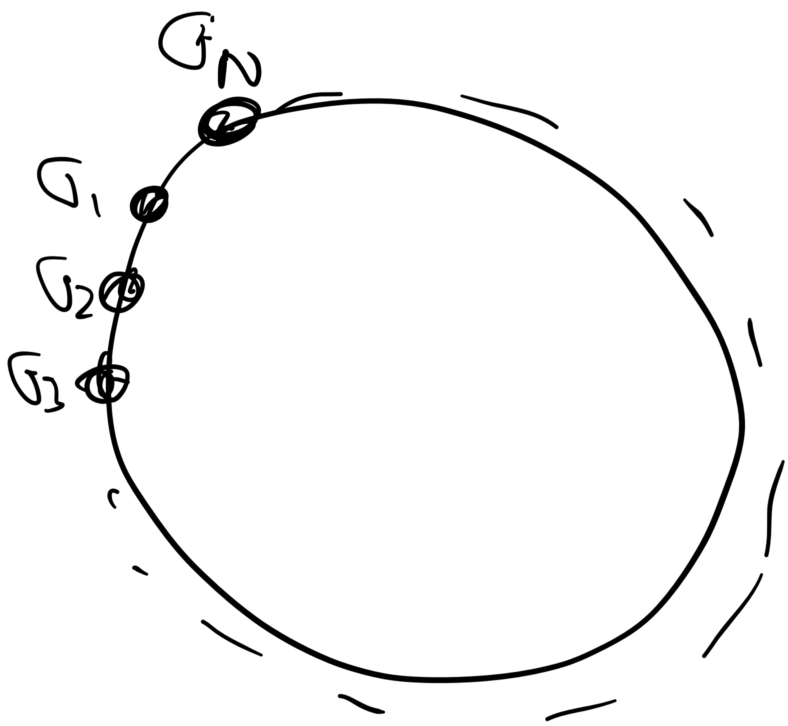


$$\sigma_{N+1} = \sigma_1$$

$$H = -J \sum_{i=1}^{N} \sigma_i \sigma_{i+1} - h \sum_{i=1}^{N} \sigma_i$$

$$K = \frac{J}{k_B T} \qquad \hat{h} = \frac{h}{k_B T}$$

$$Z = \sum_{\{\sigma\}} e^{K \sum_{i=1}^{N} \sigma_i \sigma_{i+1} + \frac{\hat{h}}{2} \sum_i \sigma_i + \frac{\hat{h}}{2} \sum_i \sigma_i}$$

$$T = \begin{pmatrix} e^{K+\hat h} & e^{-K} \\ e^{-K} & e^{K-\hat h} \end{pmatrix}$$

$\sigma_i = 1$, $\sigma_{i+1} = -1$

$\sigma_i = \sigma_{i+1} = -1$

$\sigma_i = \sigma_{i+1} = 1$

$\sigma_i = -1$, $\sigma_{i+1} = 1$

$$Z = \sum_{\substack{i_1 = 1,2 \\ i_2 = 1,2 \\ \vdots}} T_{i_1 i_2} T_{i_2 i_3} \cdots T_{i_N i_1}$$

$$= \mathrm{tr}\left\{ T^N \right\}$$

Si ϵ_1 y ϵ_2 son los autovalores de T

$$\boxed{\epsilon_{1/2} = e^{K} \mathrm{ch}(\hat h) \pm \left(e^{2K} \mathrm{sh}(\hat h) + e^{-2K} \right)^{1/2}}$$

$$\epsilon_1 > \epsilon_2 > 0$$

$$\boxed{Z = \epsilon_1^N + \epsilon_2^N}$$

límite $N \to \infty$; $N \gg 1$

$$Z = \epsilon_1^N \left(1 + \left(\frac{\epsilon_2}{\epsilon_1}\right)^N\right) \sim \epsilon_1^N$$

$$F = -k_B T \ln Z = -k_B T N \ln \epsilon_1$$

$$m(h) = \frac{1}{N} \frac{\partial}{\partial h} \ln Z = \frac{\partial}{\partial h} \ln \epsilon_1$$

$$= \frac{sh(\hat{h})}{k_B T \sqrt{sh^2(\hat{h}) + e^{-4K}}}$$

obs $\quad m(h)\Big|_{h=0} = 0 \quad \forall T$

$\Rightarrow$ No hay fase ordenada!

2) Función de correlación

el caso $h=0$

$$T = \begin{pmatrix} e^{K} & e^{-K} \\ e^{-K} & e^{K} \end{pmatrix} \qquad \begin{aligned} \epsilon_1 &= 2\,ch(K) \\ \epsilon_2 &= 2\,sh(K) \end{aligned}$$

$$\underset{i<j}{\langle \sigma_i \sigma_j \rangle} = \boxed{\frac{1}{Z} \sum_{\{\sigma\}} e^{-\beta H}\, \sigma_i \sigma_j}$$

introducimos la matriz

$$\Sigma = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\langle \sigma_i \sigma_j \rangle = \frac{1}{Z}\, \mathrm{tr}\left\{ T^{i} \underset{\uparrow}{\Sigma}\, T^{j-i} \underset{\uparrow}{\Sigma}\, T^{N-j} \right\}$$

$$O = O^t = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \qquad OO = \mathbb{1}$$

$$OTO = \begin{pmatrix} \epsilon_1 & 0 \\ 0 & \epsilon_2 \end{pmatrix} = \tilde{T}$$

$$O\Sigma O = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \tilde{\Sigma}$$

$$\mathrm{tr}\left\{ T^i \underset{OO}{\wedge} \Sigma \underset{OO}{\wedge} T^{j-i} \underset{OO}{\wedge} \Sigma \underset{OO}{\wedge} T^{N-j} \underset{OO}{\wedge} \right\}$$

$$O T^i O = O \underbrace{T . T . T \ldots T}_{i \text{ veces}} O$$

$$= O T O O T O O T \ldots -$$

$$= \tilde{T}^i$$

$$\langle \sigma_i \sigma_j \rangle = \frac{1}{Z} \mathrm{Tr} \left\{ \hat{T}^{i} \hat{\Sigma} \hat{T}^{j-i} \hat{\Sigma} \hat{T}^{N-j} \right\}$$

$$= \frac{1}{Z} \mathrm{Tr} \left\{ \begin{pmatrix} \epsilon_1^{i} & 0 \\ 0 & \epsilon_2^{i} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \epsilon_1^{j-i} & 0 \\ 0 & \epsilon_2^{j-i} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} \epsilon_1^{N-j} & 0 \\ 0 & \epsilon_2^{N-j} \end{pmatrix} \right\}$$

$$\Rightarrow \langle \sigma_i \sigma_j \rangle = \frac{\epsilon_1^{N-j+i} \epsilon_2^{j-i} + \epsilon_2^{N-j+i} \epsilon_1^{j-i}}{\epsilon_1^{N} + \epsilon_2^{N}}$$

si $N \gg 1$

distancia

$$\Rightarrow \langle \sigma_i \sigma_j \rangle \underset{N \gg 1}{\sim} \left(\frac{\epsilon_2}{\epsilon_1} \right)^{j-i}$$

$$\Rightarrow \langle \sigma_i \sigma_j \rangle \sim \underline{\left(\mathrm{tgh}(K) \right)^{j-i}}$$

$$= e^{-\frac{|i-i'|}{\xi}}$$

ξ longitud de correlación

$$\boxed{\xi = \frac{1}{\ln(\coth(K))} = \frac{1}{\ln\left(\coth\left(\frac{J}{k_B T}\right)\right)}}$$

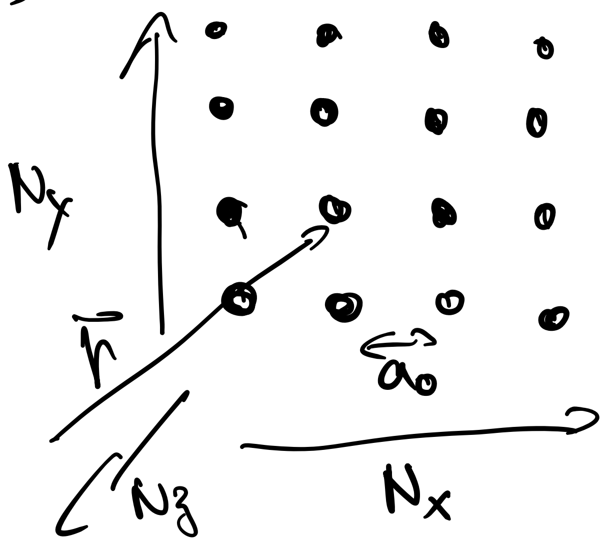
obs si $T \to \infty$; $\xi \to 0$

si $T \to 0 \Rightarrow \xi \to \infty$

V El Modelo Gaussiano

$$S = \int d\vec{x} \left[\frac{c}{2}(\vec{\nabla}\phi)^2 + \frac{a_2}{2}\phi^2\right]$$

1) Sistema discreto y finito



$N = N_x N_y$ etc...

$L_x = a_0 N_x$

$L_y = a_0 N_y$ etc...

→ Condiciones de borde periódicas.

$$\vec{r} = m_x a_0 \vec{e}_x + m_y a_0 \hat{e}_y + \dots$$

$m_x = 0, 1, \dots, N_x$

$m_y = 0, 1, \dots, N_y$

C.B.P.

$$\vec{r} + N_x a_0 \hat{e}_x \equiv \vec{r}$$

$$+ N_y a_0 \vec{e}_y \equiv \vec{r}$$

a cada punto $\vec{r} \Rightarrow \phi_{\vec{r}} \in \mathbb{R}$

Modos de Fourier

$$\underline{e^{i\vec{k}\cdot\vec{r}}} \qquad \underline{\vec{k} = \frac{2\pi n_x}{L_x}\vec{e}_x} + \frac{2\pi n_y}{L_y}\hat{e}_y + \dots$$

$$\boxed{-\frac{N_x}{2} < n_x \leq \frac{N_x}{2}} \quad \text{ou} \quad \underline{0 \leq n_x < N_x}$$

idem para n_y, n_z etc...

Zona de Brillouin.

<u>propiedades</u>

$$\alpha \neq 1 \; ; \Rightarrow \sum_{n=0}^{N-1} \alpha^n = \frac{1-\alpha^N}{1-\alpha}$$

1.D $$\sum_{x=1}^{N_x} \frac{e^{ikx}\, e^{-ik'x}}{N} = \delta_{k,k'} = \delta_{n_x,n_x'}$$

$\forall \cdot D$

$$\sum_{\vec{r}} \frac{e^{i(\vec{k}_1-\vec{k}_2)\cdot\vec{r}}}{N} = \delta_{\vec{k}_1,\vec{k}_2}$$

$$= \delta_{n_x^1, n_x^2}\,\delta_{n_y^1, n_y^2}\cdots$$

$$\sum_{\vec{k}\in Z.B} \frac{e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)}}{N} = \delta_{\vec{r}_1,\vec{r}_2}$$

$$= \delta_{m_x^1, m_x^2}\,\delta_{m_y^1, m_y^2}\cdots$$

$$\vec{r}_i = m_x^i a_0 \vec{e}_x + m_y^i a_0 \vec{e}_y + \cdots$$

Transf. de Fourier

$$\hat{\phi}_{\vec{k}} = \sum_{\vec{r}} e^{i\vec{k}\cdot\vec{r}}\,\phi_{\vec{r}}$$

<u>propiedades</u>

* $$\phi_{\vec{r}} = \sum_{\vec{k}} \frac{e^{-i\vec{k}\cdot\vec{r}}}{N} \hat{\phi}_{\vec{k}}$$

* $\phi_{\vec{r}}$ real $\Rightarrow \hat{\phi}_{-\vec{k}} = \hat{\phi}^{*}_{\vec{k}}$

* $$\sum_{\vec{r}} \phi^2_{\vec{r}} = \sum_{\vec{k}} \frac{1}{N} |\hat{\phi}_{\vec{k}}|^2$$

* $$\sum_{\vec{r}} \phi_{\vec{r}} \phi_{\vec{r}+d\hat{e}_x} = \frac{1}{2} \sum_{\vec{r}} [\phi_{\vec{r}-d\hat{e}_x} \phi_{\vec{r}} + \phi_{\vec{r}} \phi_{\vec{r}+d\hat{e}_x}]$$

$d = n a_0$

$$= \frac{1}{N} \sum_{\vec{k}} |\hat{\phi}_{\vec{k}}|^2 \cos(k_x d)$$

$$\boxed{S = \sum_{\vec{r}, \vec{r}'} \phi_{\vec{r}} A_{\vec{r}-\vec{r}'} \phi_{\vec{r}'}}$$

$A_{\vec{r}-\vec{r}'} \in \mathbb{R}$

$\hat{A}_{\vec{n}}$ $\in \mathbb{R}$

$\left(\hat{A}^{*}_{-\vec{n}}\right)$

$$A_{\vec{n}} = \sum_{\vec{k} \in Z.B}' \frac{1}{N} e^{i\vec{k}\cdot\vec{n}} \hat{A}_{\vec{k}}$$

$$S = \frac{1}{2N} \sum_{\vec{k}}' |\hat{\phi}_{\vec{k}}|^2 \left[\hat{A}_{\vec{k}} + \hat{A}^{*}_{\vec{k}}\right]$$

Fución de partición.

$$Z = \int_{-\infty}^{\infty} \prod_{n} d\phi_{\vec{n}}\, e^{-S}$$

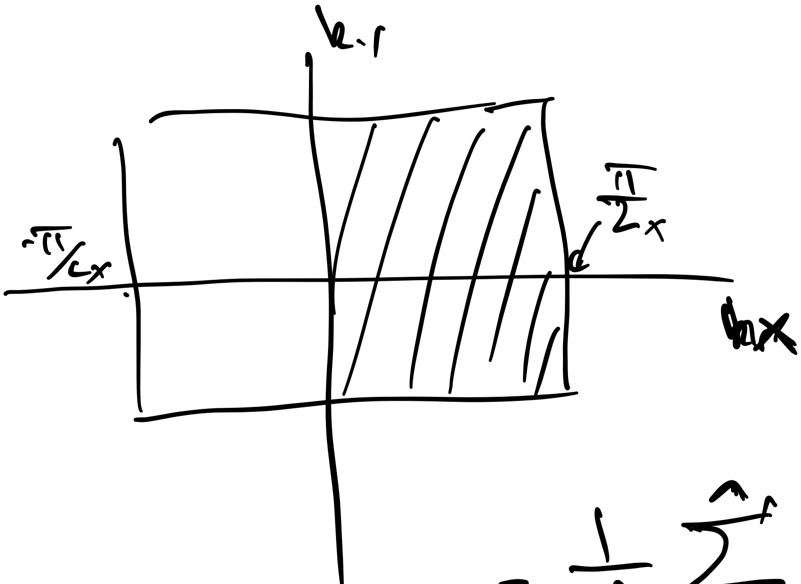
$d\phi_{\vec{n}} \rightarrow d\hat{\phi}_{\vec{k}}$ el Jacobiano es 1.

$$\hat{\phi}_{\vec{k}} = \hat{\phi}^R_{\vec{k}} + i\,\hat{\phi}^i_{\vec{k}} \quad y \quad \hat{\phi}_{\vec{k}} \quad y \quad \hat{\phi}_{-\vec{k}}$$

$$\prod_{\vec{k}} d\phi_{\vec{k}} = \prod_{\vec{k}} d\hat{\phi}^R_{\vec{k}}\, d\hat{\phi}^i_{\vec{k}} = \hat{\prod_{\vec{k}}} d\hat{\phi}^R_{\vec{k}}\, d\hat{\phi}^i_{\vec{k}}$$

$\forall\, k_x > 0$

$$= \hat{\prod_{\vec{k}}} d\hat{\phi}_{\vec{k}}$$



$$Z = \int \hat{\prod_{\vec{k}}} d\hat{\phi}_{\vec{k}}\; e^{-\frac{1}{N}\hat{\sum_{\vec{k}}} |\hat{\phi}_{\vec{k}}|^2 (\hat{A}_{\vec{k}} + \hat{A}^*_{\vec{k}})}$$

$$\int_{-\infty}^{\infty} dx\, e^{-\alpha x^2} = \sqrt{\frac{\pi}{a}} \qquad |\hat{\phi}_{\vec{k}}|^2 = \hat{\phi}^{R\,2}_{\vec{k}} + \hat{\phi}^{i\,2}_{\vec{k}}$$

$$\int_{-\infty}^{\infty} dx\, dy\, e^{-\alpha(x^2+y^2)} = \frac{\pi}{a}$$

$$\Rightarrow Z = \prod_{\vec{k}}^{\Lambda} \frac{\pi N}{2\,\mathrm{Re}\,\hat{A}_{\vec{k}}}$$

<u>Funciones de correlación:</u>

$$\langle \phi_{\vec{r}_1} \phi_{\vec{r}_2} \rangle = \frac{1}{N^2} \sum_{\vec{k}_1,\vec{k}_2} e^{i\vec{k}_1\cdot\vec{r}_1 + i\vec{k}_2\cdot\vec{r}_2}$$

$$\times \underline{\langle \phi_{\vec{k}_1} \phi_{\vec{k}_2} \rangle}$$

$$= \frac{1}{Z}\frac{1}{N^2} \sum_{\vec{k}_1,\vec{k}_2} e^{i\vec{k}_1\cdot\vec{r}_1 + i\vec{k}_2\cdot\vec{r}_2}$$

$$\times \int \prod_{\vec{k}}^{\Lambda} d\hat{\phi}_{\vec{k}}\; \hat{\phi}_{\vec{k}_1} \hat{\phi}_{\vec{k}_2}\, e^{-S}$$

$$\hat{\phi}_{\vec{k}_1} \hat{\phi}_{\vec{k}_2} = \underline{\hat{\phi}^R_{\vec{k}_1} \hat{\phi}^R_{\vec{k}_2}} + i\,\cancel{\hat{\phi}^R_{\vec{k}_1} \hat{\phi}^I_{\vec{k}_2}}$$

$$+ i\,\cancel{\hat{\phi}^I_{\vec{k}_1} \hat{\phi}^R_{\vec{k}_2}} - \underline{\hat{\phi}^I_{\vec{k}_1} \hat{\phi}^I_{\vec{k}_2}}$$

$$\frac{\int dx\,dy\, e^{-a(x^2+y^2)}\, x^2}{\int dx\,dy\, e^{-a(x^2+y^2)}} = \frac{1}{2a}$$

$$\boxed{\int dx\,dy\, e^{-a(x^2+y^2)}\, xy = 0}$$

el único caso no nulo será

$\vec{k}_2 = -\vec{k}_1 \qquad \phi^{a\,i}_{\vec{k}_2} = -\phi^{a\,i}_{\vec{k}_1}$

$$\frac{1}{\int dx\,dy\, e^{-a(x^2+y^2)}} \int dx\,dy\, (x^2+y^2) e^{-a(x^2+y^2)} = \frac{1}{a}$$

$$\boxed{\langle \phi_{\vec{n}_1} \phi_{\vec{n}_2} \rangle = \frac{1}{N^2} \sum_{\vec{k}}' e^{i\vec{k}\cdot(\vec{n}_1-\vec{n}_2)} \underline{\langle \hat{\phi}_{\vec{k}} \hat{\phi}_{-\vec{k}} \rangle}}$$

$$\Rightarrow \boxed{\langle \phi_{\vec{r}_1} \phi_{\vec{r}_2} \rangle = \frac{1}{N^2} \sum_{\vec{k}}^{\prime} \frac{N}{2\,\mathrm{Re}(A_{\vec{k}})} e^{i\vec{k}\cdot(\vec{r}_1 - \vec{r}_2)}}$$

ejemplo:

$$S = \sum_{\vec{r}}^{\prime} \left[\alpha \phi_{\vec{r}}^2 - \beta \left(\phi_{\vec{r}} \phi_{\vec{r}+a_0\hat{e}_x} + \phi_{\vec{r}} \phi_{\vec{r}+a_0\hat{e}_y} + \dots \right) \right]$$

$$S = \frac{1}{N} \sum_{\vec{k}}^{\prime} |\hat{\phi}_{\vec{k}}|^2 \left\{ \alpha - \beta \left[\cos(k_x a_0) + \cos(k_y a_0) + \dots \right] \right\}$$

$$= \frac{2}{N} \sum_{\vec{k}}^{\wedge} \dots \dots$$

$$Z = \int \prod_{\vec{k}}^{\wedge} d\hat{\phi}_{\vec{k}}\, e^{-S} = \prod_{\vec{k}}^{\wedge} \frac{\pi N}{2\left[\alpha - \beta\cos(k_x a_0) - \beta\cos(k_y a_0) \dots\right]}$$

y

$$\boxed{\langle \phi_{\vec{r}_1} \phi_{\vec{r}_2} \rangle = \frac{1}{N} \sum_{\vec{k}} \frac{e^{i\vec{k}\cdot(\vec{r}_1 - \vec{r}_2)}}{2[\alpha - \beta \cos(k_x a_0) - \beta \cos(k_y a_0) \cdots]}}$$

2) Límite continuo.

L_x, L_y, L_z finitos

$N_x, N_y, N_z \longrightarrow \infty$

$a_0 \longrightarrow 0$

$L_x = a_0 N_x$ finita

$$\vec{k} = \frac{2\pi n_x}{L_x}\hat{e}_x + \frac{2\pi n_y}{L_y}\hat{e}_y + \cdots$$

$-\infty < n_x < \infty$ idem para n_y, n_z etc. .

$L_x L_y L_z = V = N a_0^d$ finito

$$\delta_{\bar{r}_1,\bar{r}_2} \longrightarrow a_0^d\,\delta(\bar{r}_1-\bar{r}_2)$$

$$\sum_{\bar{r}}' \longrightarrow \frac{1}{a_0^d}\int d\bar{r}$$

$$N \rightarrow \frac{V}{a_0^d}$$

$$\sum_{\bar{k}}' \frac{e^{i\bar{k}\cdot(\bar{r}_1-\bar{r}_2)}}{N} = \delta_{\bar{r}_1,\bar{r}_2}$$

$$\Rightarrow \boxed{\frac{1}{V}\sum_{\bar{k}}' e^{i\bar{k}\cdot(\bar{r}_1-\bar{r}_2)} = \delta(\bar{r}_1-\bar{r}_2)}$$

$$\sum_{\bar{r}}' \frac{1}{N} e^{i(\bar{k}_1-\bar{k}_2)\cdot\bar{r}} = \delta_{\bar{k}_1,\bar{k}_2}$$

$$\Rightarrow \int d\vec{r}\, e^{i(\vec{k}_1-\vec{k}_2)\cdot\vec{r}} = V\,\delta_{\vec{k}_1,\vec{k}_2}$$

$$\phi_{\vec{r}} = \underbrace{\phi(\vec{r})} = \frac{1}{a_0^d}\,\underline{\phi_{\vec{r}}}$$

T.F.

$$\tilde{\phi}_{\vec{k}} = \sum_{\vec{r}} e^{i\vec{k}\cdot\vec{r}}\phi_{\vec{r}} \longrightarrow \int d\vec{r}\, e^{i\vec{k}\cdot\vec{r}}\phi(\vec{r})$$

$$\phi(\vec{r}) = \frac{1}{V}\sum_{\vec{k}} e^{i\vec{k}\cdot\vec{r}}\tilde{\phi}_{\vec{k}}$$

$$* \sum_{\vec{r}} \phi_{\vec{r}}^2 \longrightarrow a_0^d \int d\vec{r}\, \phi^2(\vec{r})$$

$$= \frac{a_0^d}{V}\sum_{\vec{k}} \left|\tilde{\phi}(\vec{r})\right|^2$$

$* \sum_{\vec{r}}' \phi_{\vec{r}} \phi_{\vec{r} \pm a_0 \hat{e}_x}$

$$\to a^d \int d\vec{r}\, \phi(\vec{r}) \left(\phi(\vec{r}) \pm a_0 \partial_x \phi(\vec{r}) + \frac{a_0^2}{2} \partial_x^2 \phi(\vec{r}) + \cdots \right)$$

$$+ \sum_{\vec{r}}' \frac{1}{2} \left(\phi_{\vec{r}-a_0\hat{e}_x} \phi_{\vec{r}} + \phi_{\vec{r}} \phi_{\vec{r}+a_0\hat{e}_x} \right)$$

$$\to a^d \int d\vec{r} \left[\phi^2(\vec{r}) + a_0^2 \partial_x^2 \phi(\vec{r}) + \cdots \right]$$

$$= \frac{a_0^d}{V} \sum_{\vec{k}}' |\hat{\phi}_{\vec{k}}|^2 \left[1 - a_0^2 k^2 + \cdots \right]$$

$$* \ S = \sum_{\vec{r}}' \left[\alpha \phi_{\vec{r}}^2 - \beta \left(\phi_{\vec{r}} \phi_{\vec{r}+a_0\hat{e}_x} + \cdots \right) \right]$$

$$\to \int d\vec{r} \left[\frac{a_2}{2} \phi^2(\vec{r}) + \frac{c}{2} (\vec{\nabla}\phi)^2 + \cdots \right]$$

$$\text{con } \boxed{\begin{aligned} a_2 &= 2a_0^{d}(\alpha-\beta) \\ c &= \beta a_0^{d+2} \end{aligned}}$$

$$S = \frac{1}{V}\sum_{\vec{k}} |\phi_{\vec{k}}^{d}|^2 \left(\frac{a_2}{2} + \frac{k^2 c}{2}\right)$$

$$\langle \phi(\vec{r}_1)\,\phi(\vec{r}_2)\rangle = \frac{1}{a_0^{2d}}\langle \phi_{\vec{r}_1}\,\phi_{\vec{r}_2}\rangle$$

$$\longrightarrow \frac{1}{a_0^{2d}}\,\frac{1}{\left(\frac{V}{a_0^{d}}\right)}\sum_{\vec{k}} \frac{e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)}}{2\left[\alpha-\beta(\cos k_x a_0+\dots)\right]}$$

$$\cos k_x a_0 \simeq 1 - \frac{1}{2}k_x^2 a_0^2 + \dots$$

$$\boxed{\langle \phi(\vec{r}_1)\,\phi(\vec{r}_2)\rangle = \frac{1}{V}\sum_{\vec{k}} \frac{e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)}}{a_2 + ck^2}}$$

3] límite de tamaño infinito

$L_x, L_y, L_z \to \infty$, $V \to \infty$

$\Delta k_x = \frac{2\pi}{L_x}$ idem para y, z.

$$\sum_{\vec{k}} \to \frac{1}{\Delta k_x \Delta k_y \Delta k_z} \int d\vec{k}$$

$$\boxed{\sum_{\vec{k}} \to \frac{V}{(2\pi)^d} \int d\vec{k}}$$

$$\boxed{\delta_{\vec{k}_1, \vec{k}_2} \to \frac{(2\pi)^d}{V} \delta(\vec{k}_1 - \vec{k}_2)}$$

$$\sum_{\vec{k}_1} \delta_{\vec{k}_1, \vec{k}_2} = 1 \to \int d\vec{k}_1 \, \delta(\vec{k}_1 - \vec{k}_1) = 1$$

$$\sum_{\vec{k}} \frac{e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)}}{V} = \delta(\vec{r}_1-\vec{r}_2)$$

$$\rightarrow \int \frac{d\vec{k}}{(2\pi)^d} e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)} = \delta(\vec{r}_1-\vec{r}_2)$$

$$\int d\vec{r}\, e^{i(\vec{k}_1-\vec{k}_2)\cdot\vec{r}} = V\,\delta_{\vec{k}_1,\vec{k}_2}$$

$$\longrightarrow \int d\vec{r}\, e^{i(\vec{k}_1-\vec{k}_2)\cdot\vec{r}} = (2\pi)^d\,\delta(\vec{k}_1-\vec{k}_2)$$

T.F.

$$\hat{\phi}(\vec{k}) = \int d\vec{r}\, e^{i\vec{k}\cdot\vec{r}}\,\phi(\vec{r})$$

$$\phi(\vec{r}) = \frac{1}{(2\pi)^d}\int d\vec{k}\, e^{-i\vec{k}\cdot\vec{r}}\,\hat{\phi}(\vec{k})$$

$$S = \frac{1}{V} \sum_{\bar{k}}' |\phi_{\bar{k}}|^2 \left(\frac{a_2}{2} + \frac{c}{2} k^2 \right)$$

$$\to \int \frac{d\bar{k}}{(2\pi)^d} |\phi(\bar{k})|^2 \left(\frac{a_2}{2} + \frac{c}{2} k^2 \right)$$

$$\Rightarrow \boxed{\langle \phi(\bar{r}_1) \phi(\bar{r}_2) \rangle = \int \frac{d\bar{k}}{(2\pi)^d} \frac{e^{i\bar{k}\cdot(\bar{r}_1 - \bar{r}_2)}}{a_2 + c k^2}}$$

$k_x^2 + k_y^2 + k_z^2$

$$G(\bar{r}) = \int d\bar{k} \, \frac{e^{i\bar{k}\cdot\bar{r}}}{k^2 + m^2} \quad \{ \sim \frac{1}{m}$$

$|\bar{r}| \to \infty$!