Modelo Gaussiano
Límite al continuo.

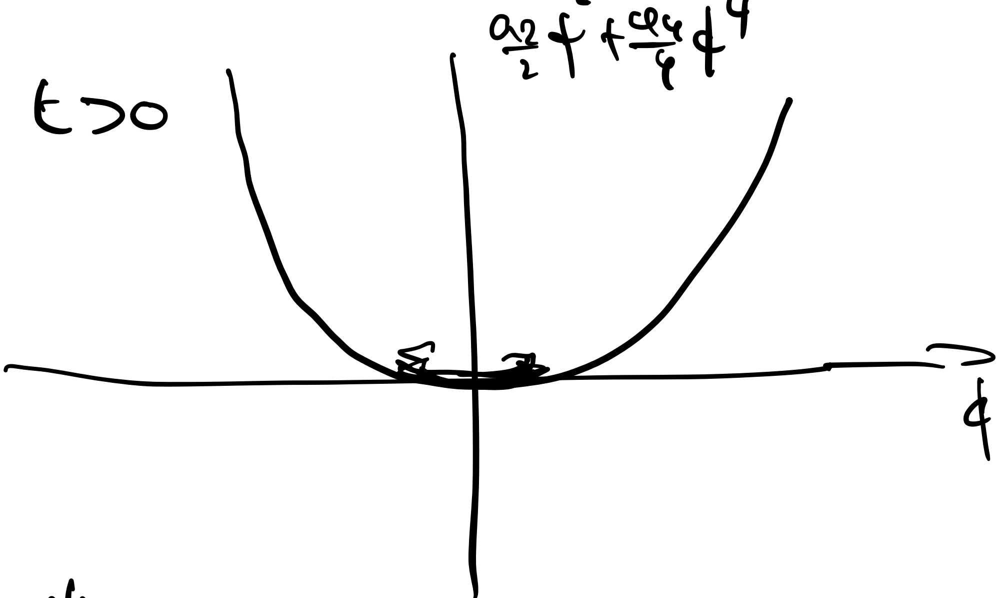
V finito C.B.P.	$V \to \infty$
$S = \frac{1}{V} \sum_{\vec{k}}' \lvert\hat{\phi}_{k}\rvert^2 \left(\frac{a_2}{2} + \frac{c}{2} k^2\right)$	$S = \int \frac{d\vec{k}}{(2\pi)^d} \lvert\hat{\phi}(\vec{k})\rvert^2 \left(\frac{a_2}{2} + \frac{c}{2} k^2\right)$
	$\Longleftrightarrow S = \int d\vec{r} \left[\frac{c}{2}(\vec{\nabla}\phi)^2 + \frac{a_2}{2}\phi^2\right]$
$\langle\phi(\vec{r}_1)\phi(\vec{r}_2)\rangle = \frac{1}{V} \sum_{\vec{k}}' \frac{e^{i\vec{k}\cdot(\vec{r}_1 - \vec{r}_2)}}{a_2 + ck^2}$	$\boxed{\langle\phi(\vec{r}_1)\phi(\vec{r}_2)\rangle = \int \frac{d\vec{k}}{(2\pi)^d} \frac{e^{i\vec{k}\cdot(\vec{r}_1 - \vec{r}_2)}}{a_2 + ck^2}}$

obs G-L.

$$S_{G.L.} = \int d\vec{r} \left[\underline{\frac{c}{2}(\vec{\nabla}\phi)^2 + \frac{a_2}{2}\phi^2} + \cancel{\frac{a_4}{4}\phi^4} + \dots\right]$$

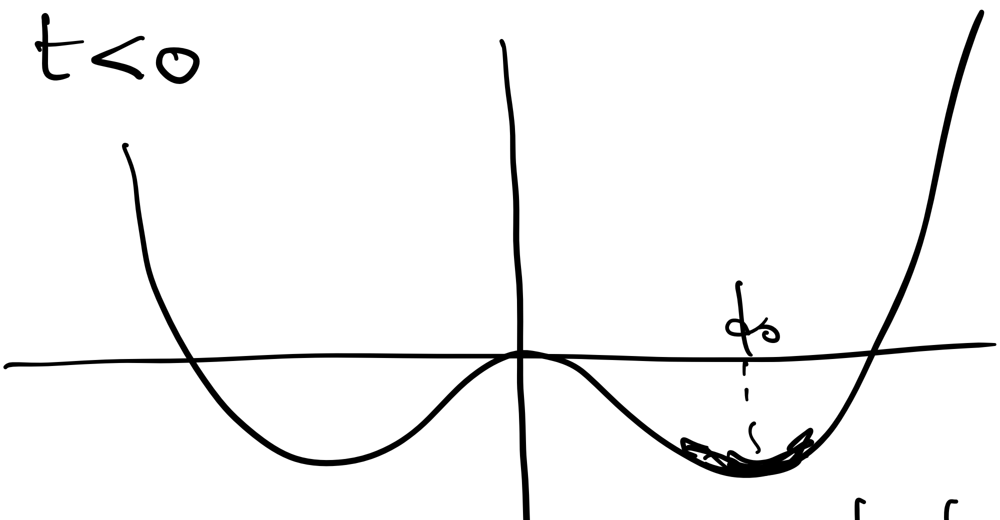
$a_2 \sim T - T_c \quad a_2 = a t \quad t = \frac{T-T_c}{T_c}$

$t>0$



$\phi^4 \sim 0$

$t<0$

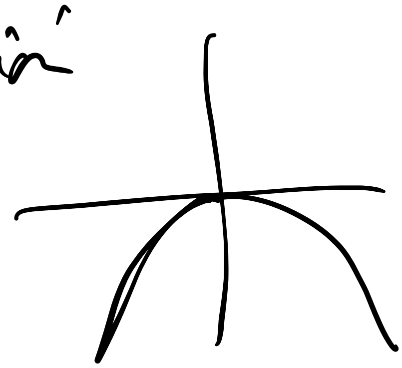


$\sim \phi = \phi_0 + \underline{\delta\phi}$

$$S = \int d\vec{r} \left[\frac{c}{2} (\vec{\nabla} \phi)^2 + \frac{a_2}{2} \phi^2 \right]$$

$$c > 0, \quad a_2 > 0$$

→ no tiene transición de fase.



para el modelo Gaussiano V finito.

$$\langle \hat{\phi}(\vec{k}_1) \hat{\phi}(\vec{k}_2) \rangle = \frac{V}{a_2 + c k_1^2} \delta_{\vec{k}_1, -\vec{k}_2}$$

$$\langle \hat{\phi}(\vec{k}) \hat{\phi}(-\vec{k}) \rangle = \boxed{\frac{V}{a_2 + c k^2} = V S(\vec{k})}$$

$$S(\vec{k}) = \frac{1}{V} \int d\vec{r}_1 \, d\vec{r}_2 \, e^{i \vec{k}_1 \cdot (\vec{r}_1 - \vec{r}_2)} \langle \phi(\vec{r}_1) \phi(\vec{r}_2) \rangle_c$$

$$S(\overline{k}=\overline{0}) = \frac{1}{V}\int d\overline{r}_1\, d\overline{r}_2 \, \langle \phi(\overline{r}_1)\, \phi(\overline{r}_2) \rangle$$

$T > T_c$

recuerden para Ising

$$\chi = \sum_{i,j} \langle \sigma_i \sigma_j \rangle - \cancel{\langle \sigma_i \rangle} \cancel{\langle \sigma_j \rangle}$$

$0 \quad\quad 0$

$$\sigma_i \Longleftrightarrow \phi(\overline{r})$$

$$\sum_i \Longleftrightarrow \int d\overline{r}$$

$$S(\overline{k}=0) = \frac{\chi}{V} \sim \lim_{k\to 0} \frac{1}{a_2 + ck^2} = \frac{1}{a_2}$$

$$\frac{\chi}{V} \sim \frac{1}{a_2} \sim \frac{1}{t} = t^{-1}$$

$$\Rightarrow \frac{\chi}{V} \sim t^{-\gamma} \Rightarrow \gamma = 1$$

La función a dos puntos para $r \to \infty$

$$\langle \phi(\vec{r}_1)\, \phi(\vec{r}_2) \rangle \propto \int d\vec{k}\, \frac{e^{i\vec{k}\cdot(\vec{r}_1 - \vec{r}_2)}}{k^2 + m^2}$$

$$m^2 = \frac{a_2}{c}$$

en 1-D $\to$ th. de residuos.

3-D $\to$ coord esféricos + Th. residuos.

el caso $|\vec{r}_1 - \vec{r}_2| \to \infty$

$$G(\vec{r}) = \langle \phi(\vec{r}_1)\, \phi(\vec{r}_2) \rangle \qquad \vec{r} = \vec{r}_1 - \vec{r}_2$$

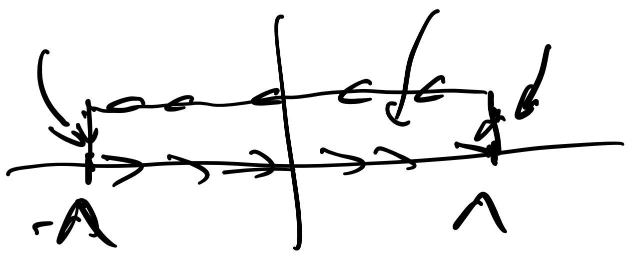
$G(\vec{r})$ para $r \to \infty$.

$$G(\vec{r}) = \int d\vec{k}\, \frac{e^{i\vec{k}\cdot\vec{r}}}{k^2 + m^2}$$

$$G(\vec{r}) = \int d\vec{k} \int_0^{\infty} d\alpha \, e^{i\vec{k}\cdot\vec{r} - \alpha(k^2 + m^2)}$$

$$e^{i k_x r_x + i k_y r_y + \dots - \alpha k_x^2 - \alpha k_y^2 \dots}$$

$$\int_{-\infty}^{+\infty} dk_x \, e^{i k_x x - \alpha k_x^2} = e^{-\frac{x^2}{4\alpha}} \int_{-\infty}^{\infty} e^{-\alpha\left(k_x - \frac{ix}{\alpha}\right)^2} dk_x$$



$\Lambda \to \infty$

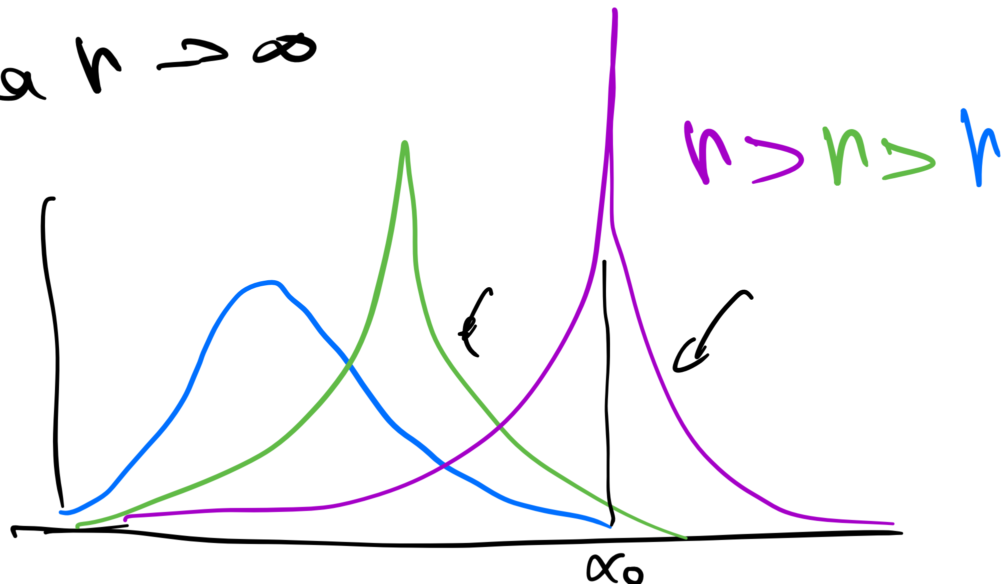
$$= \sqrt{\frac{\pi}{\alpha}} \, e^{-\frac{x^2}{4\alpha}}$$

$$\Rightarrow G(\vec{r}) = \int_0^{\infty} \left(\frac{\pi}{\alpha}\right)^{d/2} e^{-\frac{r^2}{4\alpha} - m^2\alpha} \, d\alpha$$

$$= \int_0^\infty e^{-f(\alpha)}\, d\alpha$$

$$\boxed{f(\alpha) = \frac{n^2}{4\alpha} + \alpha m^2 + \frac{d}{2}\ln\alpha}$$

para $n \to \infty$



→ método del punto de silla.

$$\boxed{f(\alpha) \simeq f(\alpha_0) + \frac{1}{2} f''(\alpha_0)\,(\alpha-\alpha_0)^2 + \dots}$$

$$\int_0^\infty e^{-f(\alpha)}\, d\alpha \simeq \int_{-\infty}^{\infty} e^{-f(\alpha)}\, d\alpha$$

$$= \int_{-\infty}^{\infty} e^{-f(\alpha_0) - \frac{1}{2} f''(\alpha_0)(\alpha-\alpha_0)^2 + \ldots} \, d\alpha$$

$$= \boxed{e^{-f(\alpha_0)} \sqrt{\frac{2\pi}{f''(\alpha_0)}}} \left(1 + O\left(\frac{1}{r}\right)\right)$$

$$f'(\alpha) = \frac{-r^2}{4\alpha^2} + m^2 + \frac{d}{2\alpha}$$

$$\Rightarrow \boxed{\alpha_0} = \frac{1}{2m^2}\left[-\frac{d}{2} + \sqrt{\frac{d^2}{4} + r^2 m^2}\right] \rightarrow \boxed{\frac{r}{2m}} + \ldots$$

$$f''(\alpha) = \frac{r^2}{2\alpha^3} - \frac{d}{2\alpha^2}$$

$$f(\alpha_0) = mr + \frac{d}{2} \ln\left(\frac{r}{2m}\right)$$

$$f''(\alpha_0) = \frac{4m^3}{r} - \frac{4md}{r^2} \underset{\max}{\sim} \frac{4m^3}{r}$$

$$\Rightarrow \boxed{G(\vec{r}) \sim \frac{\text{cte}}{r^{\frac{d-1}{2}}}\, e^{-mr}} \qquad m \neq 0$$

$$\xi = \frac{1}{m}$$

El caso $m = 0$ $(a_2 = 0)$ (el punto crítico de G.L. $T = T_c$)

$$S = \int d\vec{r} \left(\frac{a}{2} (\nabla \phi)^2 \right)$$

$$G(\vec{r}) = \int d\vec{k}\, \frac{e^{i\vec{k}\cdot\vec{r}}}{k^2} \qquad L^{-d+2}$$

$$-\Delta G(\vec{r}) = \delta(\vec{r})$$

$$G(\vec{r}) = \begin{cases} |x| & d = 1 \\ \sim \ln|\vec{r}| & d = 2 \\ \sim \dfrac{1}{r^{d-2}} & d > 2 \end{cases}$$

→ invariante de escala.

$$\vec{r} \to \lambda \vec{r}$$

$$G(\vec{r}) \to G(\lambda \vec{r}) = \lambda^{2-d} G(\vec{r})$$

$$S = \int d\vec{r} \left[\frac{c}{2} (\vec{\nabla} \phi)^2 \right]$$

$$\vec{r} \to \lambda \vec{r}$$

$$\phi \to \lambda^{\frac{2-d}{2}} \phi$$

$$c \to c$$

$$\int \left| \begin{array}{l} d\vec{r} \to \lambda^{d} d\vec{r} \\ \vec{\nabla}^2 \to \lambda^{-2} \vec{\nabla}^2 \\ \phi^2 \to \lambda^{2-d} \phi \end{array} \right| S$$

G.L.

$$Z = cte \int \mathcal{D}\phi \, e^{-S^{c}}$$

$$S = \int d\vec{r} \left[\frac{c}{2}(\nabla\phi)^2 + \frac{a_2}{2}\phi^2 + \frac{a_4}{4}\phi^4 + \dots \right]$$

ϕ_0 tq $S[\phi_0]$ es mínima

$$Z \sim e^{-S[\phi_0]}$$

$$I = \int_{-\infty}^{\infty} dx \, e^{-f(x)}$$

x_0 mínimo de f

$$f(x) = f(x_0) + \frac{1}{2} f''(x-x_0)^2 + \cdots$$

$$I = e^{-f(x_0)} \int_{-\infty}^{\infty} dx\, e^{-\frac{1}{2} f''(x-x_0)^2 + \cdots}$$

C.M.

1ra contribución de las fluctuaciones.

$$S(\phi) = \int d^d r \left[\frac{c}{2} (\nabla \phi)^2 + \frac{a_2}{2} \phi^2 + \frac{a_4}{4} \phi^4 \right]$$

$a_4 > 0$

$a_2 = at$

$$Z = \int D\phi\, e^{-S[\phi]}$$

ϕ_0 mínima de S

$$\phi_0 = \begin{cases} 0 & \text{si } t > 0 \\ \sqrt{-\frac{a_2}{a_4}} & \text{si } t < 0 \end{cases}$$

$\phi(\vec{r}) = \phi_0 + \psi(\vec{r})$

$$S(\phi) = S(\phi_0) + \int d\vec{r} \left[\frac{c}{2} (\nabla \psi)^2 + \frac{at + 3a_4\phi_0^2}{2} \psi^2 + O(\psi^3) \right]$$

$$= S(\phi_0) + \int d\vec{r} \, \frac{c}{2} \left[(\nabla \psi)^2 + \frac{1}{\xi^2} \psi^2 \right] + \ldots$$

$$\frac{c}{\xi^2} = at + 3a_4\phi_0^2 = \begin{cases} at & \text{si } t > 0 \\ -2at & t < 0 \end{cases}$$

$$\langle \psi(\vec{r}) \psi(\vec{r}') \rangle \sim \frac{e^{-\frac{|\vec{r} - \vec{r}'|}{\xi}}}{|\vec{r} - \vec{r}'|^{\frac{d-1}{2}}}$$

$$Z \sim e^{-\beta F}$$

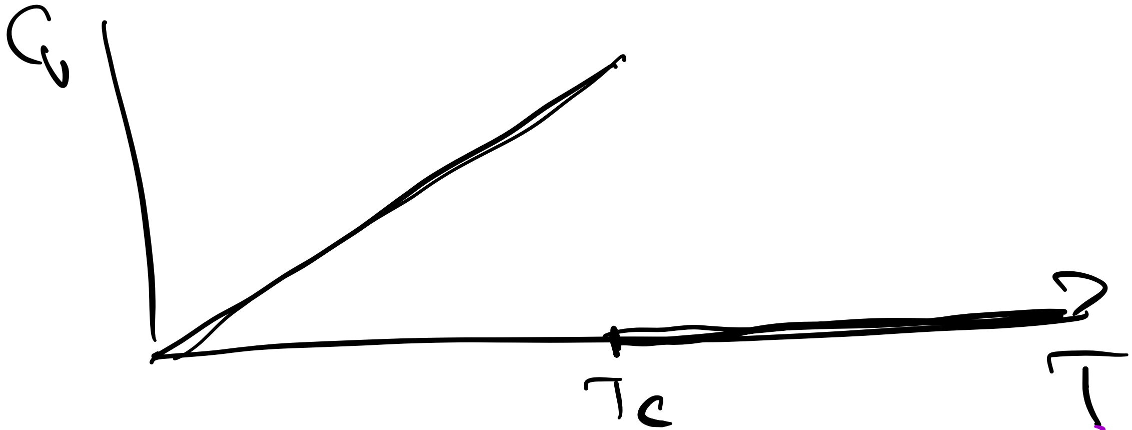
$$\text{si } Z \sim e^{-S_0}$$

$$S_0 = V\left[\frac{a t}{2}\phi_0^2 + \frac{a_4}{4}\phi_0^4\right]$$

$$\phi_0 = \begin{cases} 0 & t>0 \\ \pm\sqrt{\frac{-a_2}{a_4}} & t<0 \end{cases}$$

$$\boxed{\frac{F}{V}(\phi_0) = \frac{a t}{2}\phi_0^2 + \frac{a_4}{4}\phi_0^4 + \dots}$$

$$\frac{+}{V} C_V = -\frac{T}{V}\frac{\partial^2 F}{\partial T^2} = \begin{cases} 0 & \text{si } T>T_c \\ \frac{T a^2}{2a_4} & T<T_c \end{cases}$$



$$S(\phi) \simeq S(\phi_0) + \int d\vec{r}\, \frac{c}{2}\left[(\nabla\psi)^2 + \frac{1}{\xi^2}\psi^2\right]$$

$$Z = e^{-S(\phi_0)} \prod_{\vec{k}} \left[c\left(k^2 + \frac{1}{\xi^2}\right)\right]^{-1/2}$$

$$-\frac{1}{V}\ln Z \simeq \frac{at\phi_0^2}{2} + \frac{a_4}{4}\phi_0^4 + \frac{1}{V}\sum_{\vec{k}}' \frac{1}{2}\ln\left[c\left(k^2+\frac{1}{\xi^2}\right)\right]$$

$$\xrightarrow[V\to\infty]{} \frac{at\phi_0^2}{2} + \frac{a_4\phi_0^4}{4} + \frac{1}{2}\int \frac{d\vec{k}}{(2\pi)^d} \ln\left[c\left(k^2+\frac{1}{\xi^2}\right)\right]$$

$$C_V = \begin{cases} 0 + \dfrac{T}{2}\displaystyle\int \frac{d\vec{k}}{(2\pi)^d} \frac{1}{\left[c\left(k^2+\frac{1}{\xi^2}\right)\right]^2} \\[2ex] \dfrac{Ta^2}{8a_4} + 2T\displaystyle\int \frac{d\vec{k}}{(2\pi)^d} \frac{1}{\left[c\left(k^2+\frac{1}{\xi^2}\right)\right]^2} \end{cases}$$

$$\int \frac{d\vec{k}}{(2\pi)^d} \frac{1}{c\left(k^2+\frac{1}{\xi^2}\right)^2} \Bigg| = I$$

para $d > 4$ la integral

diverge.

→ hay que regularizar la integral

→ cut-off. k_{max}

$k_0 \sim \frac{1}{\xi_0}$; ξ_0 min.

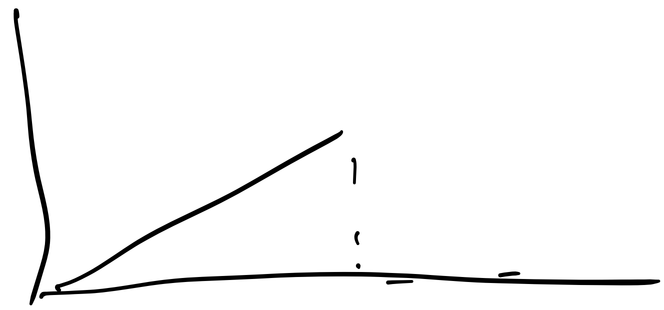
ξ_0 = escala microscópica. ($\xi_0 \sim a_0$)

si $d < 4$

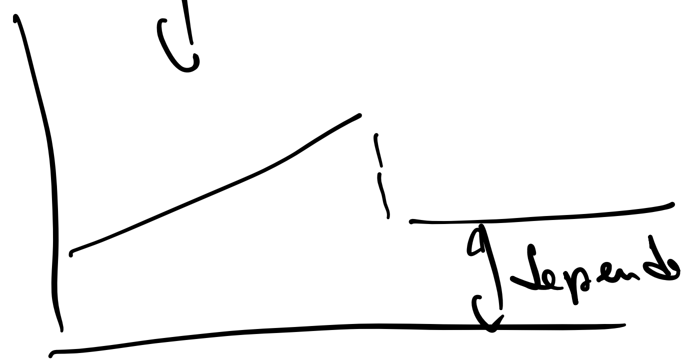
$$I \simeq cte \left(\frac{\xi^{4-d}}{c^2} + \cancel{\frac{\xi_0^{4-d}}{c^2}} \right)$$

para $d > 4$

$$C_V = \begin{cases} 0 + cte\, \xi_0^{4-d} & T > T_c \\ \frac{I a^2}{8 a_4} + cte\, \xi_0^{4-d} & T < T_c \end{cases}$$



¿!



depende de ξ_0

si $d<4$

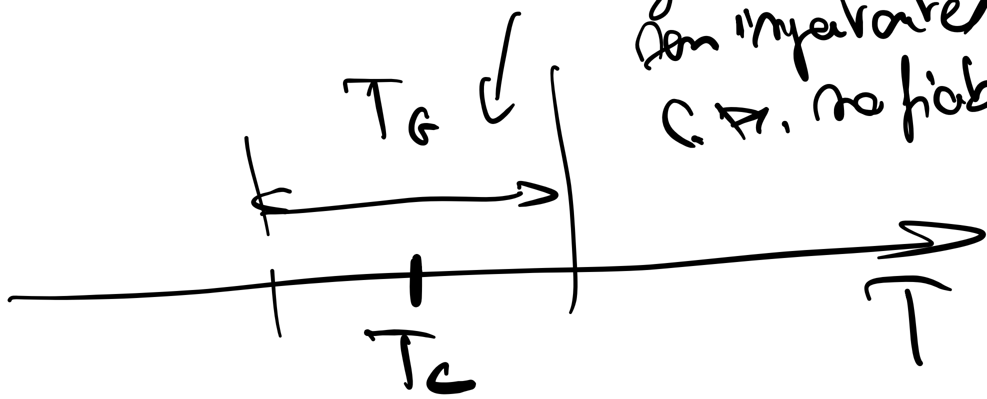
$$C_v = \begin{cases} 0 + cte\,\dfrac{\xi^{4-d}}{C^2} & T>T_c \\[2ex] \underline{\dfrac{T a^2}{8 a_4}} + \underline{cte\,\dfrac{\xi^{4-d}}{C^2}} & \end{cases}$$

si $\xi \to 0$ T lejos de T_c

→ C.M. ok.

si $T \sim T_c$, $\xi \to \infty$, $\xi^{4-d} \to \gg$

Las fluctuaciones son importantes
C.A. no fiable.



$$\frac{\xi(T)^{4-d}}{C^2} \sim \frac{T_c\, a^2}{8 a_4}$$

$$\xi \sim \xi_0 |t|^{-\nu}$$

$$\frac{\xi_0^{4-d}\, |t|^{-\nu(4-d)}}{C^2} \sim \frac{T_c\, a^2}{8 a_4} \quad ; \quad C^2 \sim \xi_0^4$$

$$\Rightarrow \xi_0^{-d}\, |t|^{-\nu(4-d)} \gtrsim \frac{T_c\, a}{8 a_4}$$

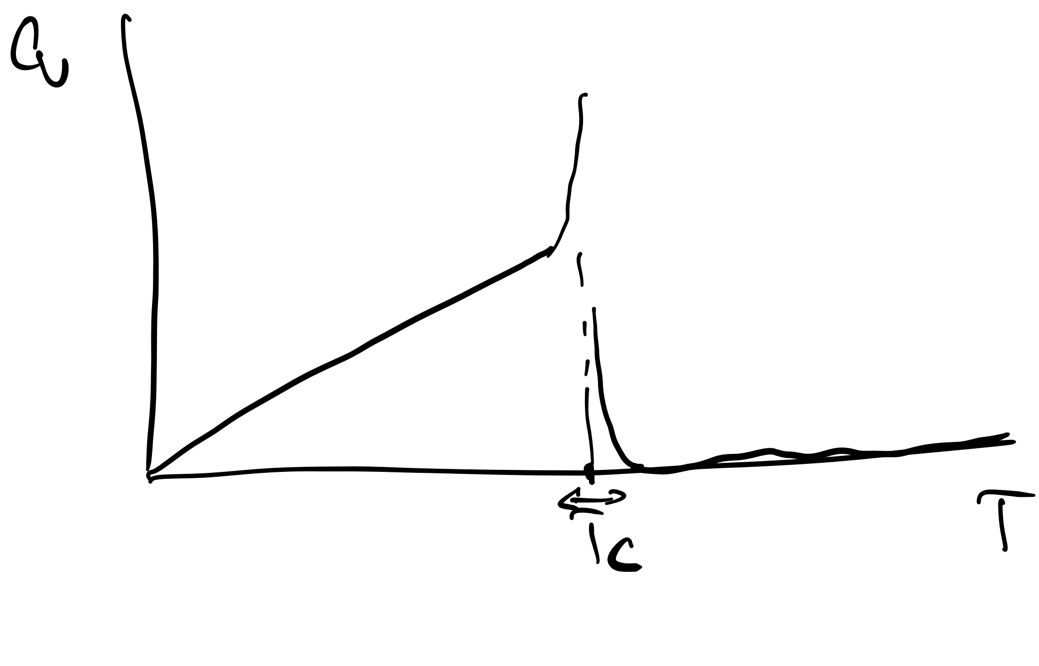
$$\Rightarrow |t| \lesssim \left(\frac{8 a_4}{T_c\, a^2}\right)^{\frac{1}{(4-d)\nu}} \xi_0^{-\frac{d}{\nu}}$$

$$\Rightarrow T_G \qquad \varepsilon \sim \frac{T-T_c}{T_c} \sim \frac{T_G}{T_c}$$

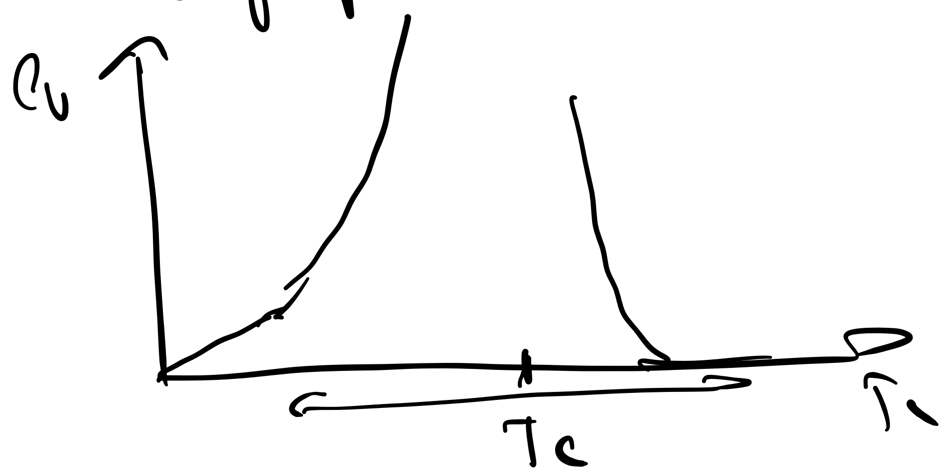
$$\Rightarrow T_G \sim T_c \left(\frac{8a_4}{T_c a^2}\right)^{\frac{1}{\nu(4-d)}} \xi_0^{\frac{-d}{2\nu(4-d)}}$$

$$T_G \sim \xi_0^{\frac{-d}{\nu(4-d)}} \left(\frac{8a_4}{T_c a^2}\right)^{\frac{1}{\nu(4-d)}} T_c$$

si ξ_0 grande $T_G \ll T_c$



si ξ_0 pequeño



$T_0 > T_c$ para 1-D.

$T \to T_c$

$\xi \to \infty$

$\to |T - T_c|^{-\nu}$

$\frac{\chi}{V} \to |T-T_c|^{-\gamma}$

$\frac{\chi}{V} = \int d\vec{r} \, \langle \phi(\vec{0}) \phi(\vec{r}) \rangle \sim \xi^{--}$

$$e^{-\frac{|r|}{\xi}} \rightarrow \frac{1}{r^{d-2}}$$

$T = T_c$