

VII

Plados de Goldstone y el teorema de Fermi-Wagner

II Ruptura espontánea de simetría continua

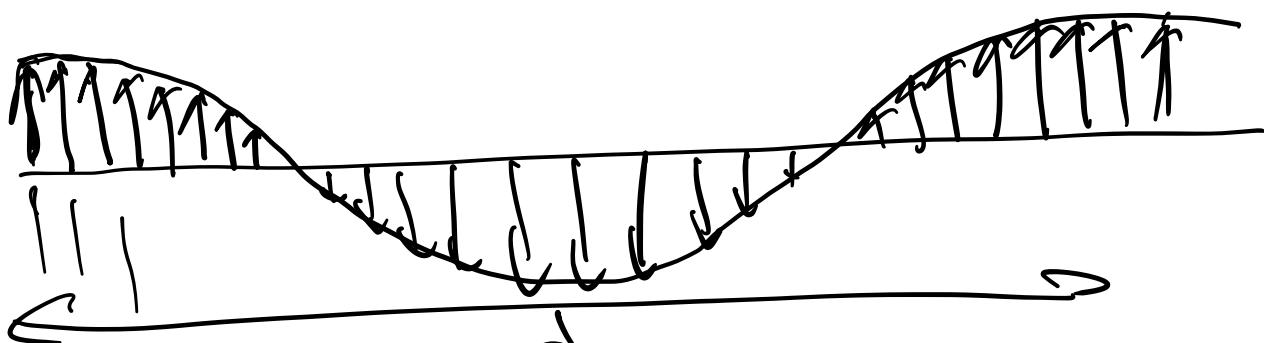
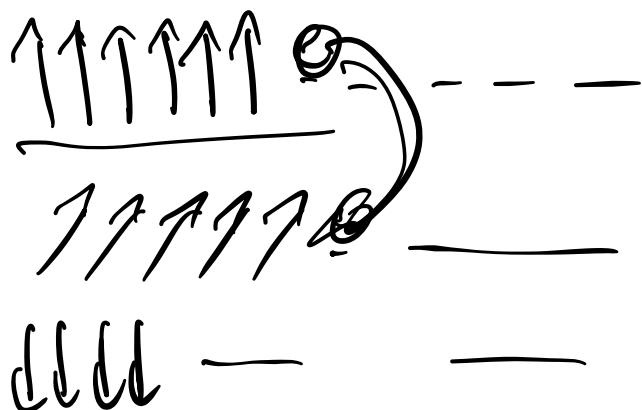
Ejemplos

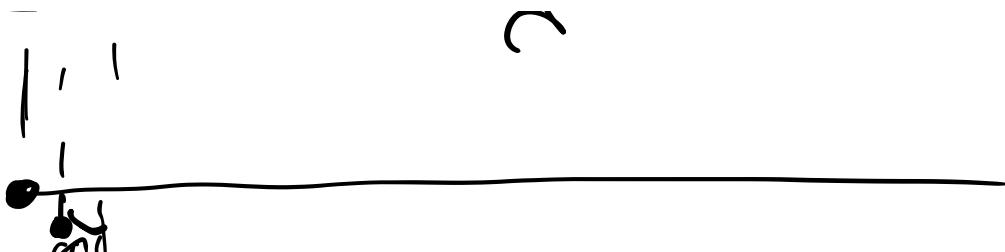
- * Plano de Heisenberg
 $\rightarrow SO(3)$
- * Plano XY $\rightarrow SO(2) \text{ ó } U(1)$
- * Cristalización \rightarrow Translación.

- en la fase "ordenada", de simetría
baja, se pueden contruir "alineaciones"
con energía anormalmente pequeña.
→ Abajo → foliolone.

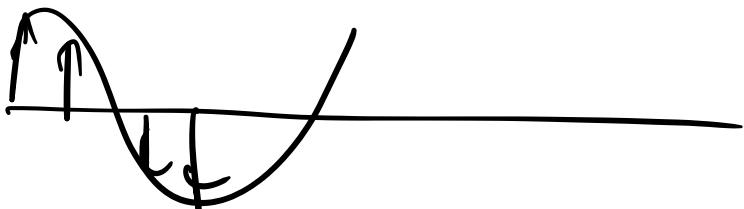
ejemplo Pauli & Heisenberg.

$$H = -J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j \quad J > 0$$





$$\Delta\theta \sim \frac{1}{2} \sim k \quad \text{bilde } k = \frac{2\pi}{L}$$



$$\underline{\underline{\epsilon_{ij}}} = -J \cos(\Delta\theta_{ij}) \sim -J + \frac{J}{2} (\Delta\theta_{ij})^2 + \dots$$

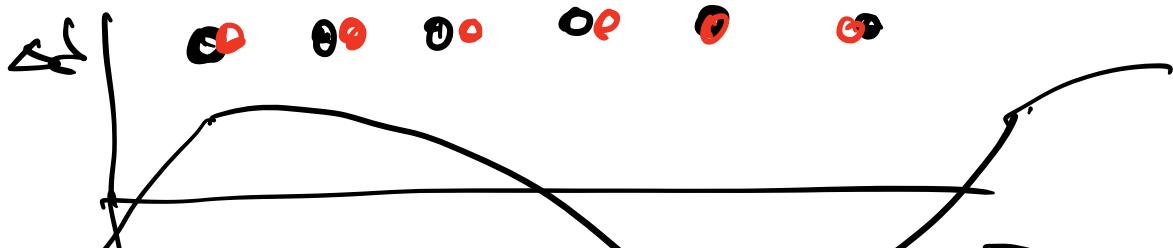
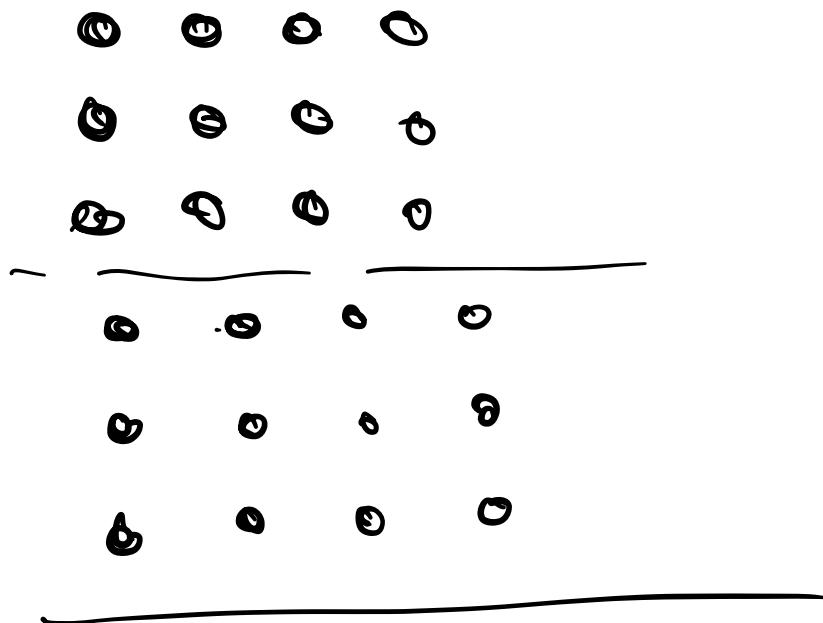
$$\Delta E = \underline{\underline{\epsilon_j}} - \epsilon_{\text{mean}} \sum_{ij} \frac{J}{2} (\Delta\theta_{ij})^2$$

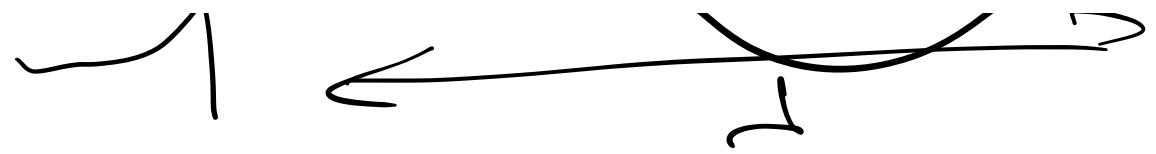
$$\sim \frac{J}{2} (\Delta\theta)^2 \sim \frac{J}{2} k^2$$

$$\underline{\underline{\Delta E(k)}} \xrightarrow[k \rightarrow 0]{} 0$$

Platos & Goldstone

- + para los modelos magnéticos
→ ondas de spin.
- + para la cristalización.
→ phonones.





2) El modelo XY como ejemplo

en la red

$$H = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

$\vec{S}_i = \begin{pmatrix} S_i^x \\ S_i^y \\ S_i^z \end{pmatrix}$
Simetría $SU(2)$

en el continuo

$$S_{C,L} = \int d\vec{r} \left[C \left((\nabla \vec{\phi}_1)^2 + (\nabla \vec{\phi}_2)^2 \right) + \frac{\alpha_2}{2} \left(\vec{\phi}_1^2 + \frac{\alpha_4}{4} \left(\vec{\phi}_1^2 \right)^2 + \dots \right) \right]$$

$$\vec{\phi}(\vec{r}) = \begin{pmatrix} \phi_1(\vec{r}) \\ \phi_2(\vec{r}) \end{pmatrix}$$

$$SO(2) \quad \vec{\phi} \mapsto \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \vec{\phi}$$

$$\psi(r) = \phi_1(r) + i\phi_2(r)$$

$$\psi \rightarrow e^{i\alpha \psi}$$

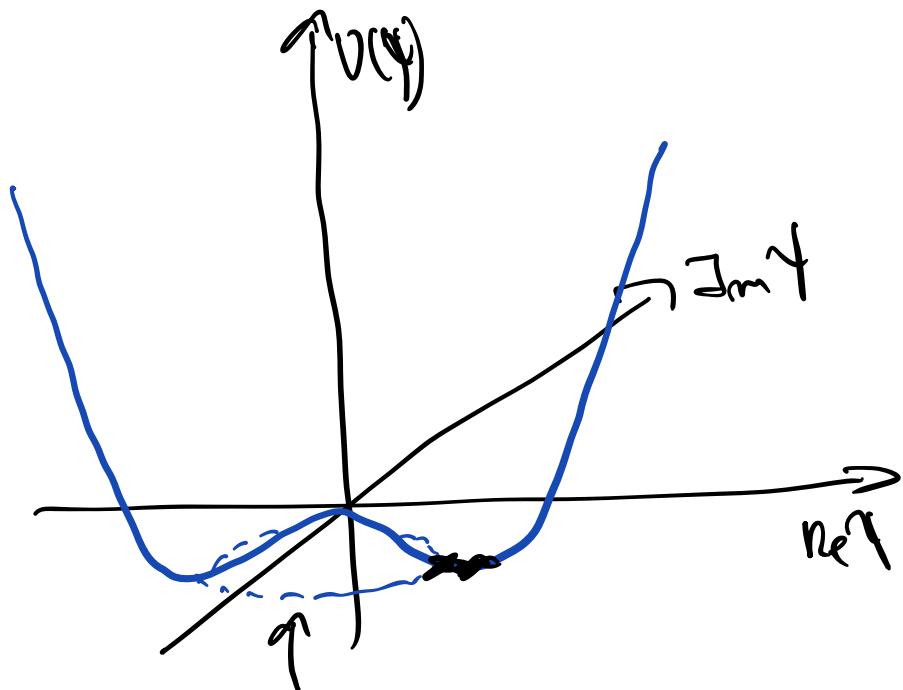
$$S_{G,L} = \int d\tau \sqrt{c \nabla^2 \psi^2 + \frac{a_2}{2} |\nabla^2 \psi|^2 + \frac{a_4}{4} |\nabla^4 \psi|^2 + \dots}$$

$\nabla^2 \psi, \nabla^4 \psi$

$$\text{Sine-Gordon O(1)} \quad \psi \rightarrow e^{i\alpha \psi}$$

$$a_2 = \alpha t, \quad t = \frac{T-T_C}{T_C}$$

$a_2 < 0$



el mínimo para S.E.L $\vec{\nabla} \chi = \vec{0}$

$$\chi = e^{i\theta_0 \sqrt{-\frac{a_2}{a_4}}} \quad a_2 < 0$$

$\uparrow \quad q$

θ_0 arbitraria.

$$a_4 = 0$$

$$\chi(\vec{r}) = (\chi_0 + f(\vec{r})) e^{\frac{i\delta(\vec{r})}{\hbar}}$$

$f(\vec{q}), \delta(\vec{r}) \in \mathbb{R}$

$$S \sim \int d\vec{r} \left\{ c \left[(\vec{\nabla}S)^2 + m^2 S^2 \right] + c V_0 (\nabla \Theta)^2 \right\} \rightarrow \hat{S}(\vec{r})^2 (k^2 + r^2) + c V_0 (\nabla \Theta)^2$$

+ ...

approx
gausione.

Vérmese de onde
superior en Θ que
depon $S \propto \Theta$.

$$m^2 > 0, m^2 = -\frac{\alpha_2}{c} > 0$$

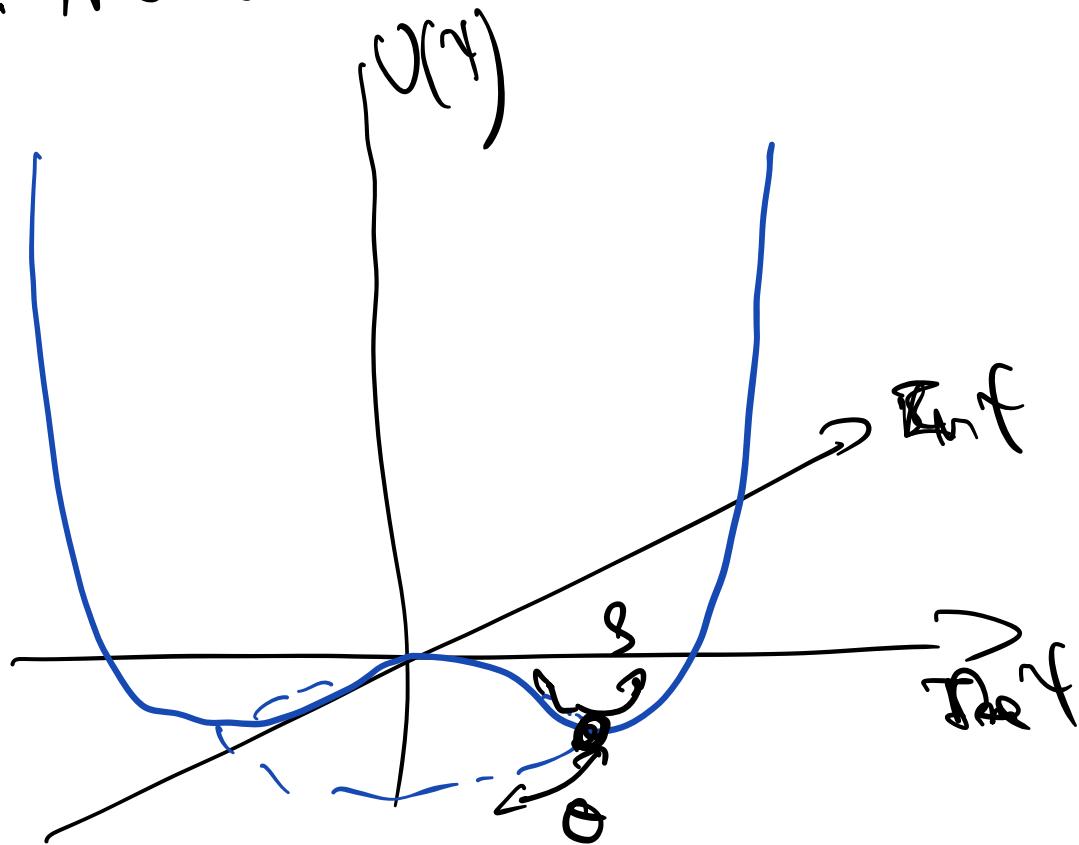
$$\langle S(0) S(r) \rangle \underset{r \rightarrow \infty}{\sim} e^{-mr}$$

$$\langle \Theta(0) \Theta(r) \rangle \sim \begin{cases} -|x| & d=1 \\ -\ln |x| & d=2 \\ \frac{c}{|x|^{d-2}} & d \geq 3 \end{cases}$$

→ δ es un campo con correlaciones
de corto alcance

→ δ es un campo con correlaciones
invariantes de escala, \rightarrow correlaciones
de largo alcance.

δ : modo de Goldstone.



$$Z = \int D\chi D\bar{\chi} e^{-S_{GL}}$$

$$= \int \overline{D\chi} D\bar{\chi} e^{-S_{GL}}$$

$$= \int D\bar{\chi} e^{-S_{eff, GL}}$$

$$S_{eff} = \int d\vec{r} \left[\frac{k(\nabla\phi)^2 + k^2(\vec{\nabla}\phi)^2}{m^2 + a\phi^2} \right]$$

Sintesi $\psi(r)$

$$\psi \rightarrow e^{i\alpha\gamma} \rightarrow \underline{\theta(\vec{r})} \rightarrow \underline{\theta(\vec{r}) + \phi}$$

$$S_{eff} = \int d\vec{r} (\theta(\vec{r}))^2 \underline{\Sigma(\vec{r})}$$

$$E(\vec{r}) \sim K(\vec{k})^2 + \tilde{K} \frac{(\vec{r})^2}{2} + \dots$$

Combinar el operador
a pequeñas distancias

Solo el término

impresa a
grandes distancias.

$$S_{eff} = \int d\vec{r} \left[K (\nabla \theta)^2 \right]$$

$$\langle \theta(\vec{r}) \theta(\vec{r}') \rangle =$$

$$\frac{-|x-x'|}{2K} \quad d=1$$

$$\frac{-1}{K\pi} \ln |\vec{r}-\vec{r}'| \quad d=2$$

$$\frac{cte}{|\vec{r}-\vec{r}'|^{d-2}} \quad d \geq 3$$

en la red. $H = -J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$

$$\langle \vec{S}_i \cdot \vec{S}_j \rangle \quad ; \quad \begin{matrix} i \\ j \end{matrix} \xrightarrow{\Delta_{ij}} \ell$$

$$\sqrt{S^2 \Phi} +$$

$$\langle \vec{\phi}(0) \cdot \vec{\phi}(\vec{r}) \rangle = \langle \psi^*(0) \psi(\vec{r}) \rangle$$

para $|\vec{r}| \rightarrow \infty$

en paralelo $\psi(\vec{r}) = \psi_0 \propto e^{i\alpha}$

$$|\psi_0|^2 = \frac{a_x}{a_y} = m^2$$

$$\langle \psi^*(0) \psi(\vec{r}) \rangle$$

$\psi(\vec{r}) \approx \psi_0 e^{i\alpha(\vec{r})}$

$$e^{i(\psi_0 + \delta(\vec{r}))} e^{i\alpha(\vec{r})}$$

$$\begin{aligned} \langle f^*(\alpha) f(\beta) \rangle &= \zeta_0^2 \langle e^{-i\alpha\phi} e^{i\beta\phi} \rangle \\ &= \zeta_0^2 \langle e^{i(\beta\phi - \alpha\phi)} \rangle \end{aligned}$$

sea $Q = \text{combinación lineal de } \phi(\vec{r})$

Como $\{ \phi \}$ es Gaussiana

$$\boxed{\langle e^{\alpha Q} \rangle = \exp\left(\frac{\alpha^2}{2} \langle Q^2 \rangle\right)}$$

$$P(x) \sim e^{-\sigma^2 x^2}$$

$$\langle Q \rangle = \int_{-\infty}^{\infty} P(x) Q dx$$

$$\text{y } Q = \alpha x$$

$$\therefore \boxed{\langle e^{-\alpha Q} \rangle = \exp\left(\frac{\alpha^2}{2} \langle Q^2 \rangle\right)}$$

$$P(x,y) = e^{-Qx^2 + \sigma_y y^2}$$

$$\langle Q \rangle = \int P(x,y) dx dy$$

$$A^* Q = ax + by$$

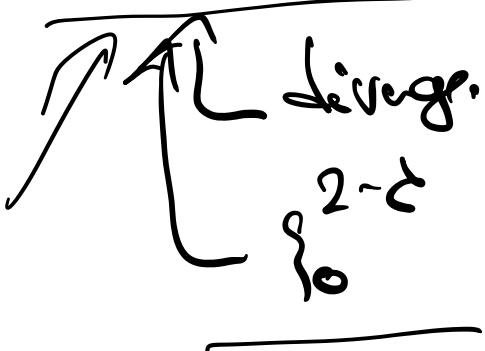
$$\Rightarrow \langle A^* Q \rangle = \frac{1}{2} e^{-\frac{1}{2} \langle Q \rangle}$$

$$\langle [Q(r) - Q(r')]^2 \rangle$$

$$= \underbrace{\langle Q(r) Q(r') \rangle}_{-2 \langle Q(r) Q(r') \rangle} + \underbrace{\langle Q(r) Q(r') \rangle}_{-2 \langle Q(r) Q(r') \rangle}$$

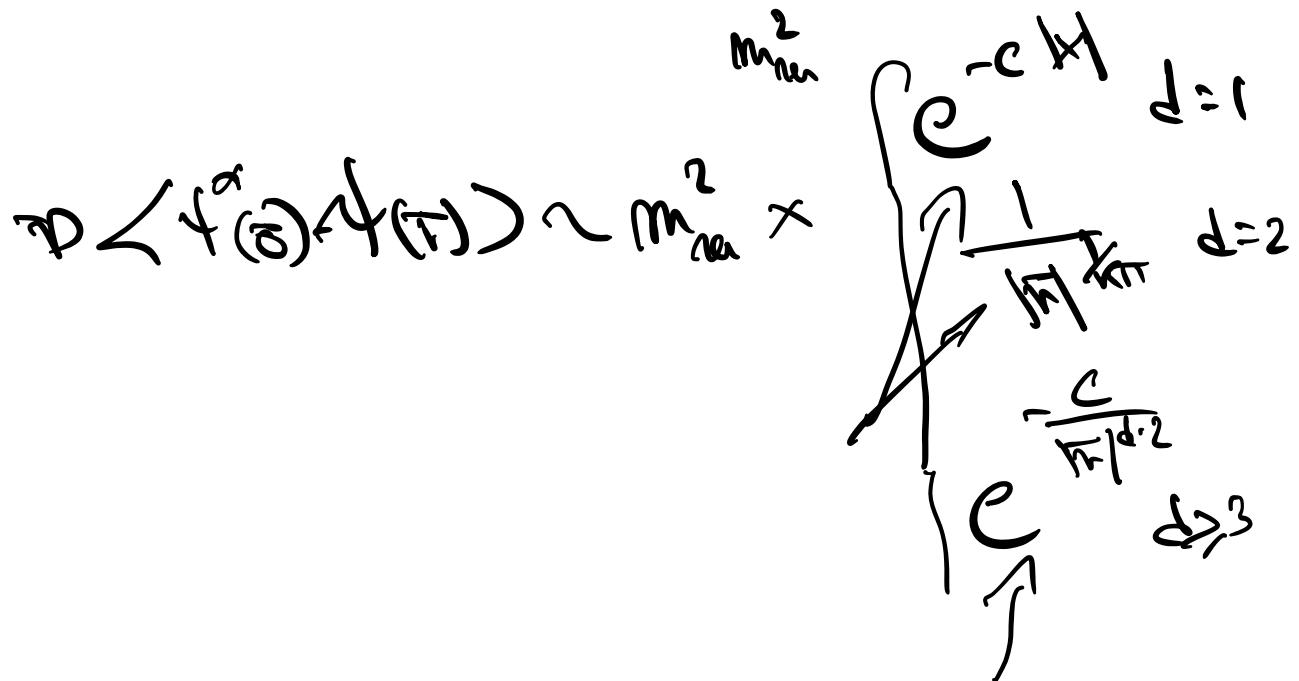
$$-2 \langle Q(r) Q(r') \rangle$$

$$= \frac{2 \langle Q(r) Q(r') \rangle}{2} - 2 \langle Q(r) Q(r') \rangle$$



 2-
 0

$$D \langle \psi^*(\vec{r}) \psi(\vec{r}) \rangle = \underbrace{\int_0^\infty e^{-\frac{c}{2} r^2} e^{\langle Q(\vec{r}) - Q(0) \rangle}}_{m_m^2}$$



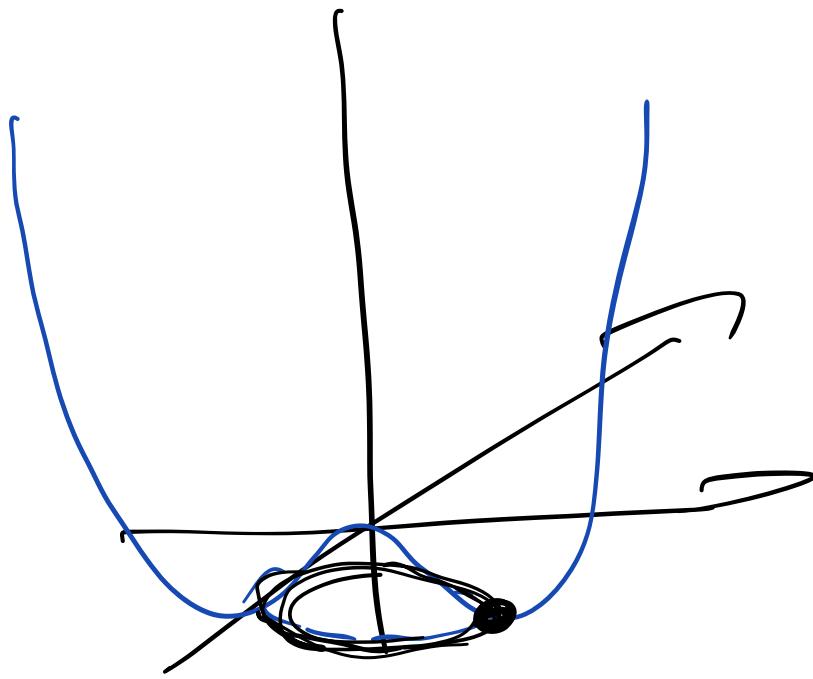
obs para $d \geq 3$

$$\langle \psi^*(\vec{r}) \psi(\vec{r}) \rangle \xrightarrow{|\vec{r}| \rightarrow \infty} m_m^2$$

para, para $d=1, 2$

$$\langle \psi^*(\vec{r}) \psi(\vec{r}) \rangle \xrightarrow{|\vec{r}| \rightarrow \infty} 0$$

$m_m = 0$!!!



Teorema de Pfefferm - Wagne.

para $d \leq 2$, no hay fase ordenada
 para en sistema de simetría continua!

Restauración de simetría para
Iosing en 1-D.

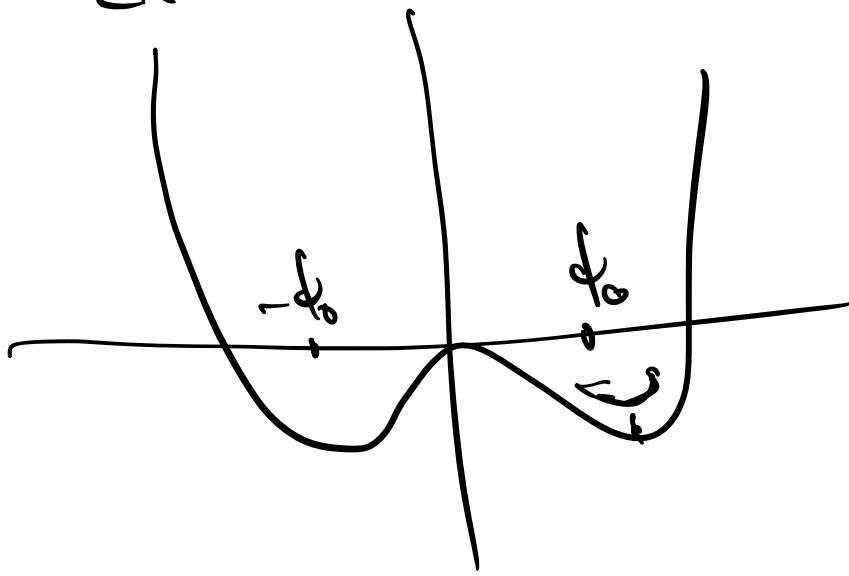
$$S_{GL} = \int dx \left\{ \frac{1}{2} \left(\frac{d\phi}{dx} \right)^2 - \frac{1}{2} \mu^2 \phi^2 + \frac{1}{4} \phi^4 \right\}$$

$\overbrace{\hspace{10em}}$

$\mu^2 > 0$

mínimo de S_{GL}

$$\frac{d\phi}{dx} = 0 \Rightarrow \phi(x) = \pm \frac{\mu}{\sqrt{2}} \quad \text{y} \quad \phi_0 = \pm \frac{\mu}{\sqrt{2}}$$



entonces $\phi(x) = \phi_0 + \eta(x)$

$$\rightarrow S_{GL} = S_0 + \int dx \left\{ \left(\frac{d\eta}{dx} \right)^2 + \mu^2 \eta^2 + \dots \right\}$$

$\overbrace{\hspace{10em}}$

$$\langle \phi(x_1) \phi(x_2) \rangle$$

$$= \left\langle \left(\frac{u}{\sqrt{n}} + \chi_{(x_1)} \right) \left(\frac{u}{\sqrt{n}} + \chi_{(x_2)} \right) \right\rangle$$

$$= \frac{1}{n^2} + \underbrace{\left\langle \frac{u}{\sqrt{n}} \chi_{(x_1)} \right\rangle}_{\frac{1}{n^2}} + \underbrace{\left\langle \frac{u}{\sqrt{n}} \chi_{(x_2)} \right\rangle}_{\frac{1}{n^2}} + \underbrace{\left\langle \chi_{(x_1)} \chi_{(x_2)} \right\rangle}_{-\frac{1}{|x_1 - x_2|}}$$

$$= m^2 + \text{etc } e^{-\frac{|x_1 - x_2|}{\zeta}} \quad \zeta = \frac{1}{m}$$

$$\rightarrow \underbrace{\langle \phi(x_1) \phi(x_2) \rangle}_{|x_1 - x_2| \rightarrow \infty} \xrightarrow{m^2 \neq 0}$$

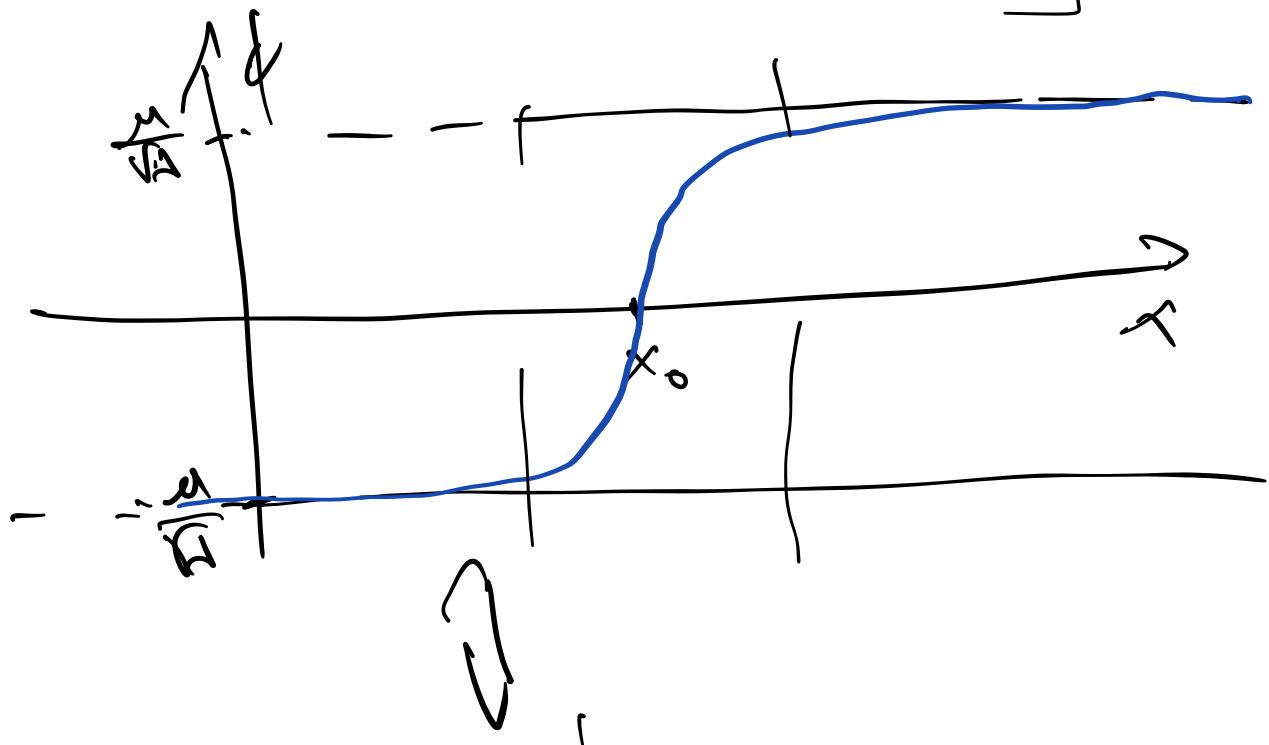
$$Z = \int D\phi \, e^{-S} \sim e^{-S_{\text{min}}}$$

fluctuaciones.

per \$S\$, tiene varios mínimos locales.

$$\frac{\delta S}{\delta \phi(x)} = 0 \Rightarrow \phi'' + \mu^2 \phi - 2\phi^3 = 0$$

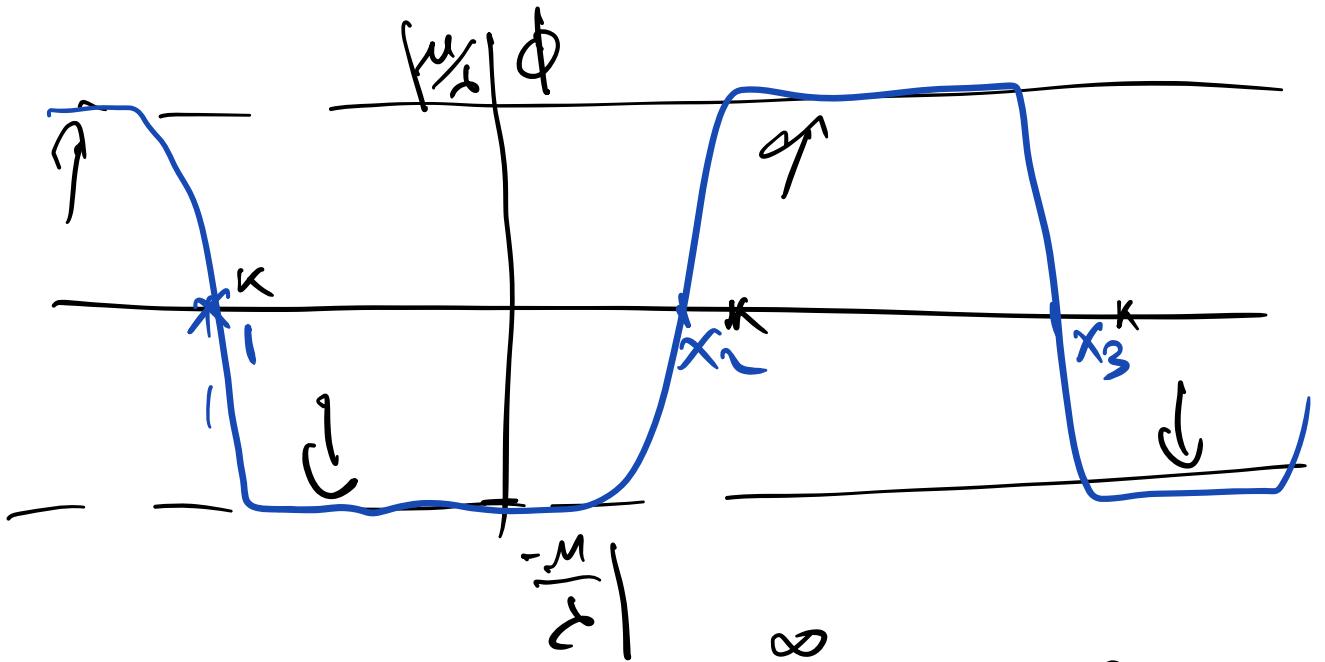
$\phi_{\text{Kink}}(x) = \frac{\mu}{\sqrt{2}} \tanh\left(\frac{\mu(x-x_0)}{\sqrt{2}}\right)$
 $\forall x_0$



$$S_{\text{Kink}} = S \{ \phi_{\text{Kink}} \} = \frac{2\sqrt{2}\mu^3}{3\epsilon}$$

k (or kink state) operators

$$S(N \text{ kinks}) \sim N S_{\text{kink}}$$



$$\langle \phi(x_1) \phi(x_2) \rangle \sim \frac{\mu}{\epsilon} \sum_{N=0}^{\infty} (-1)^N e^{-N S_{\text{kink}}} \int_{x_1}^{x_2} dx^k_1 dx^k_2 \dots dx^k_N$$

$$x_1^k < x_2^k < x_3^k$$

$$= \frac{\mu}{\epsilon} \sum_{N=0}^{\infty} (-1)^N (e^{-S_{\text{kink}}})^N \left(\int_{x_1}^{x_2} dx_N^k \right)^N \frac{1}{N!}$$

$$= \frac{m^2}{2} e^{-\frac{(x_2-x_1)}{l}} \quad \text{und} \quad \frac{1}{l} = e^{-\frac{\beta}{k_B T}}$$

$$\Rightarrow \langle \phi(x_1) \phi(x_2) \rangle \xrightarrow{x_2-x_1 \rightarrow \infty} 0$$

$$H_{BCS}, C_{\sigma, \vec{n}}, C_{\sigma}^{\dagger}$$

$$\sigma = \begin{matrix} + & \alpha \\ \downarrow & \downarrow \\ 0 & 0 \end{matrix}$$

$$H_{BCS} = \sum_{\vec{n}, \sigma} \epsilon(\vec{n}) C_{\sigma, \vec{n}}^{\dagger} C_{\sigma, \vec{n}}$$

$$+ \sum_{\vec{\epsilon}_1, \vec{\epsilon}_2} V(\vec{n}_1, \vec{n}_2) C_{\uparrow, \vec{n}_1}^{\dagger} C_{\uparrow, \vec{n}_1}^{\dagger} C_{\downarrow, \vec{n}_2} C_{\downarrow, \vec{n}_2}$$

$$\left. \begin{aligned} \forall \sigma, \forall \vec{k} \quad C_\sigma(\vec{k}) &\rightarrow e^{i\Theta} C_\sigma(\vec{k}) \\ C_\sigma^\dagger(\vec{k}) &\rightarrow e^{-i\Theta} C_\sigma^\dagger(\vec{k}) \end{aligned} \right\}$$

$$\Delta(\vec{k}) = \left\langle C_{\sigma_1}^\dagger(\vec{k}) C_{\sigma_2}^\dagger(-\vec{k}) \right\rangle \neq 0$$

$$\overbrace{\chi_{(\vec{k})}}$$