VIII El grupo de renormalización

R.G.

1) Invariancia de escala para los puntos críticos

$\xi \to \infty \to$ inv. de escala
$\to$ estado fractal.

$\to$ "Alejarse" del sistema para borrar los detalles microscópicos
$\to$ comportamiento universal del fractal.

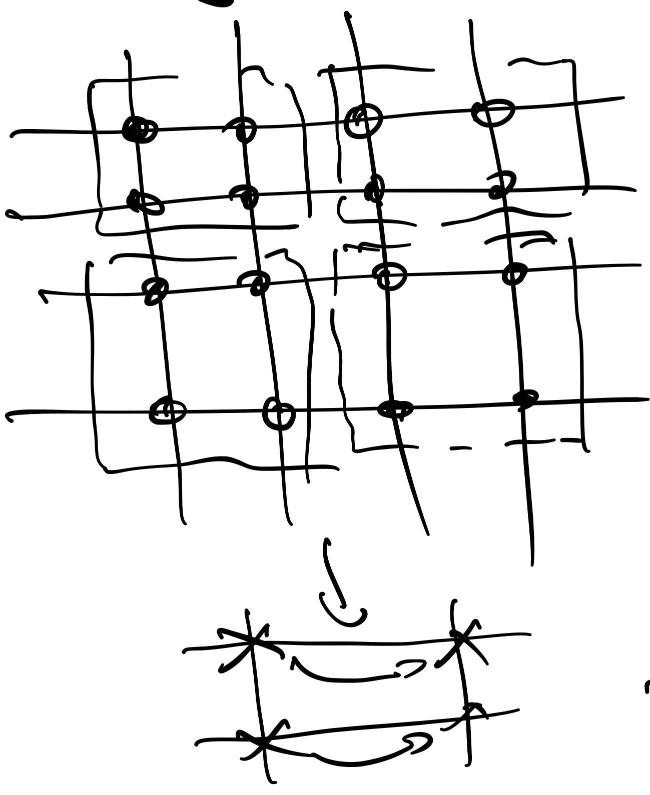
$\to$ Concepto de clase de Universalidad (K. Wilson) la clase de universalidad de un punto crítico depende de

- la dim. del espacio
- el grupo de la simetría

que se rompe

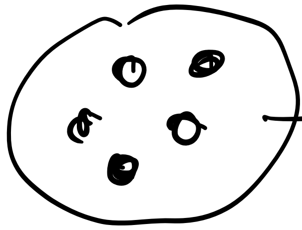
2) R.G. en el espacio real

(L. Kadanoff)



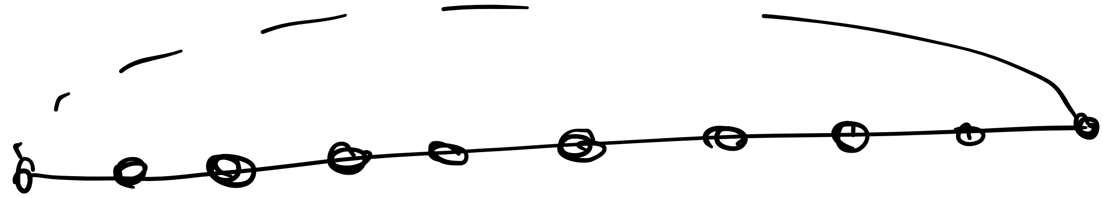
$H_1(\bullet)$

$H_2(\times)$



$$\sigma_{eff} = \begin{cases} +1 & \text{si la suma} > 0 \\ -1 & \text{si la suma} < 0 \end{cases}$$

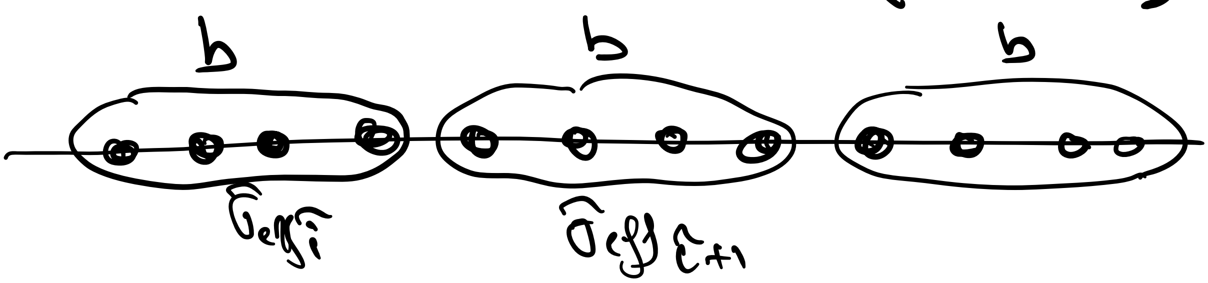
→ el caso en 1-D



$$H = -J \sum_i \sigma_i \sigma_{i+1} \, , \quad K = \frac{J}{k_B T}$$

$$Z = \mathrm{Tr}\{T^N\} \, ; \quad T = \begin{pmatrix} e^{K} & e^{-K} \\ e^{-K} & e^{K} \end{pmatrix}$$

$$O = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \qquad OTO = \begin{pmatrix} 2\,\mathrm{ch}K & 0 \\ 0 & 2\,\mathrm{sh}K \end{pmatrix}$$



$\rightarrow$ una cadena de $\underline{\hat{N} = \frac{N}{b}}$ espines

$\rightarrow$ un nuevo ~~...~~ modelo de Ising

$\tilde{T}$ tq $Z = \mathrm{tr}\{\tilde{T}^{\hat{N}}\}$

$\rightarrow \tilde{T} \neq T^b$ sino $\tilde{T} = \mathrm{cte}\, T^b$

$$\Theta \tilde{T} \Theta = \begin{pmatrix} 2\,\mathrm{ch}\,\tilde{K} & 0 \\ 0 & 2\,\mathrm{sh}\,\tilde{K} \end{pmatrix}$$

$$\begin{pmatrix} (2\,\mathrm{ch}\,K)^b & 0 \\ 0 & (2\,\mathrm{sh}\,K)^b \end{pmatrix} = \underline{\mathrm{cte}} \begin{pmatrix} 2\,\mathrm{ch}\,\underline{\tilde{K}} & 0 \\ 0 & 2\,\mathrm{sh}\,\underline{\tilde{K}} \end{pmatrix}$$

$$\| $$

$$\Theta T^b \Theta$$

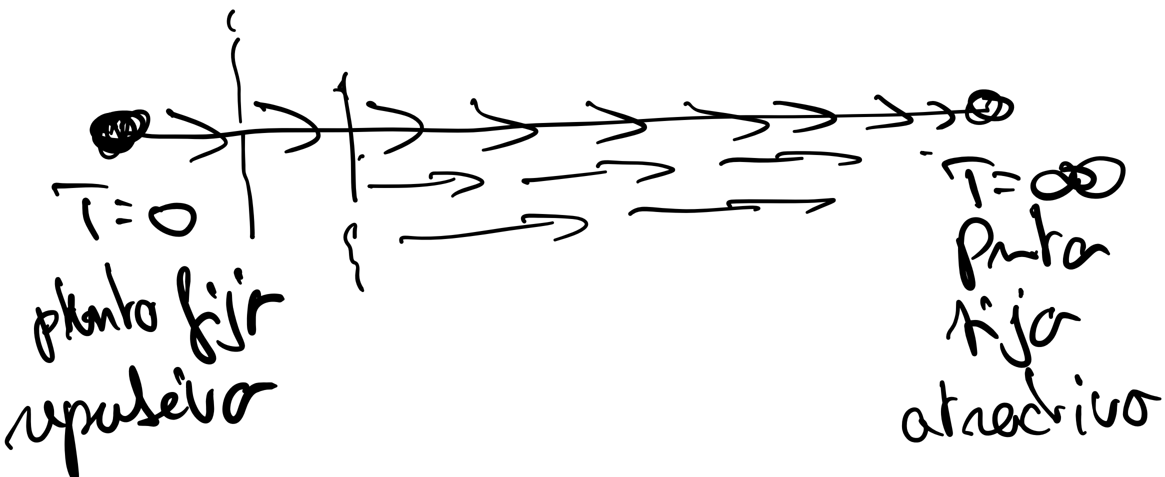
$$\mathrm{th}(\tilde{K}) = (\mathrm{th}(K))^b$$

$$\Rightarrow \boxed{\hat{K} = th^{-1}\left[th(K)^{b}\right]}$$

esta ecuación tiene 2 puntos fijos

$th\, K = 0 \quad (T = \infty)$

$th\, K = 1 \quad (T = 0)$

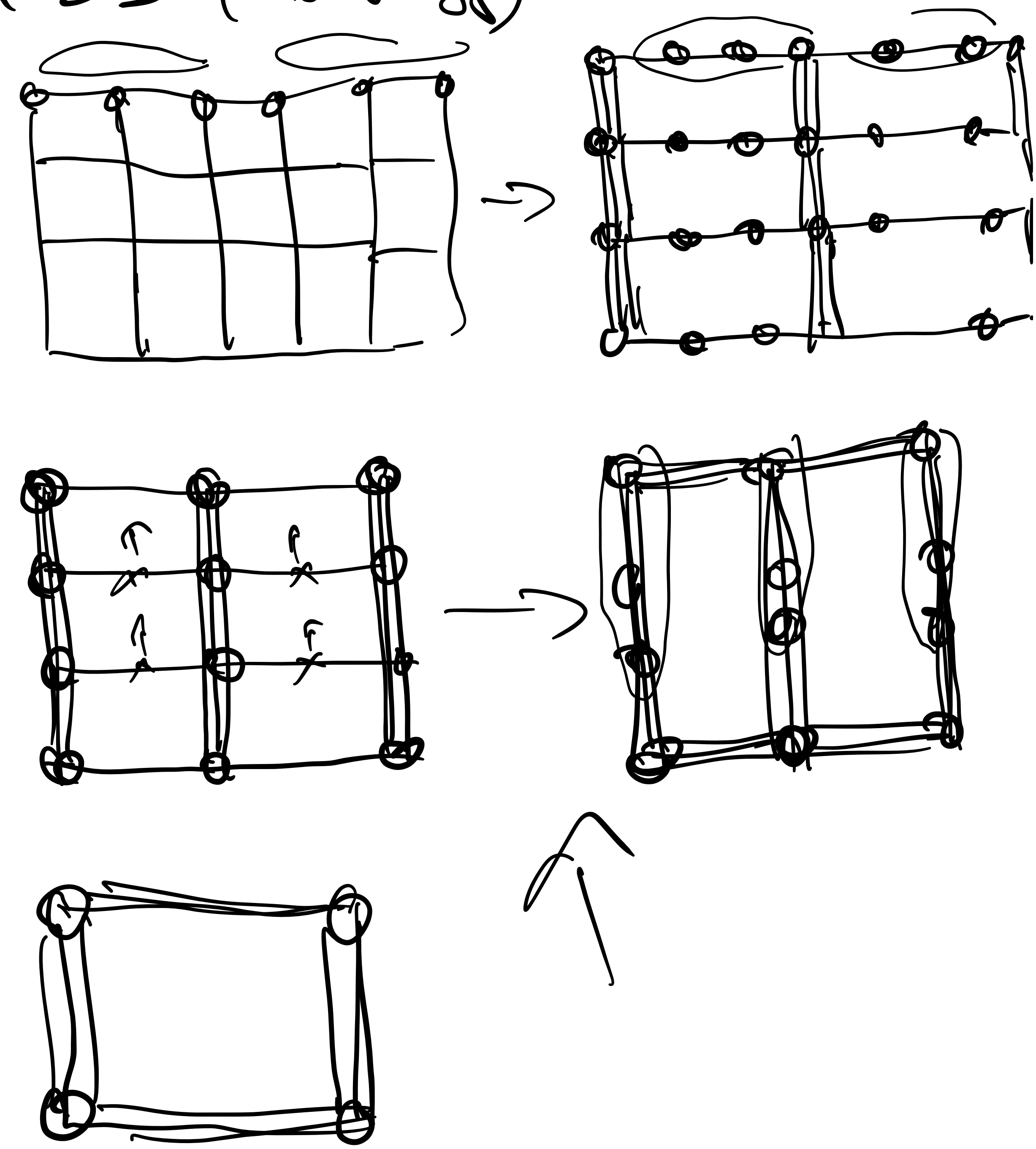


ej: mostrar que $\hat{\xi} \longleftrightarrow \hat{K}$

$\xi \longleftrightarrow K$

$$\Rightarrow \boxed{\hat{\xi} = \frac{\xi}{b}}$$

en 2-D (Kadanoff)



$K \to bK$ 1ère étape

$K \to th^{-1}\left[th(K)^b\right]$ 2e. étape

$K \to bK$

$$\boxed{\hat{K} = b\, th^{-1}\left[th(bK)^{b} \right]}$$

b entero, $\to$ prolongación

$a\, b \in \mathbb{R}$

$b \sim e^{\delta \ell} \sim (1+\delta\ell) \qquad \delta\ell \ll 1$

$$th^{-1}\left[th\left[(1+\delta\ell)K\right]^{(1+\delta\ell)} \right]$$

$$\simeq (1+\delta\ell)K + \delta\ell\left[\tfrac{1}{2}\, sh\, 2K \;\ln\left[th(K)\right]\right] + \mathcal{O}(\delta\ell^2)$$

$\hat{K} = K + \delta K$

$\frac{\delta K}{\delta \ell} \to \frac{dK}{d\ell}$

$$\to \boxed{\frac{dK}{d\ell} = K + \frac{1}{2}\left[\operatorname{sh}(2K)\ln(\operatorname{th}K)\right]}$$

para $D \geq 2$

$$\boxed{\frac{dK}{d\ell} = (D-1)K + \frac{1}{2}\left[\operatorname{sh}(2K)\ln(\operatorname{th}(K))\right]}$$

$$\epsilon = D-1$$

$$T = \frac{J}{k_B K} \quad \text{ó} \quad K = \frac{J}{k_B T}$$

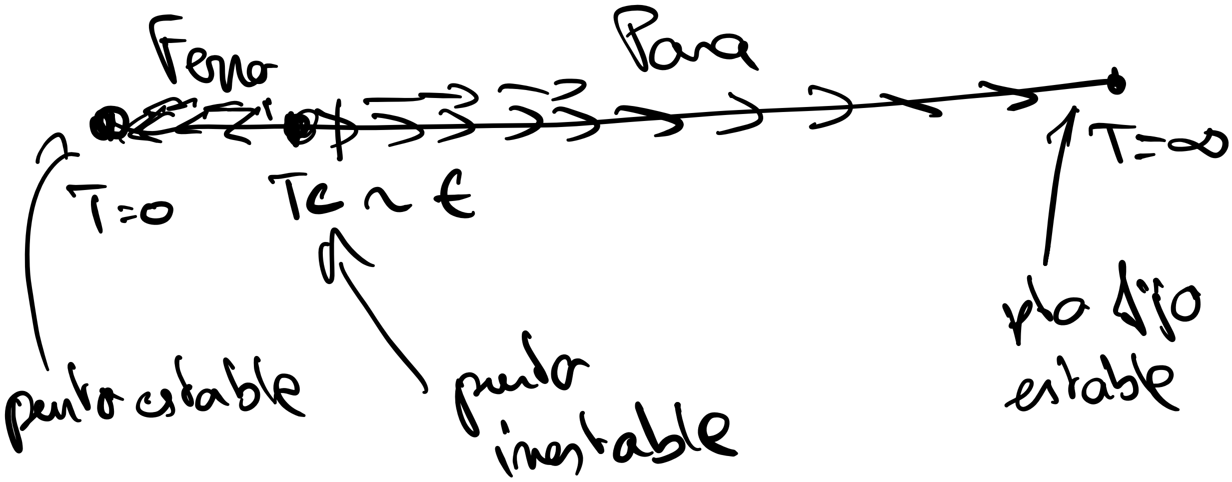
T pequeño

$$\boxed{\frac{dT}{d\ell} = -\epsilon T + \frac{k_B}{2J}T^2 + \mathcal{O}(T^3)}$$

$\to$ 2 puntos críticos para T pequeño

$* \quad T=0$

$* \quad T_c = \dfrac{2J\epsilon}{k_B}$



$T = T_c + \delta T$

$$\boxed{\frac{d\delta T}{dl} = \epsilon\, \delta T}$$

$T = 0 + \delta T$

$$\boxed{\frac{d\delta T}{dl} = -\epsilon\, \delta T}$$

$$\delta T(l) = e^{\epsilon l}\, \delta T(0)$$

$$\xi(l) = e^{-l}\, \xi(0)$$

$$\xi(\ell) \sim e^{-\ell}$$

$$\xi(\ell) \sim (\delta T)^{-\nu} \sim e^{-\ell \epsilon \nu}$$

$$\Rightarrow \boxed{\nu = \frac{1}{\epsilon}}$$

3) R.G. para la teoría de campos.

$$S_{GL} = \int d\vec{r} \left[\frac{K}{2} (\vec{\nabla}\phi)^2 + \frac{t}{2}\phi^2 + \mu\phi^4 + \mu_6\phi^6 + \mu_8\phi^8 \ldots \right]$$

$$t = \frac{T - T_c}{T_c}$$

si $t = \mu = \mu_6 = \mu_8 = \ldots = 0$

$\Rightarrow$ invariante de escala

$$\begin{cases} \vec{r} \to b\vec{r}\,' & K \to K \quad b > 1 \\ \phi \to b^{1-\frac{D}{2}}\phi' \end{cases}$$

ahora si $t \neq 0$, $u \neq 0$ etc...

$\vec{r} \to b\vec{r}'$ ó $\vec{r}' = \frac{1}{b}\vec{r}$

$\vec{q} \to b^{-1}\vec{q}'$ ó $\vec{q}' = b\vec{q}$

$\phi \to b^{1-\frac{D}{2}}\phi'$

$t \to b^{2}t = t'$

$u \to b^{(4-D)}u = u'$

$\underline{u_6} \to b^{6-2D}u_6 = u_6'$

$\underline{u_8'} = b^{8-3D}u_8$

$\underline{\text{obs}}$ si $t \neq 0$, $t' > t \to t'' > t' > t$ etc.

obs. $t \sim \frac{1}{\xi^2}$

si $t \neq 0 \to$ no hay inv. de escala

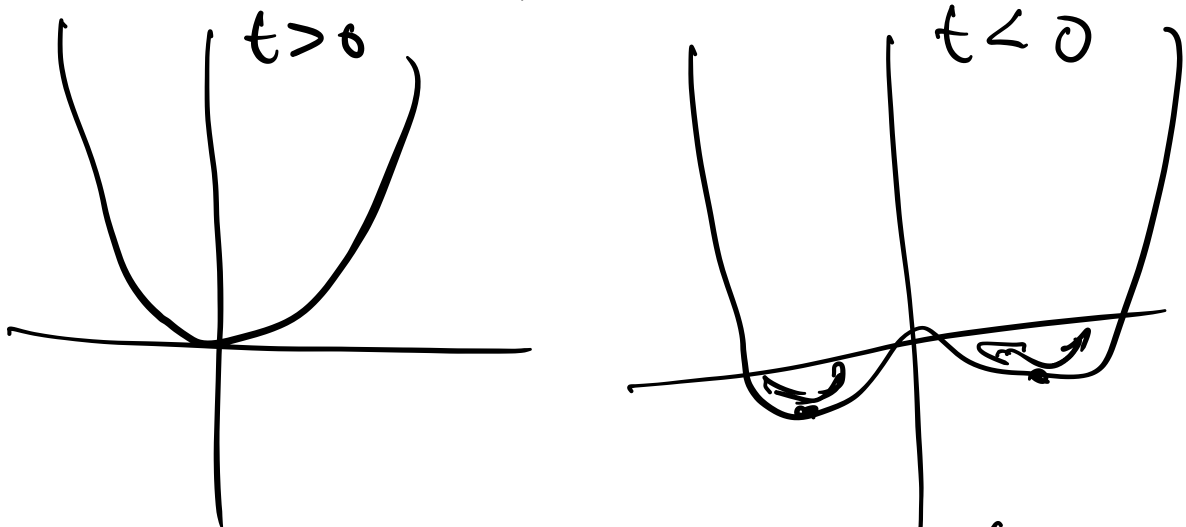
obs 2 $\mu' = b^{4-D}\mu$ $\quad b>1$

si $D > 4$ $\quad \mu' < \mu$

$\rightarrow \mu'' < \mu' < \mu$

Con el R.G. $\mu \rightarrow 0$ si $D > 4$

idem μ_6, μ_8 etc...



para $D \geq 4$ $\rightarrow$ la aproximación gaussiana se justifica.

$\rightarrow$ los resultados de campo medio son válidos!

4) R.G. perturbativo para la teoría de campos

$$Z = \int \mathcal{D}\phi \, e^{-S_{G.L.}}$$

$$= \int \mathcal{D}\phi \, e^{\underbrace{-\int d\vec{r} \left[\frac{t}{2}\phi^2 + \frac{K}{2}(\nabla\phi)^2 \right]}_{S_0} - U}$$

donde $U = u \int d\vec{r} \, \phi^4(\vec{r}) + \dots$

$$m(\vec{q}) = \int d\vec{r} \, e^{i\vec{q}\cdot\vec{r}} \phi(\vec{r})$$

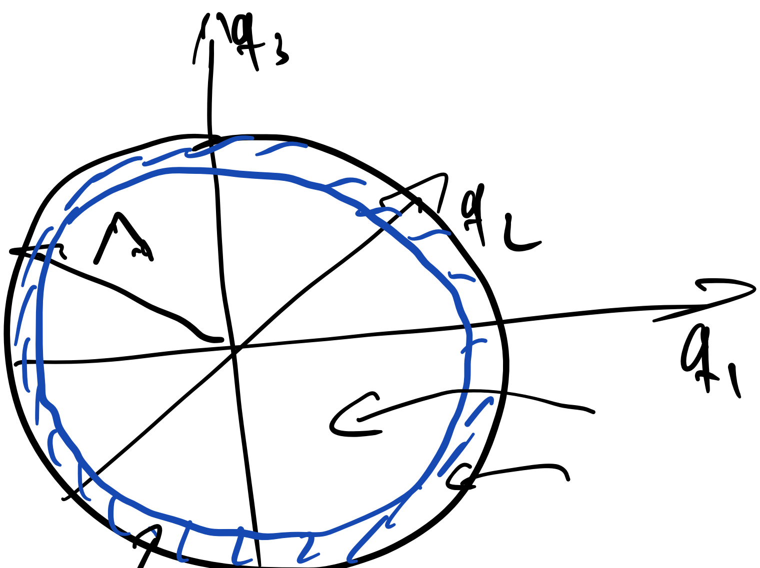
$$\phi(\vec{r}) = \int \frac{d\vec{q}}{(2\pi)^D} e^{-i\vec{q}\cdot\vec{r}} \, m(\vec{q})$$

$$S_0 = \int \frac{d\vec{q}}{(2\pi)^D} \left(\frac{t + \kappa \vec{q}^2}{2} \right) |m(\vec{q})|^2$$

$$U = \mu \int \frac{d\vec{q}_1\, d\vec{q}_2\, d\vec{q}_3}{(2\pi)^{3D}} m(\vec{q}_1)\, m(\vec{q}_2)\, m(\vec{q}_3)\, m(-\vec{q}_1 - \vec{q}_2 - \vec{q}_3)$$

$\rightarrow$ cut-off U.V.

$$|\vec{q}| < \Lambda \sim \frac{1}{a_0}$$



$$\frac{\Lambda}{b} < |\vec{q}| \leq \Lambda \qquad b > 1$$

la idea es integrar los $m(\vec{q})$ que están en la cáscara.

$$m(\vec{q}) \begin{cases} \hat{m}(\vec{q}) & \text{si } 0 < q \leq \frac{\Lambda}{b} \\ \sigma(\vec{q}) & \text{si } \frac{\Lambda}{b} < q \leq \Lambda \end{cases}$$

para S_0 hay desacople

$$S_0 = \int_{\frac{\Lambda}{b}} \frac{d\vec{q}}{(2\pi)^D} \left(\frac{t + kq^2}{2}\right) |\hat{m}(\vec{q})|^2 = S_0(\hat{m})$$

$$+ \int_{\frac{\Lambda}{b} < q < \Lambda} \frac{d\vec{q}}{(2\pi)^D} \left(\frac{t + kq^2}{2}\right) |\sigma(\vec{q})|^2 = S_0(\sigma)$$

$$Z = \int \mathcal{D}\bar{m}(\vec{q})\, \underline{\mathcal{D}\sigma(\vec{q})}$$

$$e^{-S_0(\bar{m}) - S_0(\sigma)} \times \boxed{e^{-\mathcal{U}[\bar{m},\sigma]}}$$

$$\underline{Z_\sigma = \int \mathcal{D}\sigma(\vec{q})\, e^{-S_0(\sigma)}}$$

$$\underline{\delta F_\sigma = \ln Z_\sigma}$$

$$\underline{\langle O \rangle_\sigma} = \frac{1}{Z_\sigma} \int \mathcal{D}\sigma(\vec{q})\, O\, e^{-S_0(\sigma)}$$

$$Z_{eff} = \int \mathcal{D}\bar{m}(\vec{q})\, e^{-S_0(\bar{m})} \underline{\int \mathcal{D}\sigma(\vec{q})\, e^{-S_0 \sigma}\, e^{-\mathcal{U}}}$$

$$= \underline{\int \mathcal{D}\bar{m}(\vec{q})\, e^{-S_0(\bar{m})}\, Z_\sigma \langle e^{-\mathcal{U}[\sigma,\bar{m}]} \rangle_\sigma}$$

$$Z_{eff} = \int \mathcal{D}\bar{n}(\vec{q})\, e^{-S_{eff}(\bar{n})}$$

$$S_{eff}(\bar{n}) = S_0(\bar{n}) + \cancel{\delta F_0} - \ln\langle e^{-U}\rangle_0$$

non contiene $\bar{n}$

si contiene $\bar{n}$

$$\ln\langle e^{-U}\rangle_0 = -\langle U\rangle_0 + \frac{1}{2}\left(\langle U^2\rangle_0 - \langle U\rangle_0^2\right)$$

$$\langle 1 - U + \frac{1}{2}U^2 + \dots\rangle$$