R.G. perturbativo

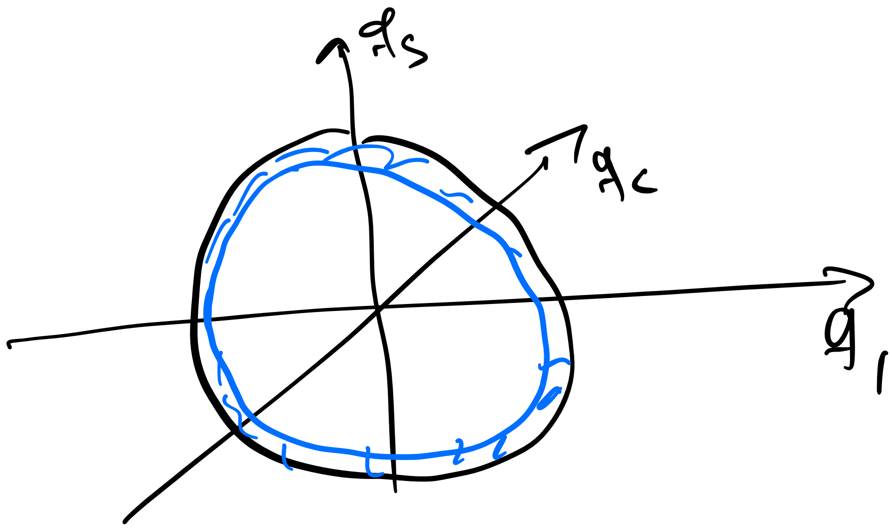
$$Z = \int \mathcal{D}m(\vec{q})\, e^{-S}$$

$$S = S_0 + U$$

$$S_0 = \int \frac{d\vec{q}}{(2\pi)^D} \left(\frac{Kq^2 + t}{2}\right) |m(\vec{q})|^2$$

$$U = \mu \int \frac{d\vec{q}_1\, d\vec{q}_2\, d\vec{q}_3\, d\vec{q}_4}{(2\pi)^{4D}} (2\pi)^D \delta(\vec{q}_1 + \vec{q}_2 + \vec{q}_3 + \vec{q}_4)$$

$$m(\vec{q}_1)\, m(\vec{q}_2)\, m(\vec{q}_3)\, m(\vec{q}_4)$$



$$|\vec{q}| \leq \Lambda \sim \frac{1}{a_0}$$

$$\frac{\Lambda}{b} < |\vec{q}| \leq \Lambda$$

$$m(\vec{q}) = \begin{cases} \tilde{m}(\vec{q}) & \text{si } q < \frac{\Lambda}{b} \\ \sigma(\vec{q}) & \text{si } \frac{\Lambda}{b} < q \leq \Lambda \end{cases}$$

$$Z = \int \mathcal{D}\tilde{m}(\vec{q})\, \mathcal{D}\sigma(\vec{q})\, e^{-S_0(\tilde{m}) - S_0(\sigma) - U(\sigma, \tilde{m})}$$

$$= \int \mathcal{D}\tilde{m}(\vec{q})\, e^{-S_{eff.}(\tilde{m})}$$

$$S_{eff} = e f e + \int_{|\vec{q}|<\frac{\Lambda}{b}} \frac{d\vec{q}}{(2\pi)^D} \left(\frac{Kq^2 + t}{2}\right) |\tilde{m}(\vec{q})|^2$$

$$- \ln \langle e^{-U} \rangle_\sigma$$

$$\ln \langle e^{-U} \rangle_\sigma = - \langle U \rangle_\sigma + \frac{1}{2}\left(\langle U^2 \rangle_\sigma - \langle U \rangle_\sigma^2 \right) + \ldots$$

$$\langle U \rangle_\sigma = \mu \int_0^\Lambda \frac{d\vec{q}_1\, d\vec{q}_2\, d\vec{q}_3\, d\vec{q}_4}{(2\pi)^{4D}} (2\pi)^D \delta(\vec{q}_1 + \vec{q}_2 + \vec{q}_3 + \vec{q}_4)$$

$$\langle [\tilde{m}(\vec{q}_1) + \sigma(\vec{q}_1)] [\quad _2] [\quad _3] [\quad _4] \rangle_0$$

obs $\langle \sigma .. \sigma .. \sigma \rangle_0 = 0$

no impar

$\to$ solo $K \langle \sigma\sigma\sigma\sigma \rangle$ ó $\langle \sigma\sigma \rangle$ ó $\langle \cdot \rangle_0$

el término sin ningún σ $\to$ hay 4 $\tilde{m}$

$$\to u \int_0^{\Lambda/b} \ldots \delta(\ldots) \, \tilde{m}(\vec{q}_1)\, \tilde{m}(\vec{q}_2)\, \tilde{m}(\vec{q}_3)\, \tilde{m}(\vec{q}_4) = \mathcal{U}(\tilde{m})$$

el término $\langle \sigma\sigma\sigma\sigma \rangle \to$ cte

$$-12u \int \frac{d\vec{q}_1\, d\vec{q}_2\, d\vec{q}_3\, d\vec{q}_4}{(2\pi)^{4D}} (2\pi)^D \delta(\ldots) \langle \sigma(\vec{q}_1)\, \sigma(\vec{q}_2) \rangle_0 \, \tilde{m}(\vec{q}_3)\, \tilde{m}(\vec{q}_4)$$

$$\left(\frac{(2\pi)^D \, \delta(\vec{q}_1 + \vec{q}_2)}{t + K q_1^2} \right.$$

$$= -12\left[\int_{\Lambda/b}^{\Lambda}\frac{d\vec{k}}{(2\pi)^D}\,\frac{1}{t+Kk^2}\right]\int_0^{\Lambda/b}\frac{d\vec{q}}{(2\pi)^D}\left|\tilde{h}(\vec{q})\right|^2$$

$$\tilde{t} = t + 12\mu\int_{\Lambda/b}^{\Lambda}\frac{d\vec{k}}{(2\pi)^D}\,\frac{1}{t+Kk^2}$$

$$t \simeq T - T_c$$

Con el re-escaleo

$$\vec{h} \to b\,\vec{h}$$

$$\vec{q} \to b^{-1}\vec{q}$$

$$t \to b^2 t$$

$$\mu \to b^{(4-D)}\mu$$

$$\Rightarrow \begin{cases} t_b' = b^2\left[t + 12\mu\displaystyle\int_{\Lambda/b}^{\Lambda}\frac{d\vec{k}}{(2\pi)^D}\,\frac{1}{t+Kk^2}\right] \\ \mu_b' = b^{4-D}\mu \end{cases}$$

$$b' = (1 + d\ell)$$

$$b^{\alpha} = (1 + \alpha\, d\ell + \dots)$$

$$t'_b = t + d\ell\, \frac{dt}{d\ell}$$

$$u'_b = u + d\ell\, \frac{du}{d\ell}$$

$$\int_{\frac{\Lambda}{1+d\ell}}^{\Lambda} \frac{d\bar{k}}{(2\pi)^D} \frac{1}{t + K k^2}$$

$$dk \simeq \Lambda\, d\ell$$

$$d^D \bar{k} = \Lambda^D d\ell\, dS_D$$

$$= \frac{S_D}{(2\pi)^D} \frac{\Lambda^D}{t + K\Lambda^2} d\ell + o(d\ell^2)$$

$$\frac{dr}{d\ell} = 2r + \frac{12u \frac{S_D}{(2\pi)^D} \Lambda^D}{r + K\Lambda^2}$$

$$\frac{du}{d\ell} = (4-D)\,u$$

ecuaciones del R.G. al 1er orden.

el 2o orden

$$\langle U^2\rangle_0 - \langle U\rangle_0^2$$

$U \to$ mmmm

$\hat{m}\langle\sigma\sigma\sigma \quad \sigma\sigma\sigma\rangle\hat{m} \to D\,\hat{m}\,\hat{m}$

$\hat{m}\hat{m}\langle\sigma\sigma \quad \sigma\sigma\rangle\hat{m}\hat{m} \to \frac{\hat{m}\hat{m}\hat{m}\hat{m}}{\delta(\cdot -)}$

~~$\hat{n}\hat{n}\hat{n}\sigma \quad \sigma\hat{n}\hat{n}\hat{n}$~~ $\rightarrow u_6$

para u_6

$$\frac{du_6}{d\ell} = (6-2D)u_6 + \dots$$

$D>3$

$$\begin{cases} \dfrac{dt}{d\ell} = 2t + \dfrac{12u \frac{S_D}{(2\pi)^D}\Lambda^D}{t+K\Lambda^2} - Au^2 \\ \dfrac{du}{d\ell} = (4-D)u - Bu^2 \end{cases}$$

$$B = \frac{36 \frac{S_d}{(2\pi)^D}\Lambda^D}{(t+K\Lambda^2)^2}$$

hay dos puntos fijos.

$t=0$ y $u=0 \rightarrow$ punto fijo gaussiano inv. de escala.

$$\begin{cases} t^* = \dfrac{-\dfrac{6u^* \frac{S_D}{(2\pi)^D}\Lambda^D}{t^*+K\Lambda^2} + \dfrac{A}{2}u^{*2}}{A} \\[2ex] u^* = \dfrac{(t^*+K\Lambda^2)^2}{36\frac{S_D}{(2\pi)^D}\Lambda^D}\,\underline{\underline{(4-D)}} \end{cases}$$

Parámetro "pequeña"

$$\epsilon = 4-D$$

$$u^* \sim \epsilon \;,\; t^* \sim u^* \sim \epsilon$$

$$\frac{S_D}{(2\pi)^D} \sim \frac{S_4}{(2\pi)^4}$$

$$t^* + K\Lambda^2 \sim K\Lambda^2 + \mathcal{O}(\epsilon)$$

punto fijo

$$\begin{cases} t^* = -\dfrac{K\Lambda^2}{6}\epsilon + \mathcal{O}(\epsilon^2) \\[2ex] u^* = \dfrac{K^2}{36\frac{S_4}{(2\pi)^4}}\epsilon + \mathcal{O}(\epsilon^2) \end{cases}$$

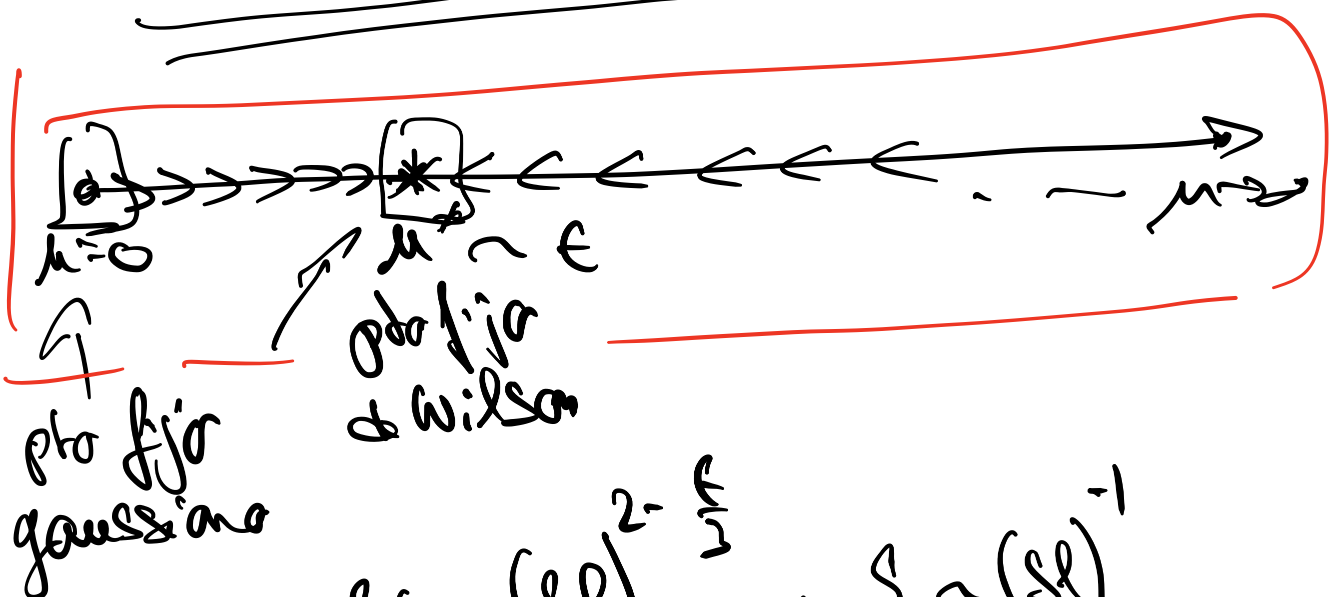
Para estudiar la estabilidad de los puntos fijos

$$t = t^* + \delta t \leftarrow$$

$$\mu = \mu^* + \delta\mu$$

$$\frac{d}{dl}\delta t = \left(2 - \frac{\epsilon}{3}\right)\delta t + O(\epsilon^2)$$

$$\frac{d}{dl}\delta\mu = -\epsilon\,\delta\mu + O(\epsilon^2)$$



$$\delta t = (\delta l)^{2-\frac{\epsilon}{3}} \; ; \; \xi \sim (\delta l)^{-1}$$

$$(\delta t)^{-\nu} = \xi \Rightarrow \nu = \frac{1}{2 - \frac{\epsilon}{3}} \simeq \frac{1}{2} + \frac{1}{12}\epsilon + O(\epsilon^2)$$

$$\nu = \frac{1}{2} + \frac{1}{12}\epsilon + \mathcal{O}(\epsilon^2)$$

para $\epsilon = 1$ $\nu_{3D} \simeq 0{,}58$ $(\nu_{\text{sim. num}} \simeq 0{,}63)$

Generalização al caso $O(n)$

$$S = \int d\vec{x} \left[\frac{K}{2} \sum_{\alpha} (\vec{\nabla}\phi_\alpha)^2 + \frac{t}{2}\vec{\Phi}^2 + u(\vec{\Phi}^2)^2 \right]$$

$$\vec{\Phi} = (\phi_1, \dots, \phi_n)$$

$$\frac{dt}{dl} = 2t + \frac{4u(n+2)K\Lambda^D}{t + K\Lambda^2} - Au^2$$

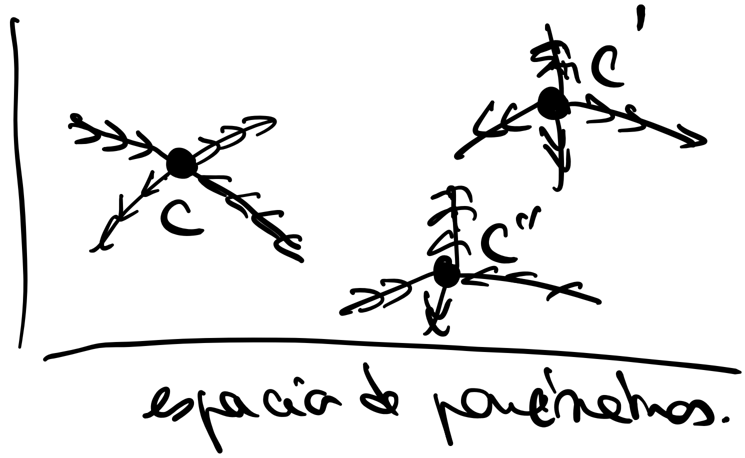
$$\frac{du}{dl} = (4-d)u - \frac{u(n+8)K\Lambda^D}{(t+K\Lambda^2)^2}u^2 + \dots$$

$$\Rightarrow \begin{cases} t^* = \dfrac{-(n+2)}{2(n+8)} K \Lambda^2 \epsilon + \mathcal{O}(\epsilon^2) \\[2ex] u^* = \dfrac{K^2}{4(n+8)} \epsilon + \mathcal{O}(\epsilon^2) \end{cases}$$

$$\nu = \frac{1}{2} + \frac{1}{4} \frac{n+2}{n+8} \epsilon + \mathcal{O}(\epsilon^2)$$

R.G. en la vecindad de un punto crítico

Modelo genérico en varios parámetros



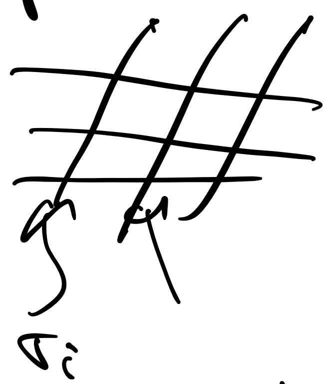
estamos en la vecindad de C.

la acción S_C

en C se le definen unos campos

"$\Phi_i(\vec{r})$"

por ejemplo para Ising: $\begin{cases} \sigma_i \leftarrow n \\ \sigma_i \sigma_j \leftarrow \varepsilon \end{cases}$



$$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j$$

$$\langle \Phi_i(\vec{r}) \, \Phi_i(\vec{r}\,') \rangle \sim \frac{1}{|\vec{r} - \vec{r}\,'|^{2x_i}}$$

x_i = dimensión de escala de Φ_i

$$[\Phi_i] = L^{-x_i}$$

si salimos de S_c

$$S = S_c + \int d\vec{r}\, g\, \Phi_i(\vec{r})$$

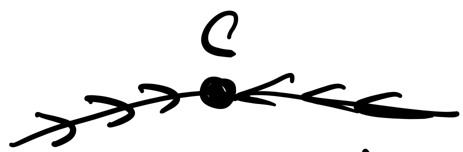
$$[g] = L^{(x_i - D)}$$

$L \to bL$ R.G.

$$g \to b^{D - x_i}\, g$$

y si $x_i > D$ $\quad g \xrightarrow[R.G.]{} 0$

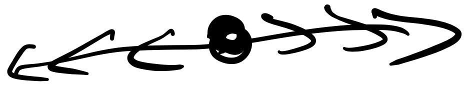
→ perturban S_c con Φ_i no llevan al sistema de la criticalidad



en ese caso la perturbación del Φ_i es una perturbación <u>irrelevante</u>

Pero si $x_i < D$

$$g \xrightarrow[R.G.]{} \infty$$



$\Rightarrow$ perturbación <u>relevante</u>

ejemplo el operador $\mathcal{E}(\vec{r})$ densidad de energía

$$\mathcal{E}(\vec{r}) \rightarrow \text{dim. de escala } x_{\mathcal{E}}$$

$$S_c \rightarrow S_c + \int d\vec{r}\, \mathcal{E}(\vec{r})\, t$$

$$t = \frac{|T - T_c|}{T_c}$$

$$t \sim \xi^{-\nu} \Rightarrow [t] = L^{-1/\nu}$$

$$[t] = L^{x_{\mathcal{E}} - D} \Rightarrow \boxed{x_{\mathcal{E}} = D - \frac{1}{\nu}}$$

Producto de operadores

O.P.E.

$$\Phi_i(\vec r_1)\,\Phi_j(\vec r_2) \xrightarrow[|\vec r_1-\vec r_2|\to 0]{} \sum_k \frac{\Phi_k\left(\frac{\vec r_1+\vec r_2}{2}\right) C_{ijk}}{|\vec r_1-\vec r_2|^{x_i+x_j-x_k}}$$

→ hacer R.G. con los O.P.E.

modelo microscópico en la red

$$S = S_c - \sum_i g_i \sum_{\vec r} a^{x_i}\, \Phi_i(\vec r)$$

$$[g_i] = 0$$

$$\sum_{\vec r} \to \frac{1}{a^D}\int d\vec r$$

$$S = S_c - \underbrace{\int d\bar{r} \sum_i a^{x_i - D} g_i \Phi_i(\bar{r})}_{S_{pert}}$$

$$Z = \int D\phi \ldots e^{-S}, \quad Z_c = \int D\phi\, e^{-S_c}$$

$$Z = Z_c \langle e^{-S_{pert}} \rangle_c$$

$$= Z_c \left(1 - \langle S_{pert} \rangle_c + \frac{1}{2} \langle S_{pert}^2 \rangle_c + \ldots\right)$$

$$= Z_c \Big[1 - \underbrace{\sum_i g_i \int \langle \phi_i(\bar{r}) \rangle \frac{d\bar{r}}{a^{D - x_i}}}$$

$$+ \frac{1}{2} \sum_{ij} g_i g_j \int \langle \phi_i(\bar{r}_1)\, \phi_j(\bar{r}_2) \rangle \frac{d\bar{r}_1\, d\bar{r}_2}{a^{2D - x_i - x_j}}$$

$$+ \ldots \Big]$$

$$\text{as } \bar{r}_1 \to \bar{r}_2 \quad \phi_i(\bar{r}_1)\, \phi_j(\bar{r}_2) \to \sum_k \frac{c_{ijk}\, \Phi_k(\bar{r}_1)}{|\bar{r}_1 - \bar{r}_2|^{x_i + x_j - x_k}}$$

y haciendo la integral
$d\bar{r}_1 d\bar{r}_2$ en $\bar{r}_2$ tq
$$a < |\bar{r}_1 - \bar{r}_2| < a(1+\delta\ell)$$
nos da $\frac{1}{2} S_d \delta\ell \sum_{ijk}' c_{ijk} \int d\bar{r}_1 \frac{\langle \phi_k(\bar{r}_1) \rangle}{a^{D-x_k}}$

$$\Rightarrow Z = Z_c \left[1 - \sum_i \hat{g}_i \int \frac{\langle \phi_i \rangle}{a^{D-x_i}} d\bar{r} + \dots \right]$$

Con $\hat{g}_k = g_k - \frac{1}{2} S_d \sum_{ij}' c_{ijk} g_i g_j \delta\ell$

+ el re-escale $a \to a(1+\delta\ell)$

$$\Rightarrow \frac{dg_k}{d\ell} = (d - x_k) g_k - \frac{1}{2} S_d \sum_{ij}' g_i g_j c_{ijk} + \mathcal{O}(g^3)$$

redefinición de los g_i

$$g_i \rightarrow \frac{2}{S_d} g_i$$

$$\Rightarrow \frac{dg_k}{d\ell} = (d - x_k) g_k - \frac{1}{2} \sum_{ij} c_{ijk} g_i g_j + O(g^3)$$

obs ojo! si $i \neq j$ por ejemplo 1 y 2

$$c_{12k} g_1 g_2 + c_{21k} g_2 g_1$$

$$c_{12k} = c_{21k}$$

pero $c_{11k} g_1^2$

ejemplo

$$S = \int d\vec{r} \left[\frac{1}{2}\sum_\mu (\partial_\mu \vec{\phi})^2 + \frac{t}{2}\vec{\phi}^2 + u(\vec{\phi}^2)^2 \right]$$

$O(n)$

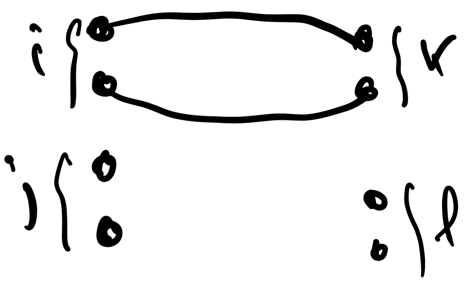
O.P.E

$(\vec{\phi}^2)^2$ con $(\vec{\phi}^2)^2$

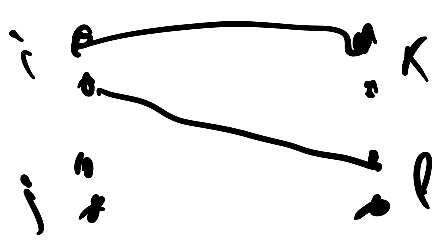
$$\vec{\phi}^2 = \sum_{i=1}^{n} \phi_i^2$$

$$(\vec{\phi}^2)^2 = \sum_{i=1}^{n} \phi_i^2 \sum_{j=1}^{n} \phi_j^2$$

$i = k = 1, 2, .., n$

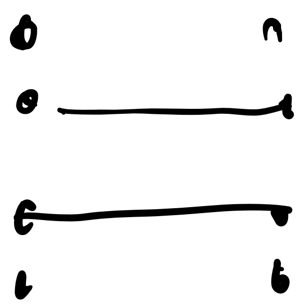


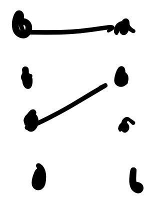
$2 \times 2 \times 2 \times n = 8n$



$j = k = l$

$\Rightarrow 64$





$\frac{(4.3)^2}{2}$

$$C_{uuu} = 8(n+8)$$

$$\frac{du}{dl} = \epsilon u - 8(n+8)u^2$$

$$u^* = \frac{\epsilon}{8(n+8)}$$

para entender la estabilidad del punto u^*

$$u = u^* + \delta u$$

$$\frac{d\delta u}{dl} = \epsilon \delta u - 16(n+8)u^* \delta u + O(\delta u^2)$$
$$= -\epsilon \delta u + O(\delta u^2)$$

De manera general.

Si tenemos varias g_i

$$\frac{dg_i}{dl} = (d - x_i) g_i - \frac{1}{2} \sum_{lj} C_{lji}\, g_l g_j + \dots$$

$\Rightarrow$ punto g_i^*

estabilidad.

$$g_i = g_i^* + \delta g_i$$

Se linealiza el sistema de eq.

$$\frac{d}{dl} \begin{pmatrix} \delta g_1 \\ \vdots \\ \delta g_N \end{pmatrix} = M^* \begin{pmatrix} \delta g_1 \\ \vdots \\ \delta g_N \end{pmatrix}$$

$$M^*_{ij} = (d - x_i)\delta_{ij} - \sum_l C_{lji}\, g_l^*$$

los autovalores > 0 → perturbaciones relevantes

los " < 0 → perturbaciones irrelevantes

$$S = \int d^n x \sum_\mu (\partial_\mu \vec{\phi})^2 + \frac{t}{2}\left[\phi_1^2 + \phi_2^2 + \lambda \phi_3^2\right]$$

$\| \; (\vec{\phi})^2$ $\quad \lambda \neq 1$

sim. ~~SO(3)~~

$$\begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\end{pmatrix} \to \begin{pmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\end{pmatrix}$$

SO(2)