Ruptura espontánea de simetría
Mecanismo Higgs
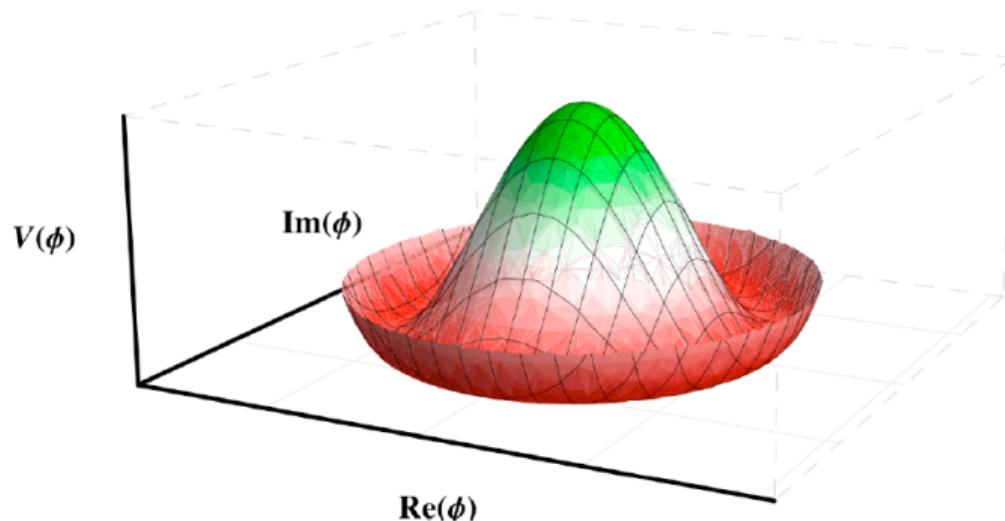
Recap Clase 11

* ruptura espontánea de simetría : $\mathcal{L}$ invariante pero el estudo de mínima energía no.

Ej. $\mathcal{L} = \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi - V(\phi)$, $V(\phi) = -\frac{1}{2} \mu^2 \phi^2 + \frac{\lambda}{4!} \phi^4$, $\mu^2 > 0$
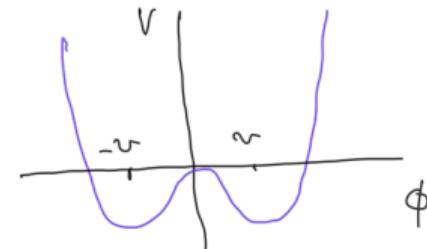
simetría $\mathbb{Z}_2$: $\phi' = -\phi$, $\mathcal{L}' = \mathcal{L}$ rota por $\langle \phi \rangle = v = \sqrt{\frac{6}{\lambda}} \mu$

$\langle \phi \rangle$: valor de expectación de vacío (vev) de ϕ

$\phi(x) = \sigma(x) + v$, $\langle \sigma \rangle = 0$

ejercicio $m_\sigma = \sqrt{2} \mu$

* ruptura de simetría global continua $\Rightarrow$ **bosones de Goldstone**
campos escalares de masa nula

Ej. $\mathcal{L} = \partial_\mu \varphi^* \partial^\mu \varphi - V(\varphi^*, \varphi)$,

simetría $U(1)$ global : $\varphi' = e^{i\alpha} \varphi$

$V(\varphi^*, \varphi) = -\mu^2 \varphi^* \varphi + \lambda (\varphi^* \varphi)^2$, $\mu^2 > 0$

potencial del "sombrero mexicano"
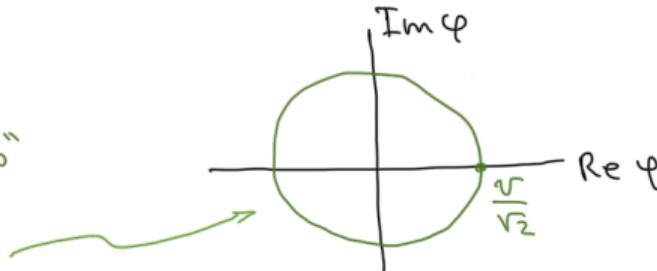
mínimo en $|\varphi|^2 = \dfrac{\mu^2}{2\lambda} = \dfrac{v^2}{2}$

$\langle \varphi \rangle = \dfrac{v}{\sqrt{2}}$ rompe la simetría

$\varphi = e^{i\xi/v} \left(\dfrac{\eta + v}{\sqrt{2}} \right)$

$\langle \xi \rangle = 0$, $\langle \eta \rangle = 0$

ejercicio $m_\eta = \sqrt{2}\,\mu$, $m_\xi = 0$

ξ es el bosón de Goldstone

* Teo. de Goldstone : por cada simetría interna global continua rota existe un bosón de Goldstone

* Simetría local $U(1)$

$$\varphi' = e^{ig\chi(x)} \varphi$$

derivada covariante $\quad D_\mu \varphi = \partial_\mu \varphi + ig B_\mu \varphi$

$$(D_\mu \varphi)' = e^{ig\chi(x)} D_\mu \varphi$$

$$B'_\mu = B_\mu - \partial_\mu \chi$$

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu = F'_{\mu\nu} \quad , \quad \mathcal{L}_B = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Grupos de simetría en el Modelo Estándar

$$U(1) = \{ e^{i\alpha} \} \qquad (e^{i\alpha})^\dagger e^{i\alpha} = 1$$

grupo Abeliano: $e^{i\alpha} e^{i\beta} = e^{i\beta} e^{i\alpha}$

1 parámetro

1 vector de calibre B_μ

$$SU(2) = \{ \text{matrices } U_{2\times2}, U^\dagger U = \mathbb{1}, \det U = 1 \}$$

$$U = \begin{pmatrix} c_0 + i c_3 & c_2 + i c_1 \\ -c_2 + i c_1 & c_0 - i c_3 \end{pmatrix}, \quad c_0^2 + c_1^2 + c_2^2 + c_3^2 = 1$$
$$c_0, c_1, c_2, c_3 \in \mathbb{R}$$

no Abeliano

3 parámetros

3 vectores de calibre
$$W_\mu^1, W_\mu^2, W_\mu^3$$

$$SU(3) = \{ \text{matrices } U_{3\times3}, U^\dagger U = \mathbb{1}, \det U = 1 \}$$

no Abeliano

8 parámetros

8 vectores de calibre
$$G_\mu^1, G_\mu^2, \ldots, G_\mu^8$$

ruptura de simetría local $SU(2) \times U(1)$ en el Modelo Estándar

$$W_\mu^1, W_\mu^2, W_\mu^3, B_\mu \longrightarrow W_\mu^\pm, Z_\mu \quad \text{masivos}, \quad A_\mu \quad \text{masa nula}$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}} \left(W_\mu^1 \mp i W_\mu^2 \right)$$

$$Z_\mu = \cos\theta_w \, W_\mu^3 - \sin\theta_w \, B_\mu$$

$$A_\mu = \sin\theta_w \, W_\mu^3 + \cos\theta_w \, B_\mu$$

SU(2)

$$U = e^{i\alpha_a T_a} = e^{i(\alpha_1 T_1 + \alpha_2 T_2 + \alpha_3 T_3)} \quad , \quad T_a = \frac{\sigma_a}{2}$$

matrices de Pauli

$$= \cos\frac{|\vec{\alpha}|}{2}\sigma_0 + i\,\text{sen}\frac{|\vec{\alpha}|}{2}\,\hat{n}\cdot\vec{\sigma}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\vec{\alpha} = (\alpha_1, \alpha_2, \alpha_3) = |\vec{\alpha}|\,\hat{n}$$

T_a : generadores

$$[T_1, T_2] = i T_3, \quad [T_2, T_3] = i T_1, \quad [T_3, T_1] = i T_2$$

$$\boxed{[T_a, T_b] = i\,\epsilon_{abc}\, T_c}$$

$T_a = \frac{\sigma_a}{2}$ son los generadores en la representación doblete ó $\mathbf{2}$

Existen otras representaciones

Ej: rep. adjunta $(T_a)_{bc} = -i\epsilon_{abc}$ $T_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \ T_2 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \ T_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

transformaciones $SU(2)$ locales

$$\Phi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} \text{ doblete} \quad , \quad \Phi' = e^{ig \chi_a(x) T_a} \Phi \quad , \quad T_a = \frac{\sigma_a}{2}$$

$$D_\mu \Phi = \partial_\mu \Phi + ig T_a W_\mu^a \Phi = \begin{pmatrix} \partial_\mu \varphi_1 \\ \partial_\mu \varphi_2 \end{pmatrix} + \frac{ig}{2} \begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$$

$$(D_\mu \Phi)' = e^{ig \chi_a T_a} D_\mu \Phi$$

$$W_\mu'^a = W_\mu^a - \partial_\mu \chi^a + g \epsilon_{abc} W_\mu^b \chi^c \quad , \quad \chi^c \text{ infinitesimal}$$

$$T_a W_\mu'^a = U \left(T_b W_\mu^b + \frac{i}{g} U^{-1} \partial_\mu U \right) U^{-1} \quad , \quad U = e^{ig \chi^a T_a}$$

* para cualquier rep. con $[T_a, T_b] = i \epsilon_{abc} T_c$, no solo $T_a = \frac{\sigma_a}{2}$

$$\Phi' = e^{ig\chi^a T_a} \Phi \quad , \quad D_\mu \Phi = \partial_\mu \Phi + ig W_\mu^a T_a \Phi$$

$$(D_\mu \Phi)' = e^{ig\chi^a T_a} D_\mu \Phi \quad , \quad W_\mu'^a = W_\mu^a - \partial_\mu \chi^a + g \epsilon_{abc} W_\mu^b \chi^c$$

χ^c infinitesimal

* tensor de campo $\quad F_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g \epsilon_{abc} W_\mu^b W_\nu^c$

Lagrangiano invariante: $\quad \mathcal{L}_W = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} \quad$ Yang-Mills SU(2)

* se generaliza a SU(3), etc, $\quad \epsilon_{abc} \rightarrow f_{abc}$

$$[T_a, T_b] = i f_{abc} T_c$$

)

Mecanismo Higgs

P.W. Higgs (1964)

F. Englert, R. Brout (1964)

G.S. Guralnik, C.R. Hagen, T.W. Kibble (1964)

Mecanismo Higgs $U(1)$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \varphi)^*(D^\mu \varphi) + \mu^2 (\varphi^* \varphi) - \lambda (\varphi^* \varphi)^2$$

$$D_\mu \varphi = \partial_\mu \varphi + i g B_\mu \varphi \quad , \quad F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$\mathcal{L}$ es invariante bajo transformaciones $U(1)$ locales

$$\varphi' = e^{ig\chi(x)} \varphi \quad , \quad B'_\mu = B_\mu - \partial_\mu \chi$$

$$V = -\mu^2 \varphi^* \varphi + \lambda (\varphi^* \varphi)^2 \quad , \quad \mu^2 > 0$$

$$\langle \varphi \rangle = \frac{v}{\sqrt{2}} \quad , \quad \varphi = e^{i\xi/v} \frac{(v+\eta)}{\sqrt{2}}$$
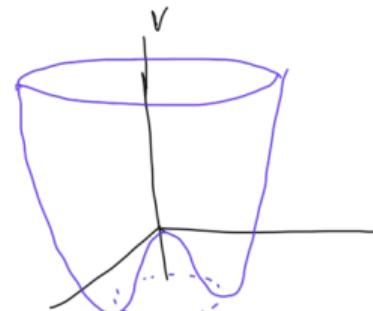
mínimo en $|\varphi|^2 = \frac{v^2}{2} = \frac{\mu^2}{2\lambda}$

$$V = \lambda v^2 \eta^2 + \lambda v \eta^3 + \frac{\lambda}{4} \eta^4 + \text{const}$$

$$\underbrace{\frac{1}{2} m_\eta^2 \eta^2} \qquad m_\eta = \sqrt{2\lambda}\, v = \sqrt{2}\, \mu$$

$$D_\mu \varphi = \frac{1}{\sqrt{2}} e^{i\xi/v} \left[\partial_\mu \eta + i \left(g B_\mu + \frac{1}{v} \partial_\mu \xi \right) (\eta + v) \right]$$

$$(D_\mu \varphi)^* D^\mu \varphi = \frac{1}{2} g^2 (\eta + v)^2 \underbrace{\left(B_\mu + \frac{1}{gv} \partial_\mu \xi \right) \left(B^\mu + \frac{1}{gv} \partial^\mu \xi \right)} + \frac{1}{2} \partial_\mu \eta \partial^\mu \eta$$

incluye $B_\mu \partial^\mu \xi$ $\overset{B}{\sim\!\!\!\sim\!\!\!\sim}\!\bullet\!-\!-\!-\, \xi$

¡Pero se puede hacer una transformación de calibre!

$$B'_\mu = B_\mu - \partial_\mu \chi = B_\mu + \frac{1}{gv} \partial_\mu \xi \quad, \quad \text{escogiendo} \quad \chi = -\frac{1}{gv} \xi$$

$$F'_{\mu\nu} = F_{\mu\nu}$$

$$(D_\mu \varphi)^* D^\mu \varphi = \frac{1}{2} g^2 (\eta + v)^2 B'_\mu B'^\mu + \frac{1}{2} \partial_\mu \eta \, \partial^\mu \eta$$

$$= \frac{1}{2} g^2 v^2 B'_\mu B'_\mu + \cdots$$

$\underbrace{\qquad\qquad\qquad}$

B'_μ es masivo, tiene $m_B = g v$

* $\mathcal{L} = -\frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} + \frac{1}{2} g^2 v^2 B'_\mu B'^\mu + \frac{1}{2} \partial_\mu \eta \, \partial^\mu \eta - \frac{1}{2} (2\lambda v^2) \eta^2$

$\qquad + v g^2 \eta B'_\mu B'^\mu + \frac{1}{2} g^2 \eta^2 B'_\mu B'^\mu - \lambda v \eta^3 - \frac{1}{4} \lambda \eta^4 + \text{const.}$

* bosón de Goldstone ξ desaparece de $\mathcal{L}$, en vez aparece B'_μ masivo

$\qquad B'_\mu$ se "come" a ξ.

$\qquad \xi$ proporciona la polarización longitudinal de B'_μ

$$B'_\mu = B_\mu + \frac{1}{gv} \partial_\mu \xi$$

* interacciones
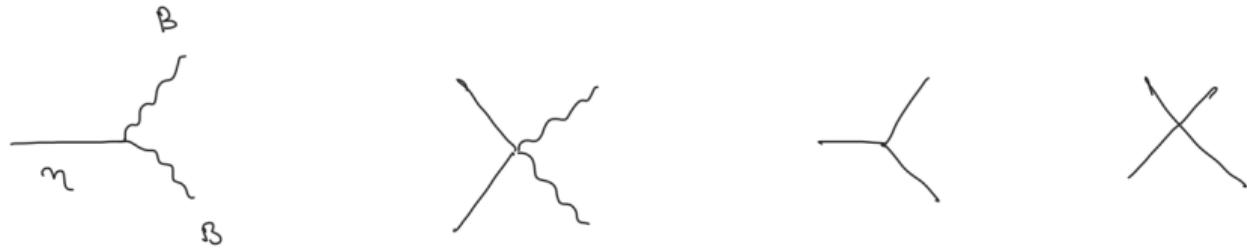
* Al hacer la transformación $B'_\mu = B_\mu - \partial_\mu \chi$, $\chi = -\frac{1}{gv}\xi$.

Se escoge el llamado <u>calibre unitario</u>, en el cual los campos en $\mathcal{L}$ corresponden a partículas físicas. En calibre unitario

$$\varphi' = e^{ig\left(-\frac{1}{gv}\xi\right)}\varphi = e^{-i\frac{\xi}{v}} e^{i\frac{\xi}{v}} \frac{(\eta+v)}{\sqrt{2}}$$

$$\varphi' = \frac{(\eta+v)}{\sqrt{2}}$$

Mecanismo Higgs en el Modelo Estándar

doblete Higgs $\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$, φ^+, φ^0 campos escalares complejos

Φ transforma bajo $SU(2) \times U(1)_Y$ Y : hipercarga

$$\Phi' = e^{i\frac{g}{2}\chi^a \sigma_a + \frac{i}{2}g'\chi} \Phi \quad , \quad (D_\mu \Phi)' = e^{i\frac{g}{2}\chi^a \sigma_a + \frac{i}{2}g'\chi} D_\mu \Phi$$

$$D_\mu \Phi = \partial_\mu \Phi + ig \frac{\sigma_a}{2} W_\mu^a \Phi + \frac{i}{2} g' B_\mu \Phi$$

en las convenciones usadas, Φ tiene $Y=1$

$SU(2)$: constante de acoplamiento g , campos de calibre $W_\mu^1, W_\mu^2, W_\mu^3$

$U(1)_Y$: constante de acoplamiento g' , campo de calibre B_μ

Lagrangiano del Modelo Estándar

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu}$$
$$+ i \bar{\psi} \slashed{D} \psi + h.c.$$
$$+ \psi_i y_{ij} \psi_j \phi + h.c.$$
$$+ |D_\mu \phi|^2 - V(\phi)$$
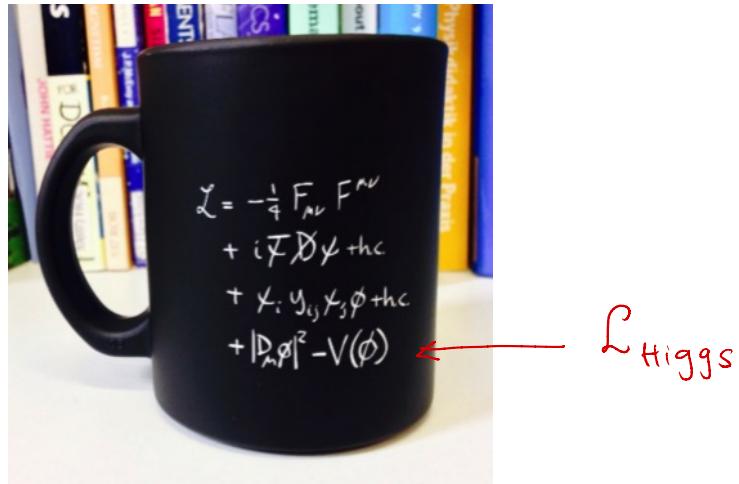
$\leftarrow \mathcal{L}_{Higgs}$

$$\mathcal{L}_{Higgs} = \left(D_\mu \Phi\right)^\dagger \left(D_\mu \Phi\right) - V(\Phi^\dagger, \Phi)$$

$$V(\Phi^\dagger, \Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2, \qquad \mu^2 > 0$$

ejercicio   mínimo en $\Phi^\dagger \Phi = |\varphi^+|^2 + |\varphi^0|^2 = \dfrac{v^2}{2}$, $\qquad v^2 = \dfrac{\mu^2}{\lambda}$

$$\langle \Phi \rangle = \begin{pmatrix} 0 \\ \dfrac{v}{\sqrt{2}} \end{pmatrix} \qquad \text{rompe la simetría}$$

calibre unitario $\Phi(x) = \begin{pmatrix} 0 \\ \dfrac{v + h(x)}{\sqrt{2}} \end{pmatrix}$ $h(x)$ campo escalar real

$$\Phi^\dagger \Phi = \tfrac{1}{2}(v+h)^2 \quad \Rightarrow \quad V = \lambda v^2 h^2 + \lambda v h^3 + \tfrac{\lambda}{4} h^4 + const.$$

$$D_\mu \Phi = \partial_\mu \Phi + ig\,\tfrac{\sigma_a}{2}\, W_\mu^a\, \Phi + i\tfrac{g'}{2}\, B_\mu \Phi$$

$$D_\mu \Phi = \begin{pmatrix} \partial_\mu + i\tfrac{g'}{2}B_\mu + i\tfrac{g}{2}W_\mu^3 & \tfrac{ig}{2}(W_\mu^1 - iW_\mu^2) \\[2mm] \tfrac{ig}{2}(W_\mu^1 + iW_\mu^2) & \partial_\mu + i\tfrac{g'}{2}B_\mu - i\tfrac{g}{2}W_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\[2mm] \dfrac{v+h}{\sqrt{2}} \end{pmatrix}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$D_\mu \Phi = \begin{pmatrix} \frac{ig}{2\sqrt{2}} (W_\mu^1 - iW_\mu^2)(v+h) \\ \frac{1}{\sqrt{2}} \partial_\mu h + \frac{i}{2\sqrt{2}} (g'B_\mu - g W_\mu^3)(v+h) \end{pmatrix}$$

$$(D_\mu \Phi)^\dagger (D_\mu \Phi) = \frac{1}{2} \partial_\mu h \partial^\mu h + \frac{1}{8} g^2 (v+h)^2 (W_\mu^1 + iW_\mu^2)(W^{1\mu} - iW^{2\mu})$$

$$+ \frac{1}{8} (v+h)^2 (g W_\mu^3 - g' B_\mu)(g W^{3\mu} - g' B^\mu)$$

$$\mathcal{L}_{Higgs} = \frac{1}{2} \partial_\mu h\, \partial^\mu h - \overbrace{\lambda v^2 h^2}^{-\frac{1}{2} m_h^2 h^2} - \lambda v h^3 - \frac{\lambda}{4} h^4$$

$$+ \frac{1}{8} g^2 (v+h)^2 (W_\mu^1 + i W_\mu^2)(W^{1\mu} - i W^{2\mu})$$

$$+ \frac{1}{8} (v+h)^2 (g W_\mu^3 - g' B_\mu)(g W^{3\mu} - g' B^\mu)$$

$$\boxed{m_h = \sqrt{2\lambda}\, v}$$

$h(x)$ es el bosón Higgs

Masas para vectores de calibre

$$\mathcal{L}_{Higgs} \supset \frac{1}{8} g^2 v^2 \left(W_\mu^1 + i W_\mu^2 \right) \left(W^{1\mu} - i W^{2\mu} \right)$$

$$= \frac{1}{2} m_W^2 \left(W_\mu^1 W^{1\mu} + W_\mu^2 W^{2\mu} \right) = m_W^2 W_\mu^+ W^{-\mu}$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}} \left(W_\mu^1 \mp i W_\mu^2 \right)$$

$$\boxed{m_W = \frac{1}{2} g v}$$

$$\mathcal{L}_{Higgs} \supset \frac{1}{8} v^2 \left(g W_\mu^3 - g' B_\mu \right) \left(g W^{3\mu} - g' B^\mu \right)$$

$$= \frac{1}{8} \frac{v^2 g^2}{\cos^2 \theta_w} Z_\mu Z^\mu = \frac{1}{2} m_Z^2 Z_\mu Z^\mu$$

$$\boxed{\frac{g'}{g} = tg\,\theta_w}$$

$$Z_\mu = \cos\theta_w W_\mu^3 - sen\,\theta_w B_\mu$$

$$\boxed{m_Z = \frac{v g}{2 \cos\theta_w}}$$

$$A_\mu = sen\,\theta_w W_\mu^3 + \cos\theta_w B_\mu$$

$$\boxed{m_A = 0}$$

A_μ es el fotón

Acoplamientos $h, W_\mu^\pm, Z_\mu$

$$\mathcal{L}_{Higgs} \supset \frac{g^2}{8\cos^2\theta_W} (2vh + h^2) Z_\mu Z^\mu$$

$$+ \frac{1}{4} g^2 (2vh + h^2) W_\mu^+ W^{-\mu}$$

$\supset$ acoplamientos cúbicos proporcionales a masas
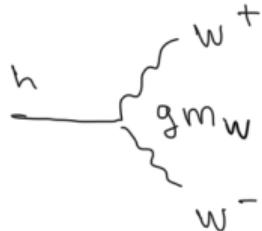
$h \to W^+ W^-$
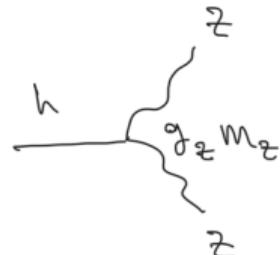
$h \to ZZ$

$$g_z = \frac{g}{\cos\theta_W}$$

Derivada covariante redux

Con hipercarga Y genérica

$$D_\mu = \partial_\mu + ig \frac{\sigma_a}{2} W_\mu^a + i\frac{g'}{2} Y B_\mu$$

Ejercicio

$$D_\mu = \partial_\mu + \frac{ig}{\sqrt{2}} \left(T^+ W_\mu^+ + T^- W_\mu^- \right) + \frac{ig}{\cos\theta_w} \left(T_3 - \text{sen}^2\theta_w Q \right) Z_\mu + ieQ A_\mu$$

$$T^+ = \frac{1}{2}(\sigma_1 + i\sigma_2) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$T^- = \frac{1}{2}(\sigma_1 - i\sigma_2) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad T_3 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

$$Q = T_3 + \frac{Y}{2}$$

$$e = g\,\text{sen}\,\theta_w$$

Resumen 1

grados de libertad

Antes	gdl	Después	gdl
φ^+, φ^0	$2 + 2$	h	1
$W_\mu^1, W_\mu^2, W_\mu^3$	$2+2+2$	W_μ^+, W_μ^-	$3 + 3$
B_μ	2	Z_μ	3
	$\overline{\quad 12 \quad}$	A_μ	$\overline{\dfrac{2}{12}}$

$$SU(2) \times U(1)_Y$$

$$U(1)_{EM}$$

Resumen 2

$$m_w = \frac{1}{2} g v, \qquad m_z = \frac{g v}{2 \cos\theta_w}, \qquad m_h = \sqrt{2\lambda}\, v, \qquad e = g \sin\theta_w$$

dependen de g, g', λ, μ en $\mathcal{L}_{Higgs}$

$$g' = g \, tg\,\theta_w, \qquad \mu^2 = \lambda v^2$$

W, Z descubiertos en 1983, $\qquad$ h descubierto en 2012

$m_w = 80.379$ GeV

$m_z = 91.1876$ GeV

$\dfrac{g^2}{m_w^2} = 4\sqrt{2}\, G_F$, $\quad G_F = 1.166 \times 10^{-5}\, GeV^{-2}$

$m_h = 125.10$ GeV

$\Rightarrow$

$v = 246$ GeV

$\lambda = 0.13$
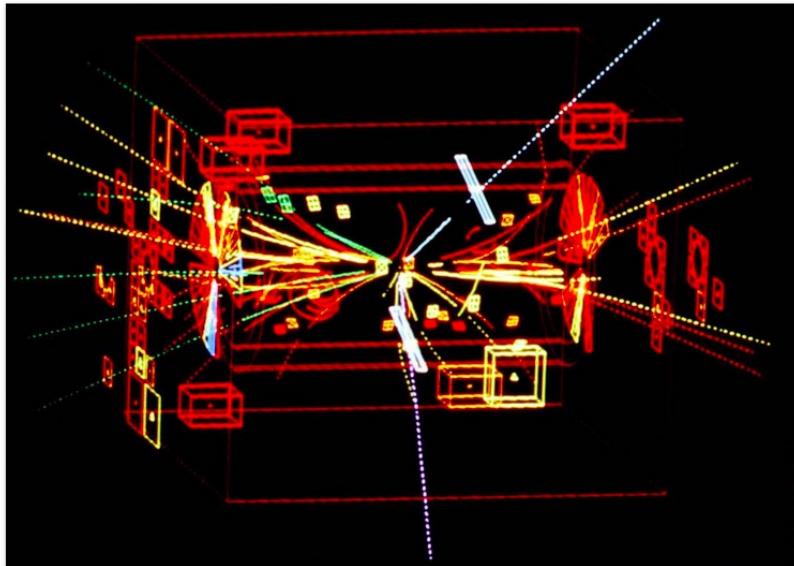
This image taken by the UA1 experiment at CERN on 30 April 1983 was later confirmed to be the first detection of a Z particle (Image: UA1/CERN)
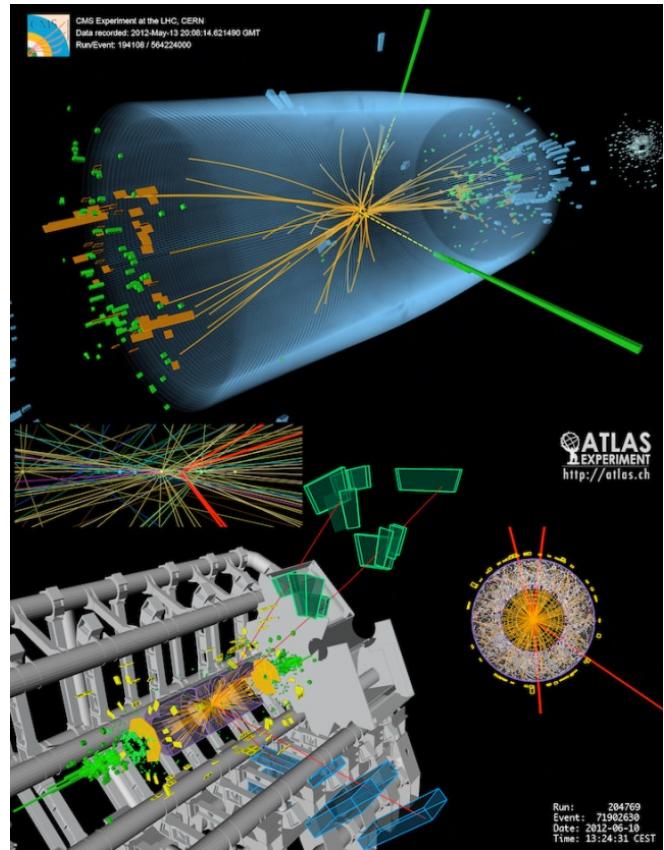
Candidate Higgs boson events from collisions between protons in the LHC. The top event in the CMS experiment shows a decay into two photons (dashed yellow lines and green towers). The lower event in the ATLAS experiment shows a decay into four muons (red tracks) (Image: CMS/ATLAS/CERN)