Física de Partículas

Modelo GWS

Carlos Sandoval

Latin American alliance for Capacity buildiNG in Advanced physics
LA-CoNGA physics

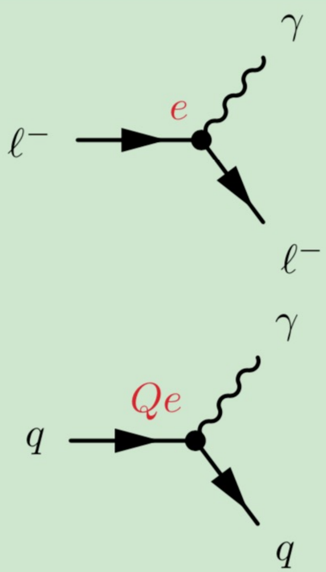
Cofinanciado por el programa Erasmus+ de la Unión Europea

Where we are so far

- The main elements of the Standard Model of particle physics have been described
- There are 12 fundamental spin-half fermions (particles and anti-particles): Dirac equation
- The interactions between particles are described by the exchange of spin-1 gauge bosons : local gauge principle
- Underlying gauge symmetry of the Standard Model is $U(1)_Y \times SU(2)_L \times SU(3)_c$: EM and weak interactions described by the unified electroweak theory
- The predictions of the Electroweak theory were confronted with precision measurements at LEP

Where we are so far

Electromagnetic (QED)

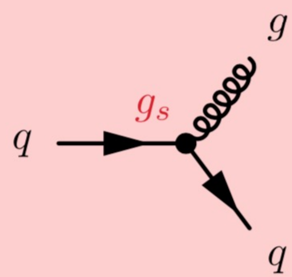


$$\alpha = \frac{e^2}{4\pi}$$

$q = u, d, s, c, b, t$

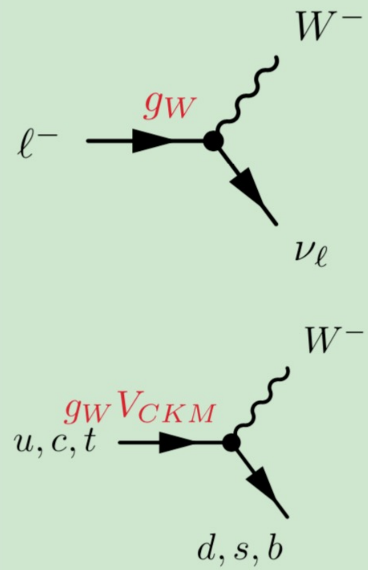
+ antiparticles

Strong (QCD)



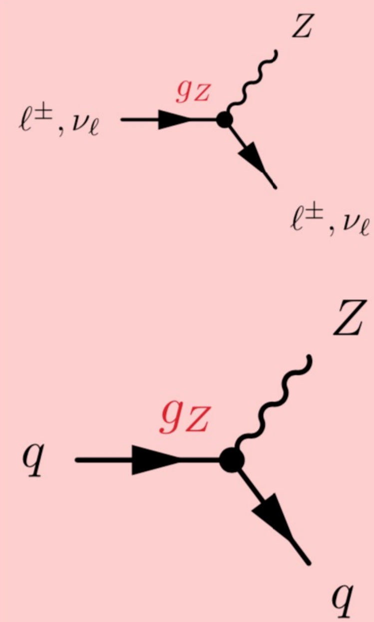
$$\alpha_s = \frac{g_s^2}{4\pi}$$

Weak CC



$$\alpha_W = \frac{g_W^2}{4\pi}$$

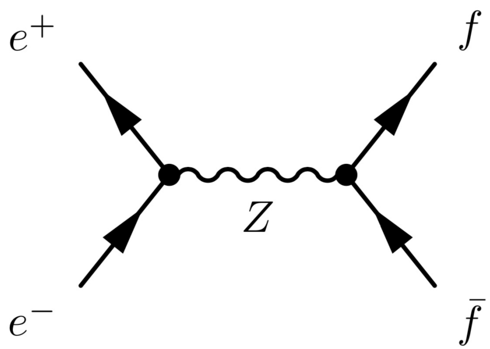
Weak NC

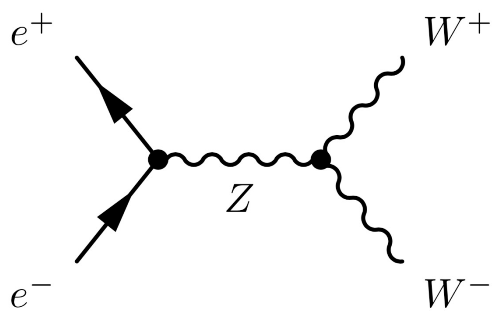


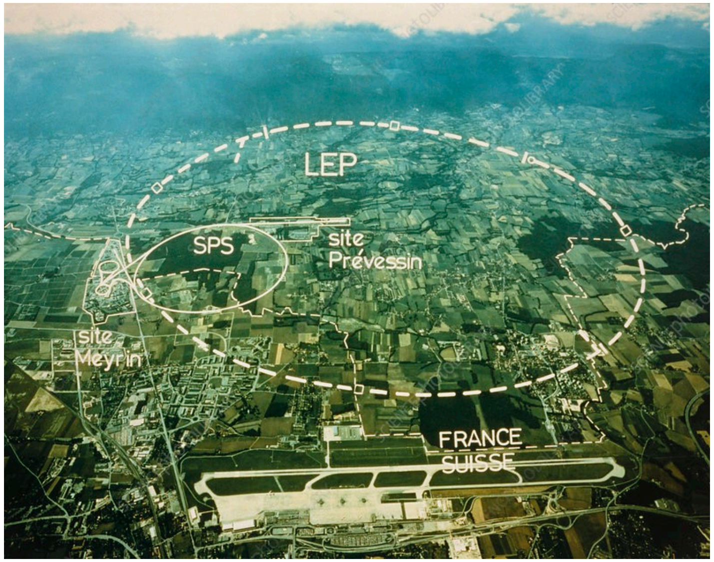
$$g_Z = \frac{g_W}{\cos\theta_W}$$

LEP

- Large Electron Positron collider at CERN (1989-2000)
- Designed as a Z and W boson factory



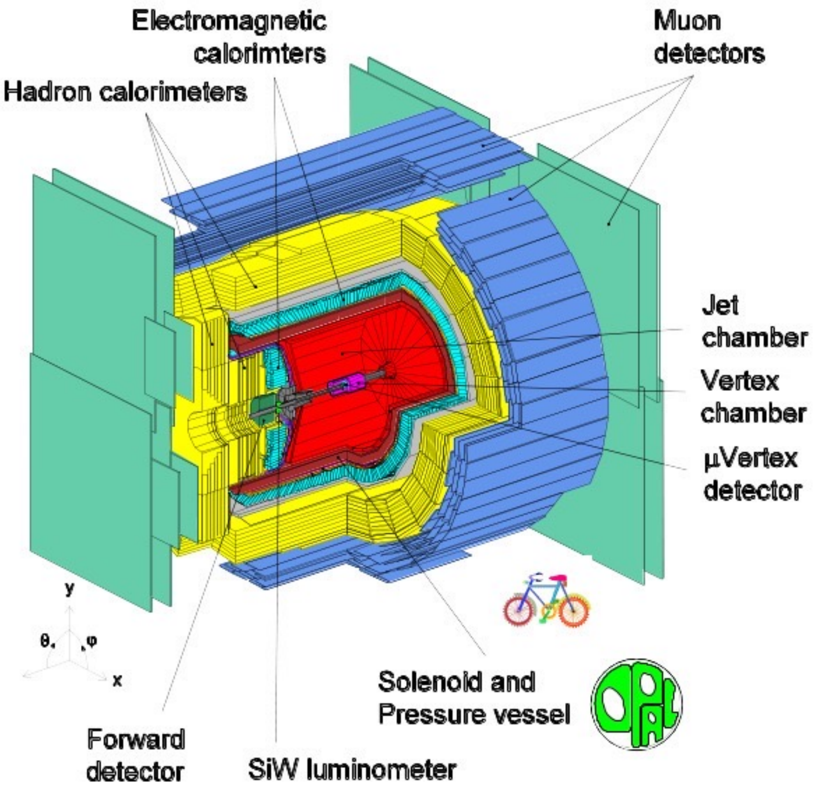


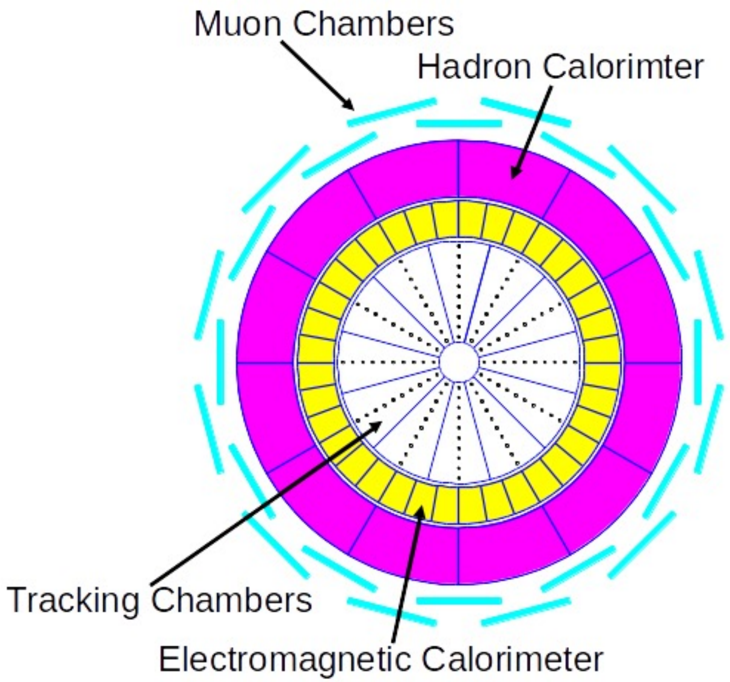


- Highest energy e^+e^- collider ever built: $\sqrt{s}$ = 90-209 GeV
- Circumference: 27 km (LEP tunnel used for the LHC)
- The 4 experiments combined: 16×10^6 Z events, 30×10^3 W events

OPAL

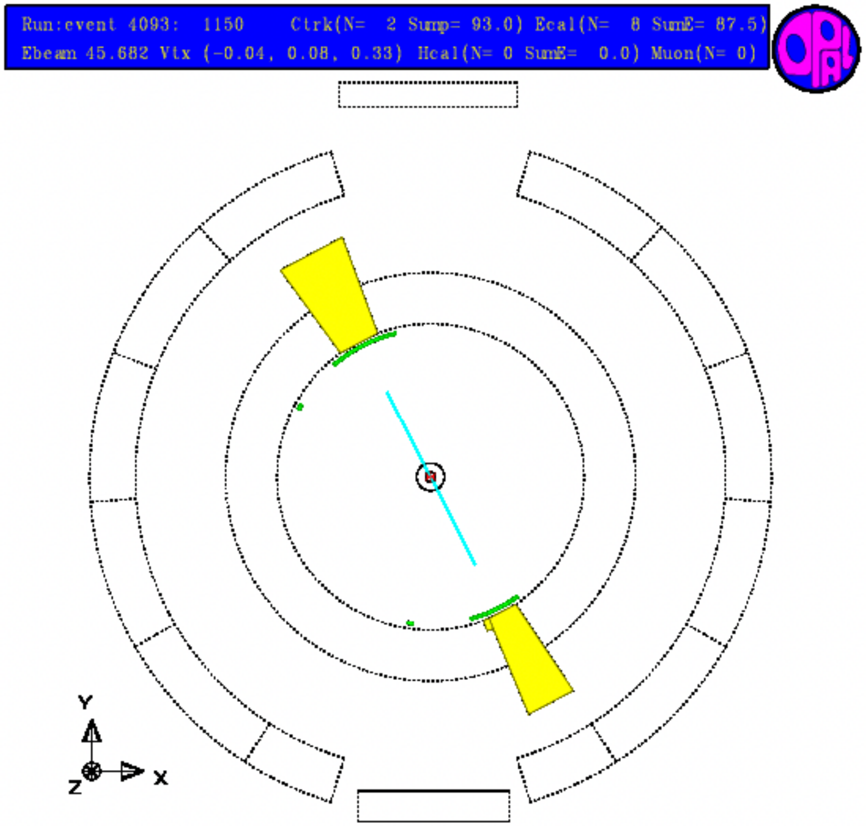
- One of the 4 experiments at LEP



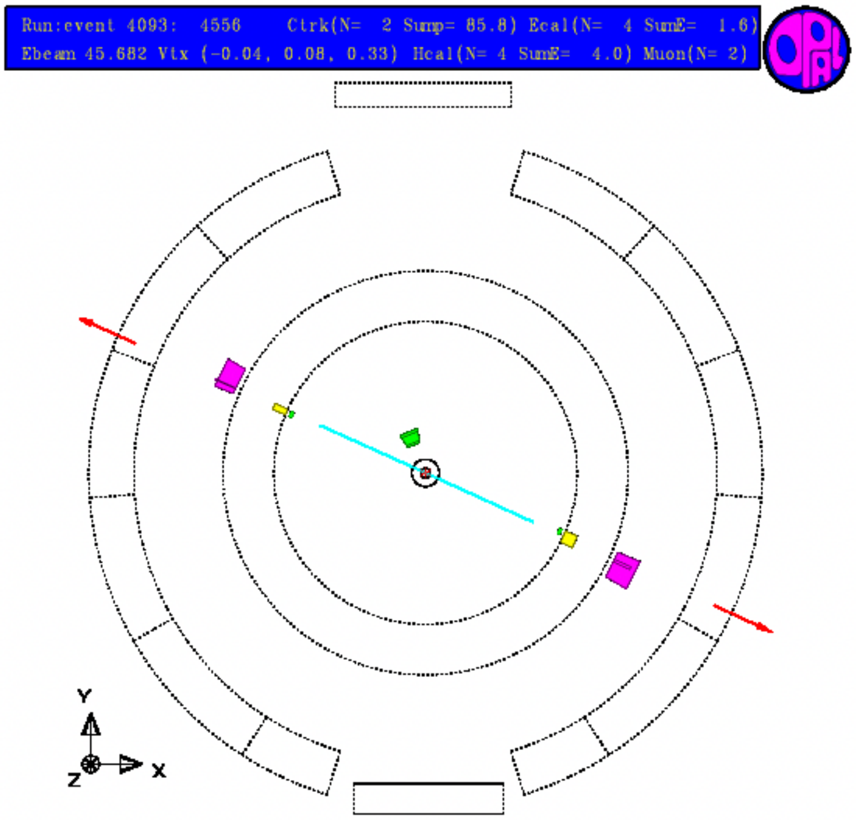


Typical Z events

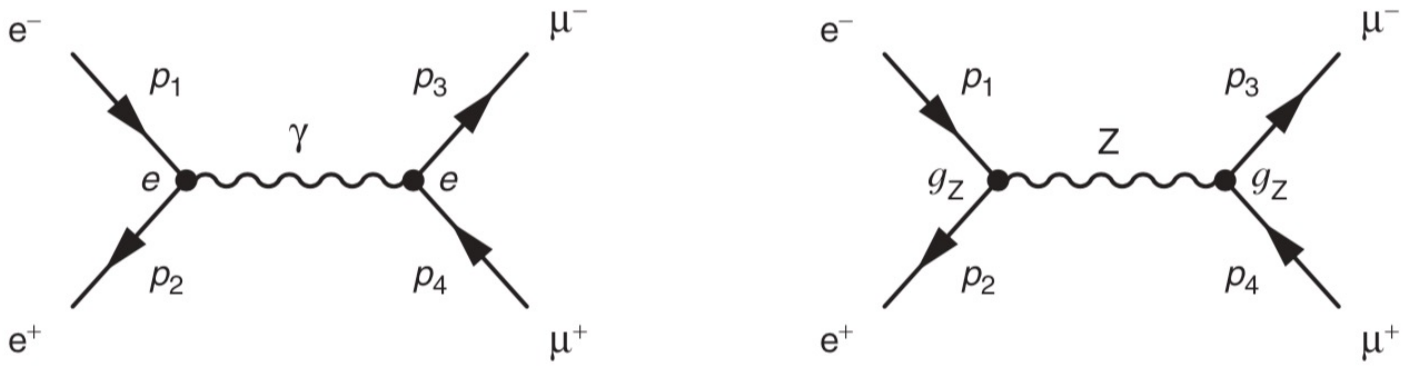
$$e^+e^- \to Z \to e^+e^-$$



$$e^+e^- \to Z \to \mu^+\mu^-$$



Z resonance

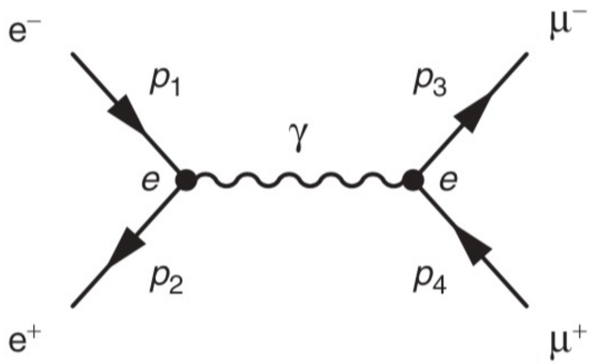


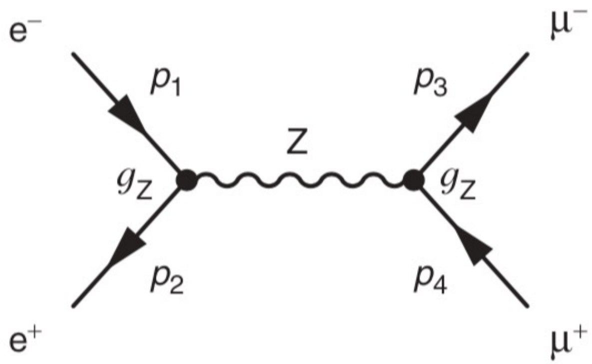
- We already calculated the QED process
- The matrix elements:

$$\mathcal{M}_\gamma \propto \frac{e^2}{q^2} \qquad \mathcal{M}_Z \propto \frac{g_Z^2}{q^2 - m_Z^2}$$

- The QED process dominates at low centre of mass energy ($q^2 = s$)
- In the region $\sqrt{s} \sim m_Z$ the Z-boson process dominates

Z resonance





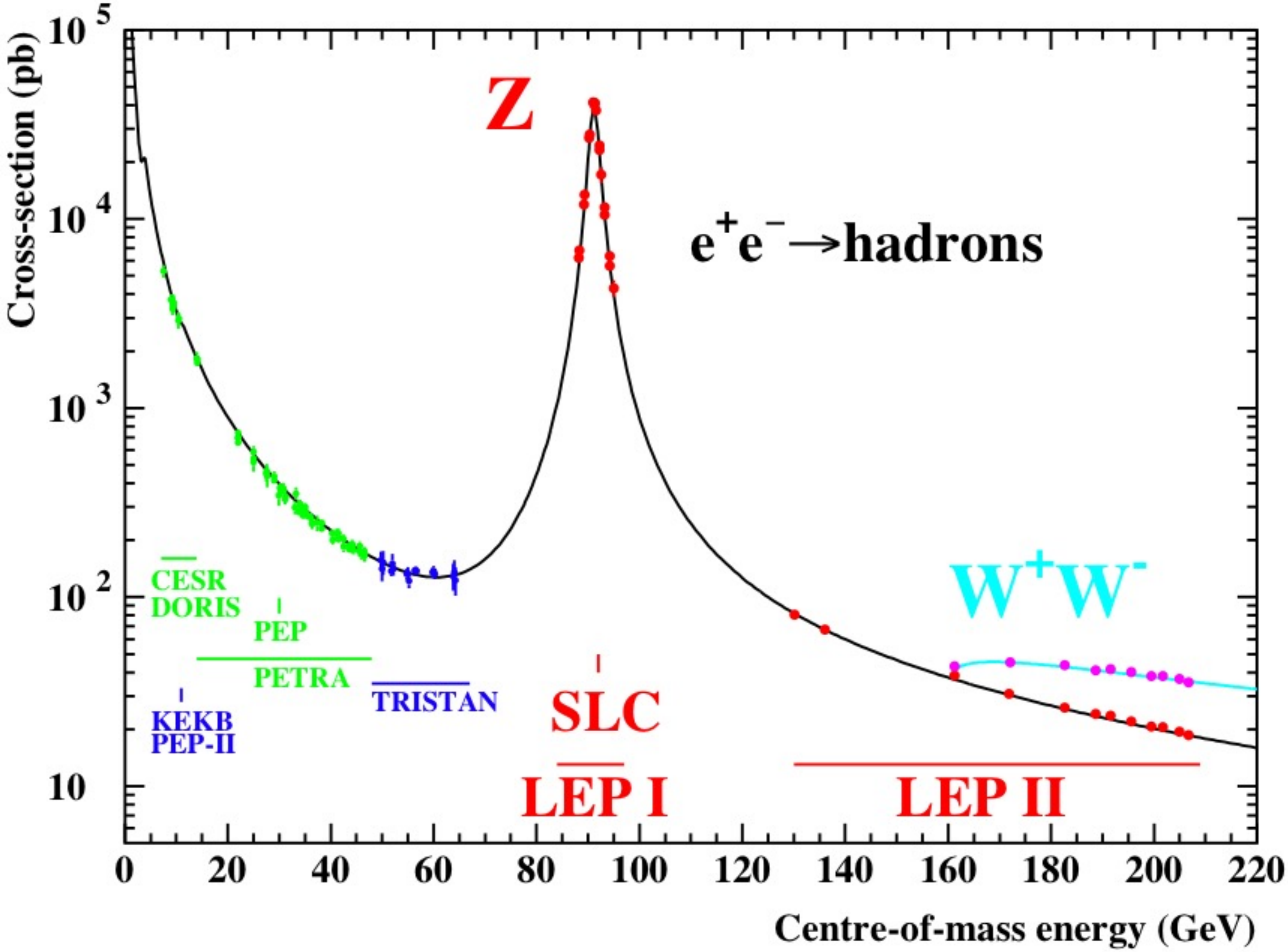
- The Z boson is not a stable particle: propagator modified

$$\frac{1}{q^2 - m_Z^2} \to \frac{1}{q^2 - m_Z^2 + i m_Z \Gamma_Z}$$

- And the cross section is proportional to:

$$\sigma \propto \frac{1}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2}$$

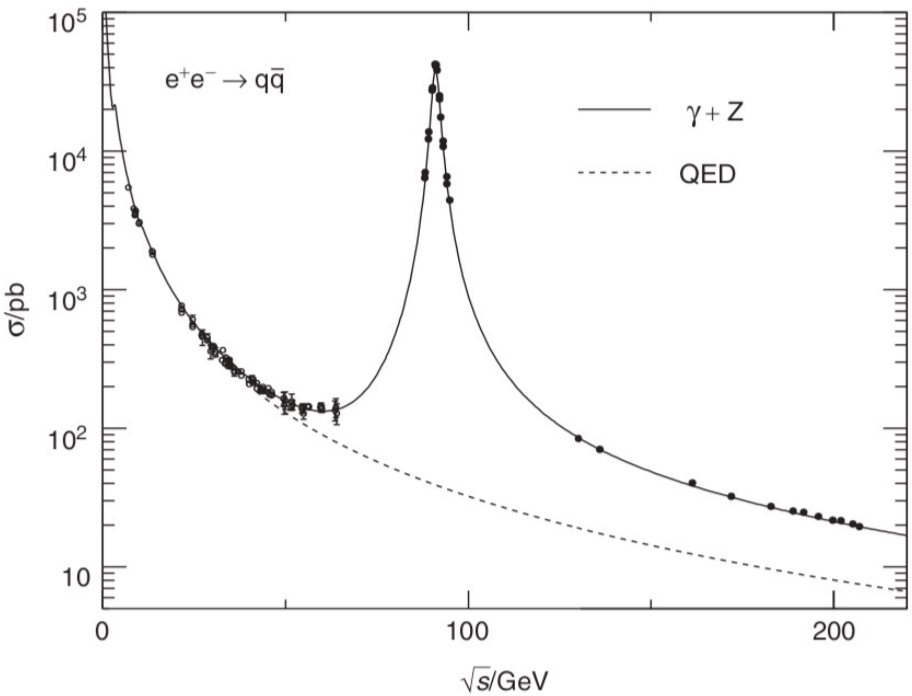
Breit-Wigner resonance



Cross-section (pb)
10^5
10^4
10^3
10^2
10
Z
e+e−→hadrons
CESR
DORIS
PEP
PETRA
KEKB
PEP-II
TRISTAN
SLC
LEP I
LEP II
W+W-
0
20
40
60
80
100
120
140
160
180
200
220
Centre-of-mass energy (GeV)

Z resonance

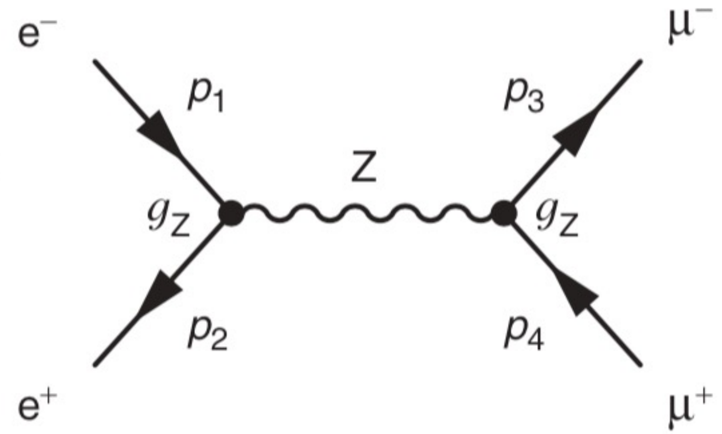
- Below 40 GeV: QED process dominates
- Between 50 and 80 GeV: contributions from both processes
- Around the resonance: Z boson process dominates
- Away from the resonance: both processes of the same order of magnitude (EW unification)



Z production cross section

- The cross section calculation around the resonance would only need the Z boson contribution

$$\mathcal{M}_{fi} = -\frac{g_Z^2}{(s - m_Z^2 + i m_Z \Gamma_Z)} g_{\mu\nu} \left[\overline{v}(p_2)\gamma^\mu \tfrac{1}{2}\left(c_V^e - c_A^e \gamma^5\right) u(p_1)\right] \times \left[\overline{u}(p_3)\gamma^\nu \tfrac{1}{2}\left(c_V^\mu - c_A^\mu \gamma^5\right) v(p_4)\right],$$



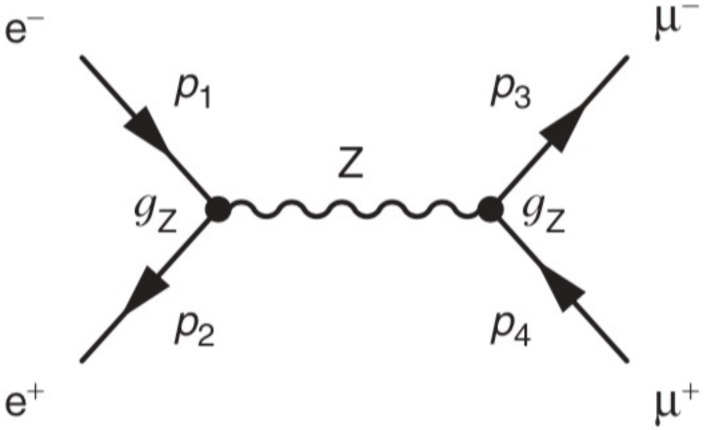
with the corresponding vector and axial couplings to the Z boson

- The rest of the procedure is similar to the QED calculation
- Around the resonance, the masses of the leptons can be ignored
- In this scenario, the helicity and chiral states are the same

Z production cross section

- The cross section calculation around the resonance would only need the Z boson contribution

$$\mathcal{M}_{fi} = -\frac{g_Z^2}{(s - m_Z^2 + i m_Z \Gamma_Z)} g_{\mu\nu} \left[\bar{v}(p_2)\gamma^\mu \tfrac{1}{2}\left(c_V^e - c_A^e \gamma^5\right) u(p_1)\right] \times$$
$$\left[\bar{u}(p_3)\gamma^\nu \tfrac{1}{2}\left(c_V^\mu - c_A^\mu \gamma^5\right) v(p_4)\right],$$



with
$$c_V = (c_L + c_R)$$
$$c_A = (c_L - c_R)$$

$$\mathcal{M}_{fi} = -\frac{g_Z^2}{(s - m_Z^2 + i m_Z \Gamma_Z)} g_{\mu\nu} \left[c_L^e \bar{v}(p_2)\gamma^\mu P_L u(p_1) + c_R^e \bar{v}(p_2)\gamma^\mu P_R u(p_1)\right] \times$$
$$\left[c_L^\mu \bar{u}(p_3)\gamma^\nu P_L v(p_4) + c_R^\mu \bar{u}(p_3)\gamma^\nu P_R v(p_4)\right]$$

Z production cross section

- The cross section calculation around the resonance would only need the Z boson contribution

$$\mathcal{M}_{fi} = -\frac{g_Z^2}{(s - m_Z^2 + i m_Z \Gamma_Z)} g_{\mu\nu} \left[c_L^{\mathrm{e}} \overline{v}(p_2)\gamma^{\mu} P_L u(p_1) + c_R^{\mathrm{e}} \overline{v}(p_2)\gamma^{\mu} P_R u(p_1)\right] \times$$
$$\left[c_L^{\mu} \overline{u}(p_3)\gamma^{\nu} P_L v(p_4) + c_R^{\mu} \overline{u}(p_3)\gamma^{\nu} P_R v(p_4)\right]$$

here $m_Z \gg m_\mu$, so:

$$P_L u = u_\downarrow \quad P_R u = u_\uparrow \quad P_L v = v_\uparrow \quad P_R v = v_\downarrow$$

and only 4 contributions to the matrix element are non zero:

$$\mathcal{M}_{RR} = -P_Z(s)\, g_Z^2\, c_R^{\mathrm{e}} c_R^{\mu}\, g_{\mu\nu} \left[\overline{v}_\downarrow(p_2)\gamma^{\mu} u_\uparrow(p_1)\right] \left[\overline{u}_\uparrow(p_3)\gamma^{\nu} v_\downarrow(p_4)\right]$$
$$\mathcal{M}_{RL} = -P_Z(s)\, g_Z^2\, c_R^{\mathrm{e}} c_L^{\mu}\, g_{\mu\nu} \left[\overline{v}_\downarrow(p_2)\gamma^{\mu} u_\uparrow(p_1)\right] \left[\overline{u}_\downarrow(p_3)\gamma^{\nu} v_\uparrow(p_4)\right]$$
$$\mathcal{M}_{LR} = -P_Z(s)\, g_Z^2\, c_L^{\mathrm{e}} c_R^{\mu}\, g_{\mu\nu} \left[\overline{v}_\uparrow(p_2)\gamma^{\mu} u_\downarrow(p_1)\right] \left[\overline{u}_\uparrow(p_3)\gamma^{\nu} v_\downarrow(p_4)\right]$$
$$\mathcal{M}_{LL} = -P_Z(s)\, g_Z^2\, c_L^{\mathrm{e}} c_L^{\mu}\, g_{\mu\nu} \left[\overline{v}_\uparrow(p_2)\gamma^{\mu} u_\downarrow(p_1)\right] \left[\overline{u}_\downarrow(p_3)\gamma^{\nu} v_\uparrow(p_4)\right]$$

Z production cross section

- The cross section calculation around the resonance would only need the Z boson contribution

$$\mathcal{M}_{RR} = -P_Z(s)\, g_Z^2\, c_R^e c_R^\mu\, g_{\mu\nu}\, [\bar{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)]\, [\bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$
$$\mathcal{M}_{RL} = -P_Z(s)\, g_Z^2\, c_R^e c_L^\mu\, g_{\mu\nu}\, [\bar{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)]\, [\bar{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$
$$\mathcal{M}_{LR} = -P_Z(s)\, g_Z^2\, c_L^e c_R^\mu\, g_{\mu\nu}\, [\bar{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)]\, [\bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$
$$\mathcal{M}_{LL} = -P_Z(s)\, g_Z^2\, c_L^e c_L^\mu\, g_{\mu\nu}\, [\bar{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)]\, [\bar{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$

with $P_Z(s) = 1/(s - m_Z^2 + i m_Z \Gamma_Z)$

- The combinations are identical to the ones derived for QED:

$$|\mathcal{M}_{RR}|^2 = |P_Z(s)|^2\, g_Z^4 s^2 (c_R^e)^2 (c_R^\mu)^2 (1 + \cos\theta)^2$$
$$|\mathcal{M}_{RL}|^2 = |P_Z(s)|^2\, g_Z^4 s^2 (c_R^e)^2 (c_L^\mu)^2 (1 - \cos\theta)^2$$
$$|\mathcal{M}_{LR}|^2 = |P_Z(s)|^2\, g_Z^4 s^2 (c_L^e)^2 (c_R^\mu)^2 (1 - \cos\theta)^2$$
$$|\mathcal{M}_{LL}|^2 = |P_Z(s)|^2\, g_Z^4 s^2 (c_L^e)^2 (c_L^\mu)^2 (1 + \cos\theta)^2$$

Z production cross section

- Analogous to the QED calculation, averaging over the initial state spin configurations:

$$\langle |\mathcal{M}|^2 \rangle = \frac{1}{4}\left(|\mathcal{M}_{RR}|^2 + |\mathcal{M}_{LL}|^2 + |\mathcal{M}_{LR}|^2 + |\mathcal{M}_{RL}|^2\right)$$

$$\langle |\mathcal{M}|^2 \rangle = \frac{1}{4}\frac{g_Z^4 s^2}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2} \times \Big\{ \Big[(c_R^e)^2 (c_R^\mu)^2 + (c_L^e)^2 (c_L^\mu)^2 \Big] (1+\cos\theta)^2 + \Big[(c_R^e)^2 (c_L^\mu)^2 + (c_L^e)^2 (c_R^\mu)^2 \Big] (1-\cos\theta)^2 \Big\}$$

- Going back to the vector and axial couplings, the differential cross section:

$$\frac{d\sigma}{d\Omega} = \frac{1}{256\pi^2 s} \cdot \frac{g_Z^4 s^2}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2} \times \left\{ \frac{1}{4}\left[(c_V^e)^2 + (c_A^e)^2\right]\left[(c_V^\mu)^2 + (c_A^\mu)^2\right]\left(1+\cos^2\theta\right) + 2c_V^e c_A^e c_V^\mu c_A^\mu \cos\theta \right\}$$

Z production cross section

- And the total cross section:

$$\sigma(e^+e^- \to Z \to \mu^+\mu^-) = \frac{1}{192\pi} \frac{g_Z^4 s}{(s - m_Z^2)^2 + m_Z^2\Gamma_Z^2} \left[(c_V^e)^2 + (c_A^e)^2\right]\left[(c_V^\mu)^2 + (c_A^\mu)^2\right]$$

- This can be expressed in terms of the partial decay rates of the Z boson:

$$\Gamma_{ee} = \frac{g_Z^2 m_Z}{48\pi}\left[(c_V^e)^2 + (c_A^e)^2\right] \quad \Gamma_{\mu\mu} = \frac{g_Z^2 m_Z}{48\pi}\left[(c_V^\mu)^2 + (c_A^\mu)^2\right]$$

$$\sigma(e^+e^- \to Z \to \mu^+\mu^-) = \frac{12\pi s}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{\mu\mu}}{(s - m_Z^2)^2 + m_Z^2\Gamma_Z^2}$$

Z production cross section

- The cross section calculation around the resonance would only need the Z boson contribution

$$\sigma(\mathrm{e^+e^-} \to \mathrm{Z} \to \mu^+\mu^-) = \frac{12\pi s}{m_\mathrm{Z}^2} \frac{\Gamma_\mathrm{ee}\Gamma_{\mu\mu}}{(s - m_\mathrm{Z}^2)^2 + m_\mathrm{Z}^2\Gamma_\mathrm{Z}^2}$$

- The cross section for other final state fermions can be obtained replacing the partial decay rates into muons with the corresponding partial decay rates: Γ_ff

- The maximum value for the cross section:

$$\sigma_\mathrm{ff}^0 = \frac{12\pi}{m_\mathrm{Z}^2} \frac{\Gamma_\mathrm{ee}\Gamma_\mathrm{ff}}{\Gamma_\mathrm{Z}^2}$$

Z production cross section

- The cross section calculation around the resonance would only need the Z boson contribution

$$\sigma(e^+e^- \to Z \to \mu^+\mu^-) = \frac{12\pi s}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{\mu\mu}}{(s - m_Z^2)^2 + m_Z^2\Gamma_Z^2}$$

- The maximum value for the cross section:

$$\sigma^0_{ff} = \frac{12\pi}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}$$

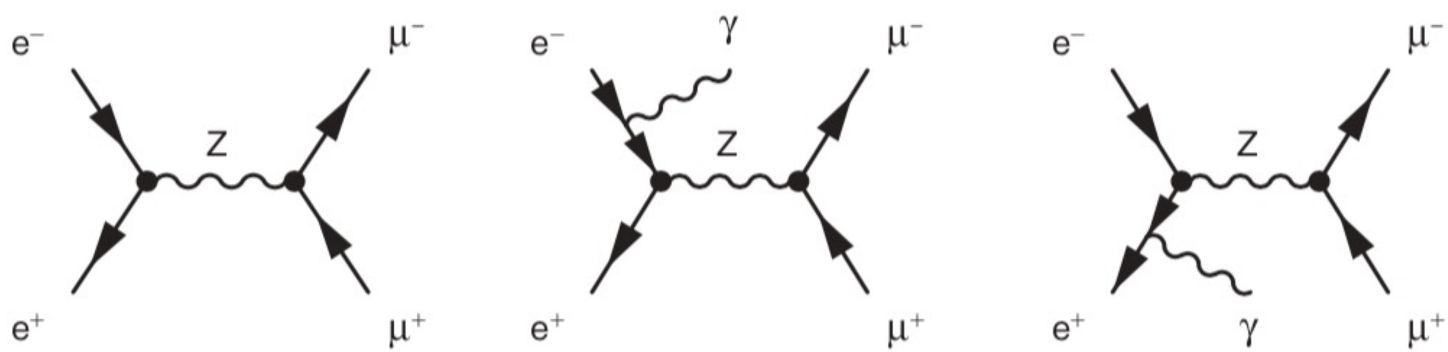
- The cross section falls to half the maximum value at:

$$\sqrt{s} = m_Z \pm \Gamma_Z/2$$

- So the total decay rate is the width of the cross-section distribution

Z mass measurement

- In practice, initial state radiation diagrams should be included
- This reduces the centre of mass energy of the collision and smears out the resonance



- At LEP, the mass of the Z boson and its decay width were measured by measuring the cross section for $e^+e^- \rightarrow Z \rightarrow q\bar{q}$ at different centre-of-mass energies

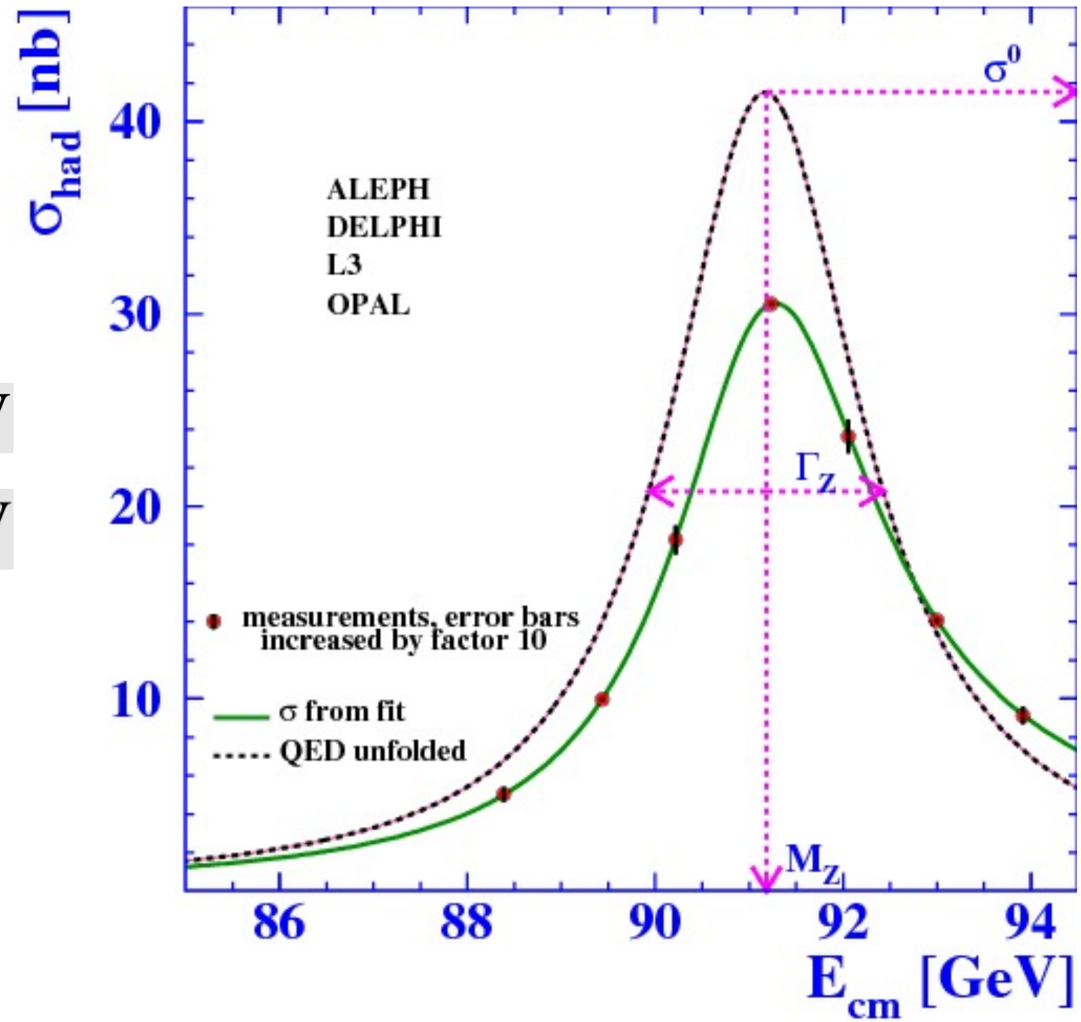
Z mass measurement

- Mass of the Z boson
- Total decay width
- Peak cross-section

$$m_Z = 91.1875 \pm 0.0021\,\text{GeV}$$

$$\Gamma_Z = 2.4952 \pm 0.0023\,\text{GeV}$$

- Measured with high level precision



Number of generations

- The Z boson couples to all fermions
- The total decay width has contributions from all fermions

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\mathrm{hadrons}} + \Gamma_{\nu_e\nu_e} + \Gamma_{\nu_\mu\nu_\mu} + \Gamma_{\nu_\tau\nu_\tau}$$

- If there were more generations, the decay width of the Z boson would be affected by it

Number of generations

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- If there were more generations, the decay width of the Z boson would be affected by it

$$\Gamma_Z = 3\Gamma_{\ell\ell} + \Gamma_{\text{hadrons}} + N_\nu\Gamma_{\nu\nu}$$

$$N_\nu = \frac{(\Gamma_Z - 3\Gamma_{\ell\ell} - \Gamma_{\text{hadrons}})}{\Gamma_{\nu\nu}^{\text{SM}}}$$

Number of generations

- The Z boson couples to all fermions
- The total decay width has contributions from all fermions

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\mathrm{hadrons}} + \Gamma_{\nu_e\nu_e} + \Gamma_{\nu_\mu\nu_\mu} + \Gamma_{\nu_\tau\nu_\tau}$$

- If there were more generations, the decay width of the Z boson would be affected by it
- The decay width to fermions can be calculated using the vertex coupling:

$$\Gamma(\mathrm{Z} \to \mathrm{f}\bar{\mathrm{f}}) = \frac{g_Z^2 m_Z}{48\pi}(c_V^2 + c_A^2)$$

- In particular for neutrinos:

$$\Gamma(\mathrm{Z} \to \nu_e\bar{\nu}_e) = \frac{g_Z^2 m_Z}{48\pi}\left(\frac{1}{4} + \frac{1}{4}\right)$$

Number of generations

- The Z boson couples to all fermions
- The total decay width has contributions from all fermions

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\text{hadrons}} + \Gamma_{\nu_e\nu_e} + \Gamma_{\nu_\mu\nu_\mu} + \Gamma_{\nu_\tau\nu_\tau}$$

- If there were more generations, the decay width of the Z boson would be affected by it

$$\Gamma_Z = 3\Gamma_{\ell\ell} + \Gamma_{\text{hadrons}} + N_\nu \Gamma_{\nu\nu}$$

$$N_\nu = \frac{(\Gamma_Z - 3\Gamma_{\ell\ell} - \Gamma_{\text{hadrons}})}{\Gamma_{\nu\nu}^{\text{SM}}}$$

measured calculated

Number of generations

- The Z boson couples to all fermions
- The total decay width has contributions from all fermions

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\text{hadrons}} + \Gamma_{\nu_e\nu_e} + \Gamma_{\nu_\mu\nu_\mu} + \Gamma_{\nu_\tau\nu_\tau}$$

- If there were more generations, the decay width of the Z boson would be affected by it

$$\Gamma_Z = 3\Gamma_{\ell\ell} + \Gamma_{\text{hadrons}} + N_\nu \Gamma_{\nu\nu}$$

$$N_\nu = \frac{(\Gamma_Z - 3\Gamma_{\ell\ell} - \Gamma_{\text{hadrons}})}{\Gamma_{\nu\nu}^{\text{SM}}}$$

- And from the cross section peak of the Z resonance:

$$\sigma^0(e^+e^- \to Z \to f\bar{f}) = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}$$

Number of generations

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$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\text{hadrons}} + \Gamma_{\nu_e\nu_e} + \Gamma_{\nu_\mu\nu_\mu} + \Gamma_{\nu_\tau\nu_\tau}$$

- If there were more generations, the decay width of the Z boson would be affected by it

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- And from the cross section peak of the Z resonance:

$$\sigma^0(e^+e^- \to Z \to f\bar{f}) = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}$$

Number of generations

- The Z boson couples to all fermions
- The total decay width has contributions from all fermions

$$N_\nu = \frac{(\Gamma_Z - 3\Gamma_{\ell\ell} - \Gamma_{\text{hadrons}})}{\Gamma_{\nu\nu}^{\text{SM}}}$$

$$\Gamma(Z \to \nu_e \bar{\nu}_e) = \frac{g_Z^2 m_Z}{48\pi}\left(\frac{1}{4} + \frac{1}{4}\right) = 167\,\text{MeV}$$

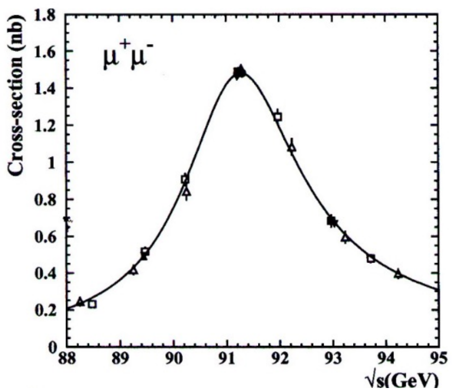
$$N_\nu = 2.9840 \pm 0.0082$$

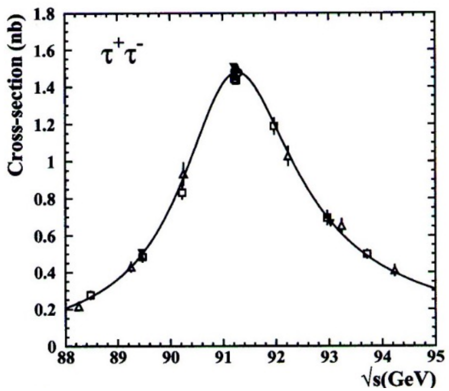
Γ_Z	**2495.2±2.3 MeV**
Γ_{ee}	83.91±0.12 MeV
$\Gamma_{\mu\mu}$	83.99±0.18 MeV
$\Gamma_{\tau\tau}$	84.08±0.22 MeV
Γ_{qq}	1744.4±2.0 MeV
$N_\nu\Gamma_{\nu\nu}$	499.0±1.5 MeV

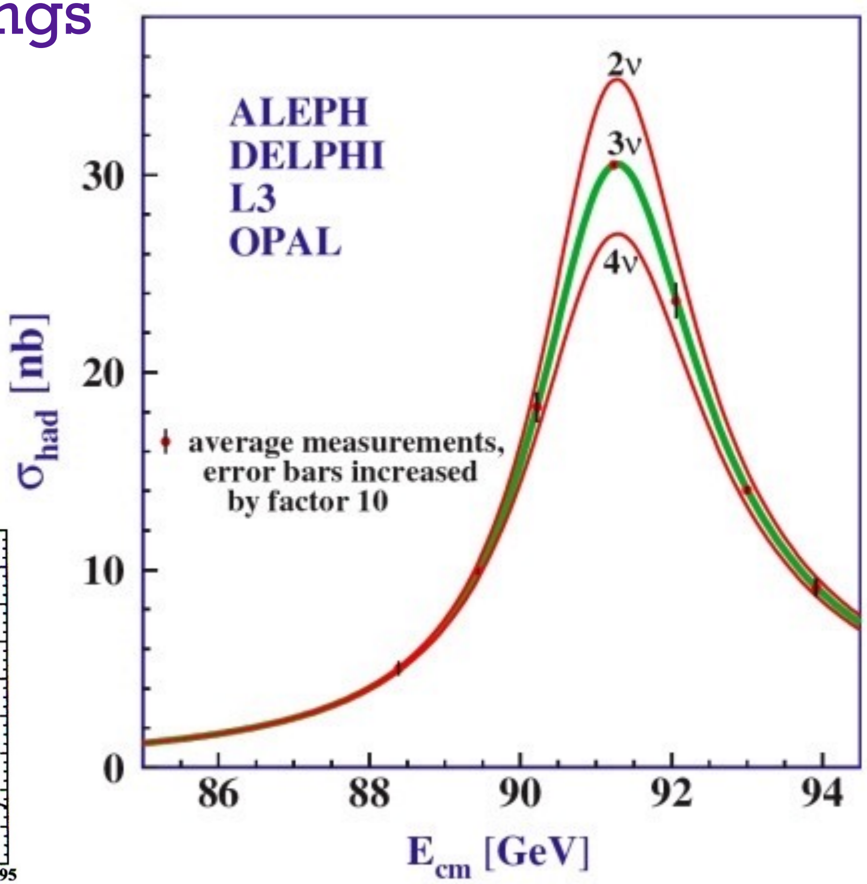
Number of generations

- Most likely, only 3 generations of fermions
- Universality of lepton couplings
- Calculated cross section assumes 3 colours

$$N_\nu = 2.9840 \pm 0.0082$$







Weak mixing angle

- The weak mixing angle relates all couplings in the EW model and is therefore a fundamental parameter
- It can be measured through the ratio between vector and axial couplings:

$$\frac{c_V}{c_A} = 1 - \frac{2Q\sin^2\theta_W}{I_W^{(3)}}$$

For charged leptons:

$$\frac{c_V^\ell}{c_A^\ell} = 1 - 4\sin^2\theta_W$$

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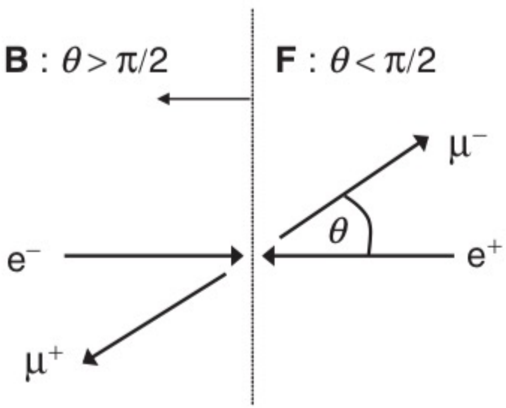

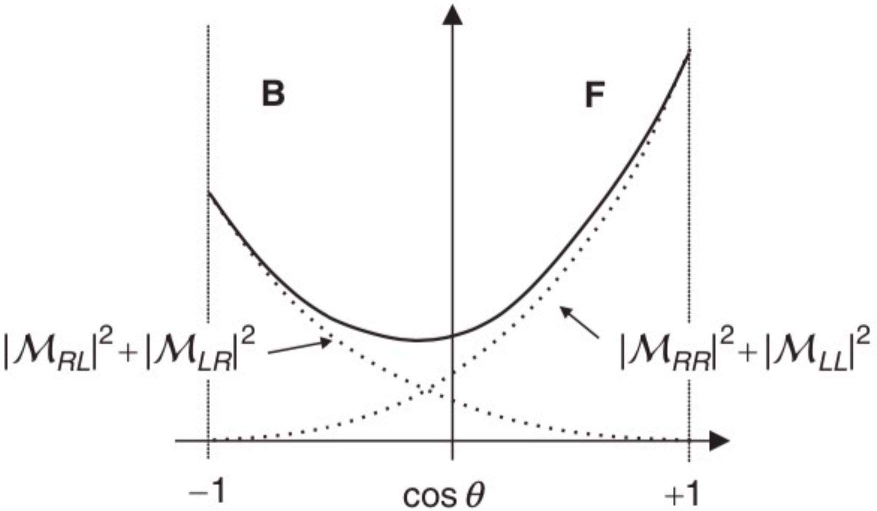

- The cross section has the form:

$$\frac{d\sigma}{d\Omega} \propto a(1 + \cos^2\theta) + 2b\cos\theta$$

if the couplings were the same, b would be zero and the angular distribution would have the same form as in QED

Weak mixing angle

- At LEP this could be measured by measuring the forward-backward asymmetry of leptons produced

$$A^{\ell}_{\mathrm{FB}} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

where the forward and backward cross sections can be obtained integrating in different hemispheres:

$$\sigma_F \equiv 2\pi \int_0^1 \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\mathrm{d}(\cos\theta) \qquad \sigma_B \equiv 2\pi \int_{-1}^0 \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\mathrm{d}(\cos\theta)$$

$$\sigma_F \propto \int_0^1 \left[a(1+\cos^2\theta) + 2b\cos\theta\right]\mathrm{d}(\cos\theta) = \int_0^1 \left[a(1+x^2) + 2bx\right]\mathrm{d}x = \left(\tfrac{4}{3}a + b\right)$$

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$$a = \left[(c_L^{\mathrm{e}})^2 + (c_R^{\mathrm{e}})^2\right]\left[(c_L^{\mu})^2 + (c_R^{\mu})^2\right] \qquad b = \left[(c_L^{\mathrm{e}})^2 - (c_R^{\mathrm{e}})^2\right]\left[(c_L^{\mu})^2 - (c_R^{\mu})^2\right]$$

$$A_{\mathrm{FB}} = \frac{3}{4}\left[\frac{(c_L^{\mathrm{e}})^2 - (c_R^{\mathrm{e}})^2}{(c_L^{\mathrm{e}})^2 + (c_R^{\mathrm{e}})^2}\right] \cdot \left[\frac{(c_L^{\mu})^2 - (c_R^{\mu})^2}{(c_L^{\mu})^2 + (c_R^{\mu})^2}\right]$$

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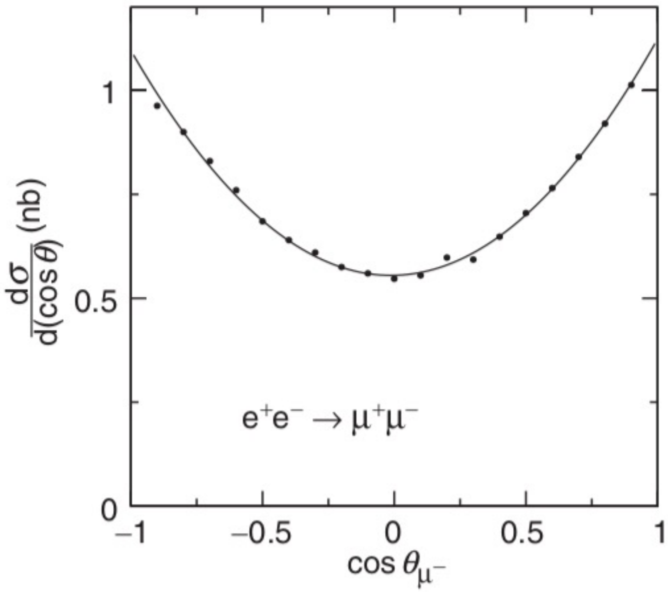
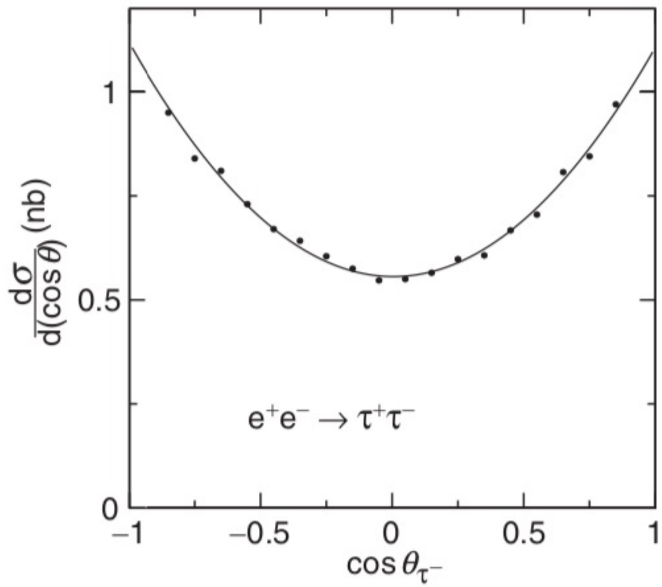
- In terms of the coupling constants: $A_{\mathrm{FB}} = \frac{3}{4}\mathcal{A}_{\mathrm{f}}\mathcal{A}_{\mathrm{f'}}$

$$\mathcal{A}_{\mathrm{f}} = \frac{2c_V^{\mathrm{f}}c_A^{\mathrm{f}}}{(c_V^{\mathrm{f}})^2 + (c_A^{\mathrm{f}})^2}$$

Weak mixing angle

- At LEP the cleanest way of measuring A_{FB} is in lepton final states





$$A^{e}_{FB} = 0.0145 \pm 0.0025 \qquad A^{\mu}_{FB} = 0.0169 \pm 0.0013 \qquad A^{\tau}_{FB} = 0.0188 \pm 0.0017$$

$$A^{e}_{FB} = \frac{3}{4}\mathcal{A}_e^2 \qquad A^{\mu}_{FB} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_\mu \qquad A^{\tau}_{FB} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_\tau$$

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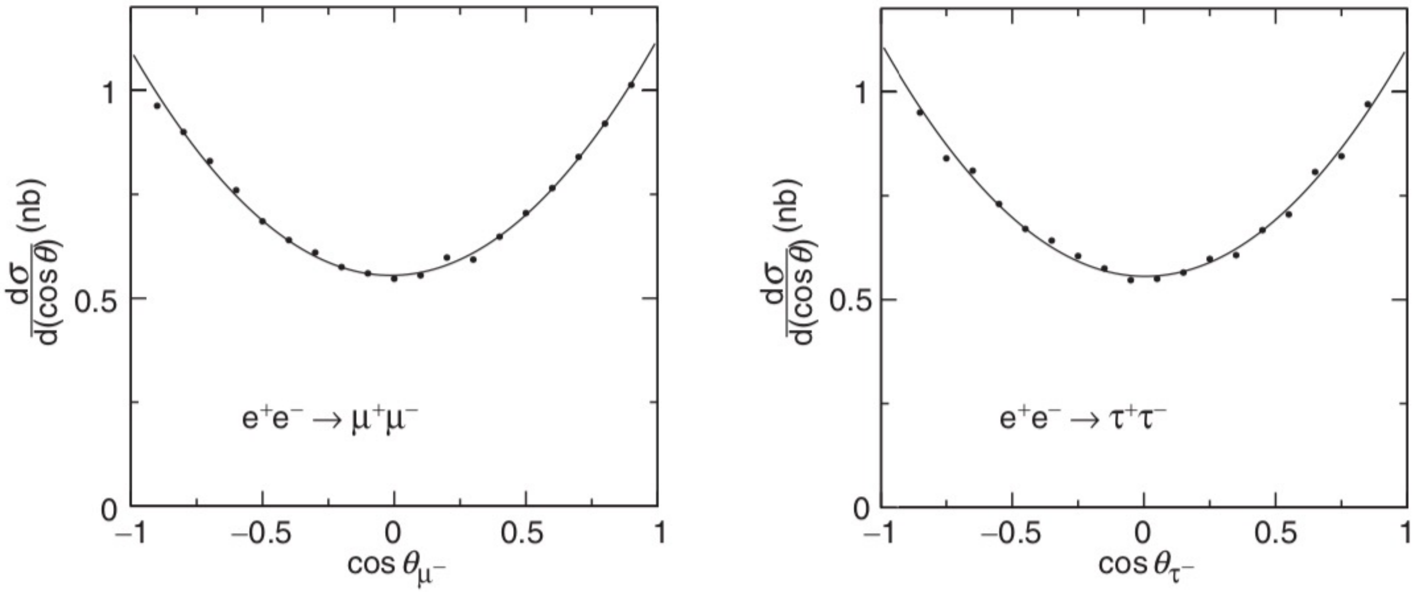

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$$\mathcal{A}_{e} = 0.1514 \pm 0.0019 \qquad \mathcal{A}_{\mu} = 0.1456 \pm 0.0091 \qquad \mathcal{A}_{\tau} = 0.1449 \pm 0.0040$$

Weak mixing angle

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$\mathcal{A}_{e} = 0.1514 \pm 0.0019$ $\mathcal{A}_{\mu} = 0.1456 \pm 0.0091$ $\mathcal{A}_{\tau} = 0.1449 \pm 0.0040$

$$\mathcal{A} = \frac{2c_V/c_A}{1 + (c_V/c_A)^2}$$

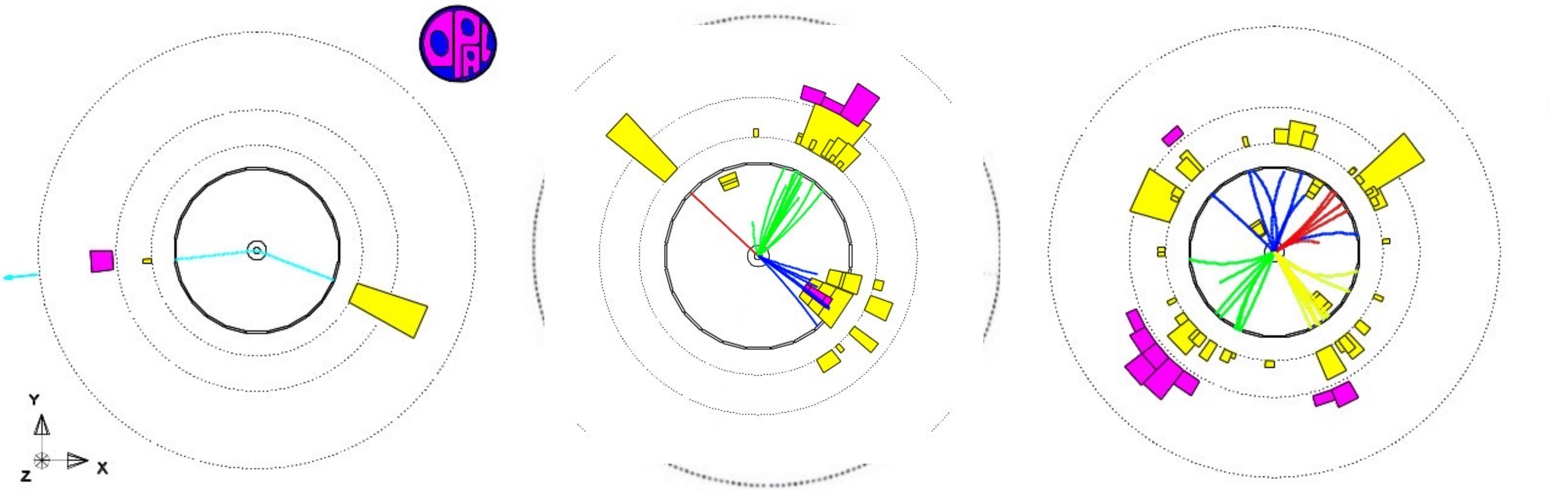
$$\frac{c_V}{c_A} = 1 - 4\sin^2\theta_{\mathrm{W}}$$

- Combining all measurements:

$$\sin^2\theta_{\mathrm{W}} = 0.23146 \pm 0.00012$$

W bosons at LEP

- W bosons were produced in pairs at LEP



$W^+W^- \to e^-\bar{\nu}_e\mu^+\nu_\mu$ | $W^+W^- \to e^-\bar{\nu}_e q_1\bar{q}_2$ | $W^+W^- \to q_1\bar{q}_2 q_3\bar{q}_4$

leptonic | semi-leptonic | hadronic

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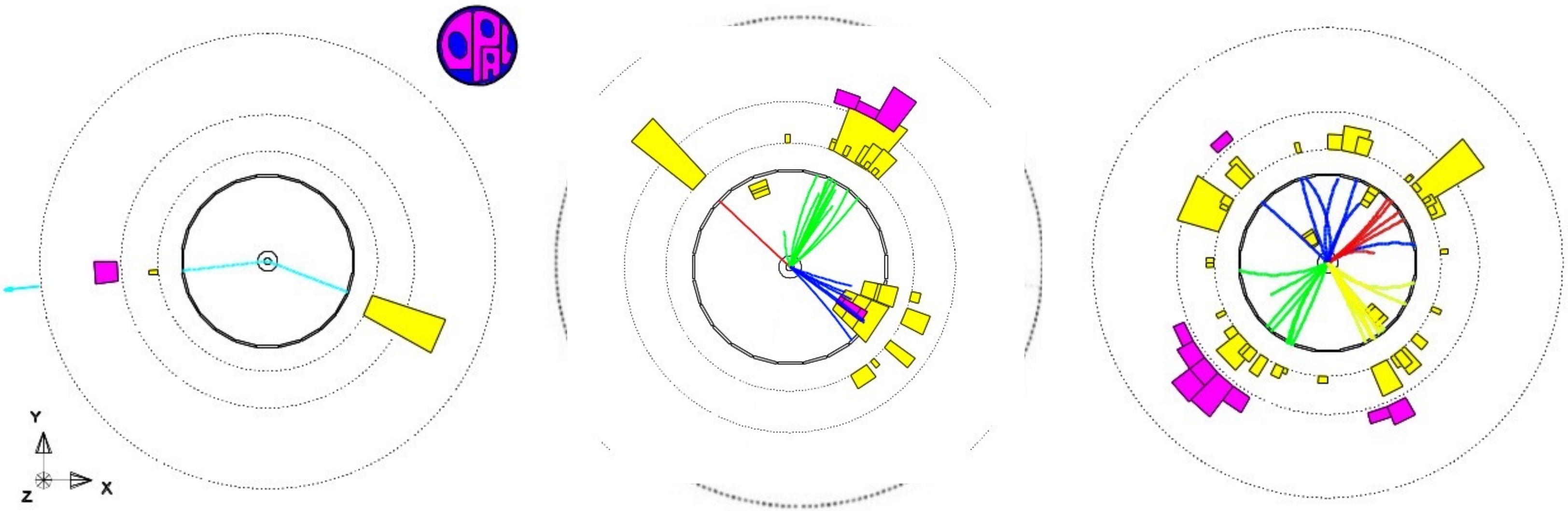

$W^+W^- \to e^-\bar{\nu}_e\mu^+\nu_\mu$ $\quad$ $W^+W^- \to e^-\bar{\nu}_e q_1\bar{q}_2$ $\quad$ $W^+W^- \to q_1\bar{q}_2 q_3\bar{q}_4$

leptonic $\quad$ semi-leptonic $\quad$ hadronic

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- From the EW Model:

$$\boxed{\Gamma(\mathrm{W}^- \to \mathrm{e}^-\bar{\nu}_\mathrm{e}) = \frac{g_\mathrm{W}^2 m_\mathrm{W}}{48\pi}}$$

$$\Gamma(\mathrm{W}^- \to \mathrm{e}^-\bar{\nu}_\mathrm{e}) = \Gamma(\mathrm{W}^- \to \mu^-\bar{\nu}_\mu) = \Gamma(\mathrm{W}^- \to \tau^-\bar{\nu}_\tau)$$

$$\Gamma(\mathrm{W}^- \to \mathrm{d\bar{u}}) = 3|V_{\mathrm{ud}}|^2\,\Gamma_{\mathrm{e}\nu} \qquad \Gamma(\mathrm{W}^- \to \mathrm{d\bar{c}}) = 3|V_{\mathrm{cd}}|^2\,\Gamma_{\mathrm{e}\nu}$$

$$\Gamma(\mathrm{W}^- \to \mathrm{s\bar{u}}) = 3|V_{\mathrm{us}}|^2\,\Gamma_{\mathrm{e}\nu} \qquad \Gamma(\mathrm{W}^- \to \mathrm{s\bar{c}}) = 3|V_{\mathrm{cs}}|^2\,\Gamma_{\mathrm{e}\nu}$$

$$\Gamma(\mathrm{W}^- \to \mathrm{b\bar{u}}) = 3|V_{\mathrm{ub}}|^2\,\Gamma_{\mathrm{e}\nu} \qquad \Gamma(\mathrm{W}^- \to \mathrm{b\bar{c}}) = 3|V_{\mathrm{cb}}|^2\,\Gamma_{\mathrm{e}\nu}$$

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$$N_{\mathrm{qqqq}} \propto \left[BR(\mathrm{W} \to \mathrm{q\overline{q}'})\right]^2 \qquad N_{\ell\nu\ell\nu} \propto \left[1 - BR(\mathrm{W} \to \mathrm{q\overline{q}'})\right]^2$$

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- From the EW Model:

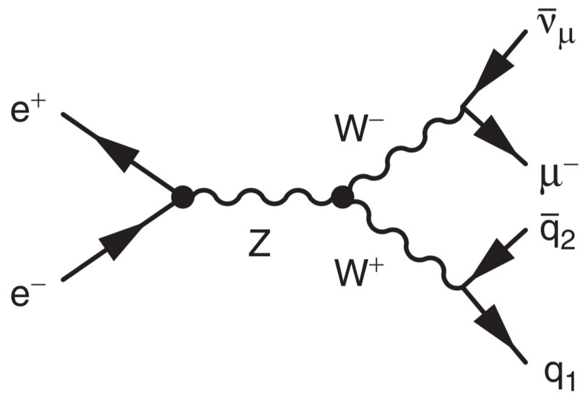
$$\Gamma(\mathrm{W}^- \to \mathrm{q\overline{q}'}) = 6\,\Gamma(\mathrm{W}^- \to \mathrm{e}^-\overline{\nu}_\mathrm{e})$$

$$\kappa_{QCD} = \left[1 + \frac{\alpha_S(m_\mathrm{W})}{\pi}\right] \approx 1.038$$

$$\Gamma_\mathrm{W} = (3 + 6\,\kappa_{QCD})\,\Gamma(\mathrm{W}^- \to \mathrm{e}^-\overline{\nu}_\mathrm{e}) \approx 9.2 \times \frac{g_\mathrm{W}^2 m_\mathrm{W}}{48\pi} = 2.1\,\mathrm{GeV}$$

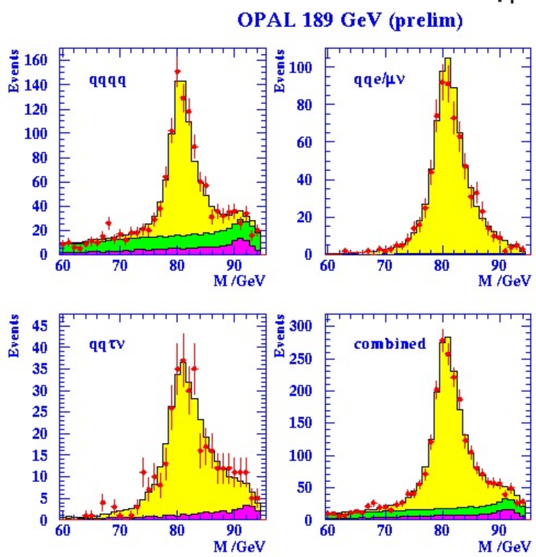
$$BR(\mathrm{W} \to \mathrm{q\overline{q}'}) = \frac{6\,\kappa_{QCD}}{3 + 6\,\kappa_{QCD}} = 67.5\%$$

- The W-pair production at LEP is not a resonant process, like the Z boson production
- The mass and width can be obtained through direct reconstruction of the invariant masses of the W decays



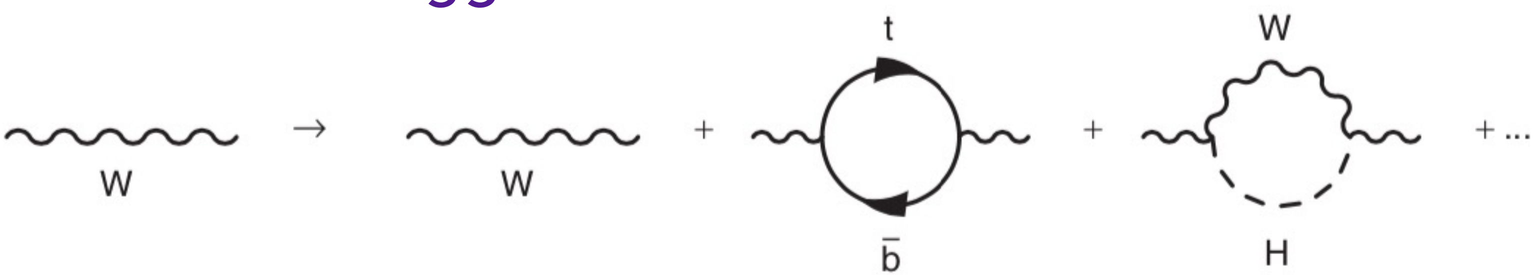
$$m_W = 80.376 \pm 0.033\,\text{GeV}$$

$$\Gamma_W = 2.195 \pm 0.083\,\text{GeV}$$



W mass loop corrections

- When comparing to the precise measurements from LEP, higher order corrections must be taken into account
- For example, the W mass has corrections related to the top quark and the Higgs boson



$$m_{\mathrm{W}} = m_{\mathrm{W}}^{0} + a\, m_{\mathrm{t}}^{2} + b \ln\left(\frac{m_{\mathrm{H}}}{m_{\mathrm{W}}}\right) + \cdots$$

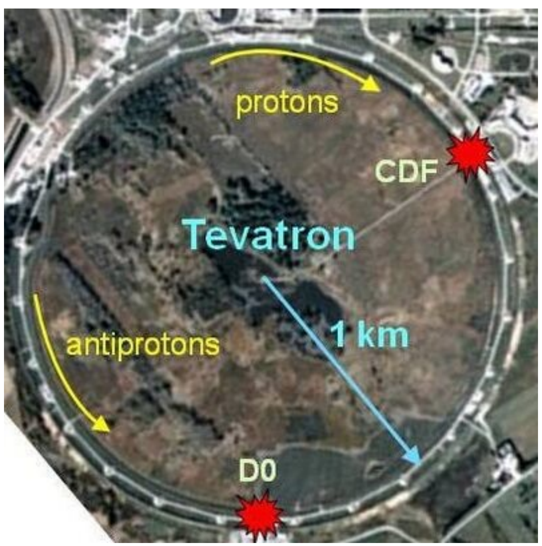
- The measurements from LEP for the EW parameters, together with the quantum loop effects, predict a top quark mass of 175 ± 11 GeV…

The Top quark

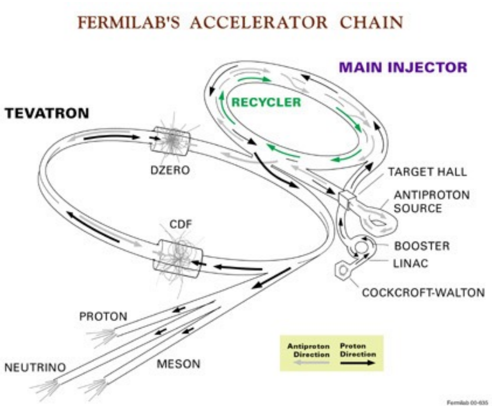
- The top quark is the "heaviest" fundamental particle we know
- It could not be observed at LEP, it was discovered at the Tevatron in 1994

- It is possible to achieve higher centre of mass energies with hadron colliders than with e^+e^- colliders
- They are central in the production of new heavy particles
- Underlying process: parton-parton scattering

TeVatron (1987-2010)



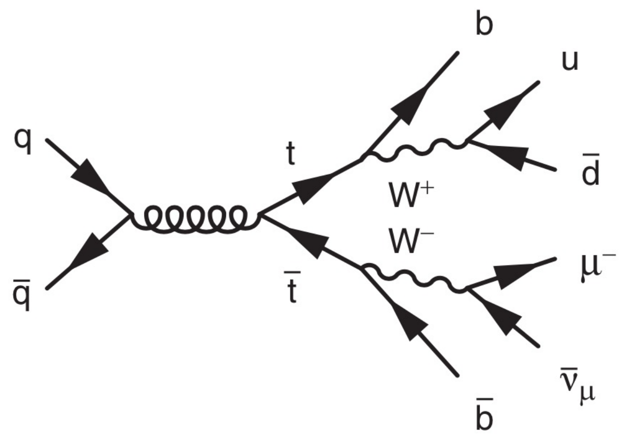
- Located at Fermilab, Chicago, USA
- $p\bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV
- Two main experiments: CDF and D0



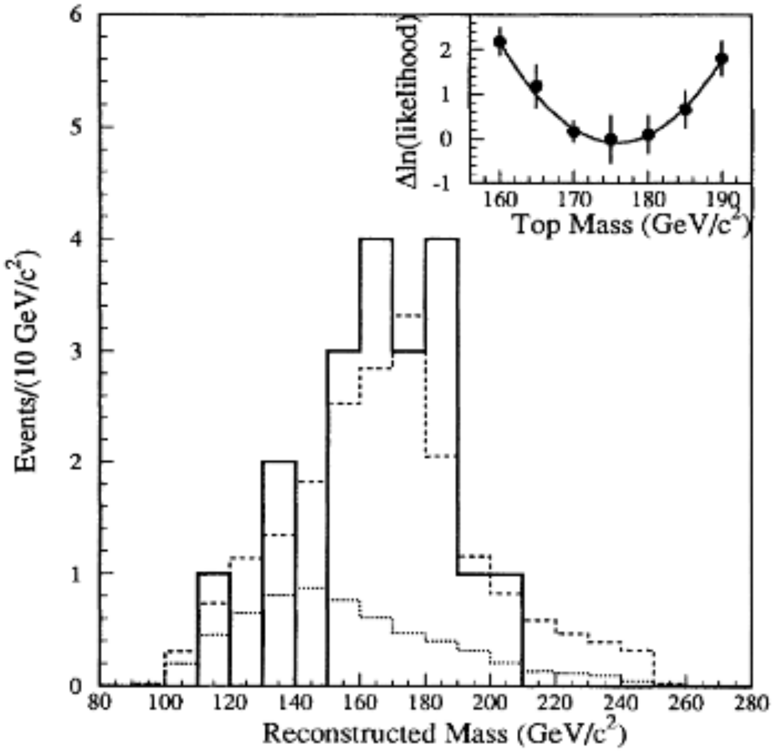
The Top quark

- The top quark is the "heaviest" fundamental particle we know
- It could not be observed at LEP, it was discovered at the Tevatron in 1994
- It has a short lifetime and decays before hadronisation
- It decays almost exclusively into a W boson and a *b* quark
- At hadron colliders it is easier to look for the semi-leptonic channel:

$$q\bar{q} \rightarrow t\bar{t} \rightarrow bW^{+}\,\bar{b}W^{-}$$



- At hadron colliders it is easier to look for the semi-leptonic channel
- First observation of top quark (CDF)



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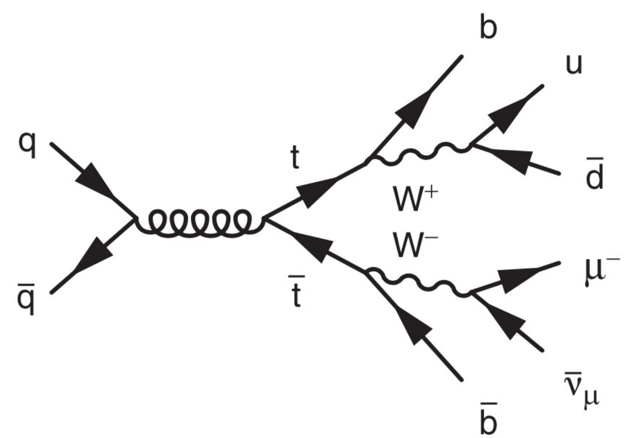
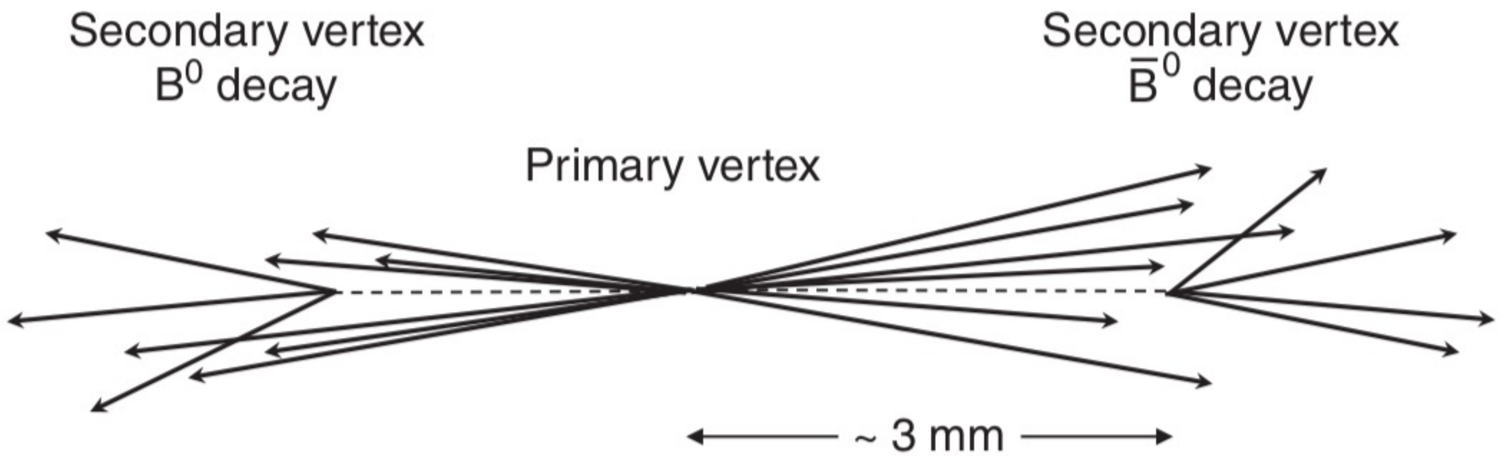


The Top quark

- To measure the top quark mass we reconstruct the invariant mass of its decay products (as with the W)
- Need to identify the b-jet: b-tagging
- b quarks have a longer lifetime than the other quarks
- The b quark travels some distance from the interaction point before decaying: secondary vertex



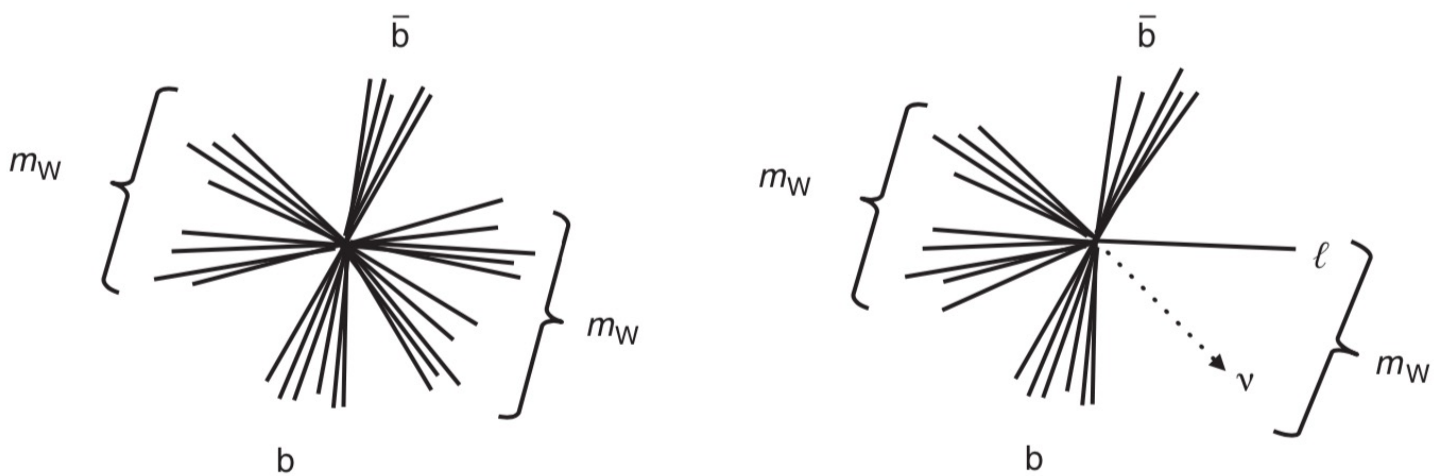
The Top quark

- To measure the top quark mass we reconstruct the invariant mass of its decay products (as with the W)

$$t\bar{t} \to (bW^+)(\bar{b}W^-) \to (b\,q_1\bar{q}_2)\,(\bar{b}\,q_3\bar{q}_4) \to 6 \text{ jets},$$

$$t\bar{t} \to (bW^+)(\bar{b}W^-) \to (b\,q_1\bar{q}_2)\,(\bar{b}\,\ell^-\bar{\nu}_\ell) \to 4 \text{ jets} + 1, \text{ charged lepton} + 1\nu$$

$$t\bar{t} \to (bW^+)(\bar{b}W^-) \to (b\,\ell^+\nu_\ell)\,(\bar{b}\,\ell'^-\bar{\nu}_{\ell'}) \to 2 \text{ jets} + 2 \text{ charged leptons} + 2\nu\text{s}$$



Tevatron average result: $m_t = 173.5 \pm 1.0\,\mathrm{GeV}$

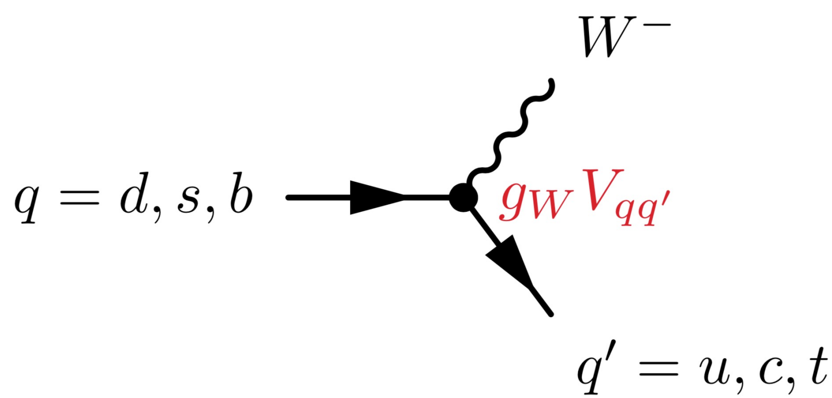
Summary

- The weak current has a V-A structure
- W boson coupling to leptons constant (lepton universality)

$$\frac{-ig_{\mathrm{W}}}{\sqrt{2}}\frac{1}{2}\gamma^{\mu}(1-\gamma^{5})$$

$$\alpha_W = \frac{g_{\mathrm{W}}^2}{4\pi} = \frac{8m_{\mathrm{W}}^2 G_{\mathrm{F}}}{4\sqrt{2}\pi} \approx \frac{1}{30}$$

$$G_{\mathrm{F}} = 1.166\,38 \times 10^{-5}\,\mathrm{GeV}^{-2}$$



- W boson coupling to quarks depends on CKM matrix
- The W boson always changes quark flavour
- Mixing between different families can occur but less likely
- Parity and Charge conjugation is violated by the charged weak current: only couples to LH particles (RH anti-particles)

Summary

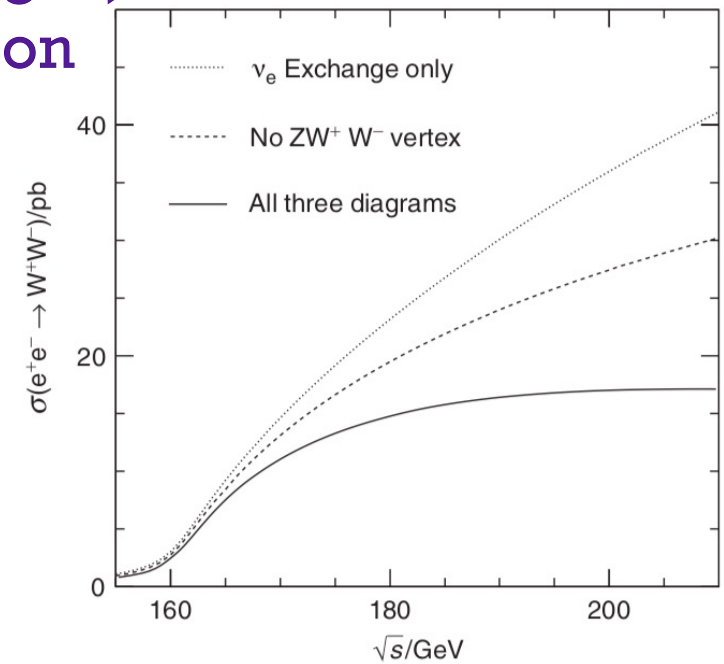
- The charged weak interaction alone does not fully explain W-pair production (cross section diverges)
- Neutral gauge boson needed: Z boson
- Unified $SU(2)_L \times U(1)_Y$ gauge theory
- One new parameter relates all couplings:

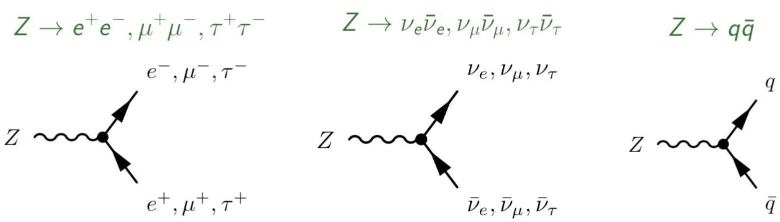
$$\sin^2\theta_W = 0.23146 \pm 0.00012$$

$$\frac{\alpha}{\alpha_W} = \frac{e^2}{g_W^2} = \sin^2\theta_W \sim 0.23$$

$$g_Z = \frac{g_W}{\cos\theta_W} \equiv \frac{e}{\sin\theta_W \cos\theta_W}$$

$$-i\tfrac{1}{2}g_Z\gamma^\mu\left[c_V - c_A\gamma^5\right]$$





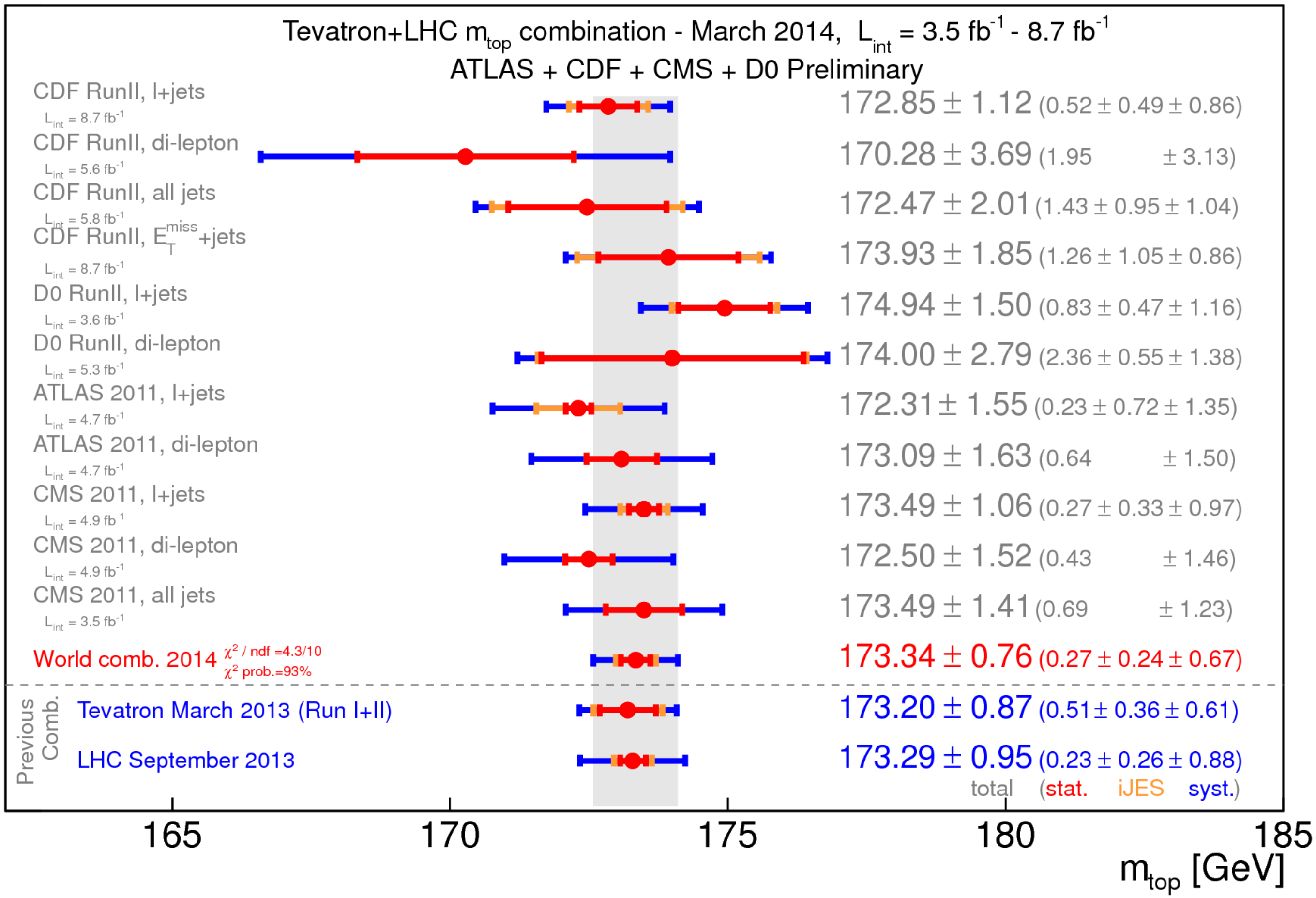
Summary

- LEP made some impressive precise tests of the EW Model:

$$m_Z = 91.1875 \pm 0.0021\,\text{GeV}$$
$$m_W = 80.385 \pm 0.015\,\text{GeV}$$
$$\alpha(m_Z^2) = \frac{1}{128.91 \pm 0.02}$$
$$G_F = 1.166\,378\,7(6) \times 10^{-5}\,\text{GeV}^{-2}$$
$$\sin^2\theta_W = 0.23146 \pm 0.00012$$
$$N_\nu = 2.9840 \pm 0.0082$$

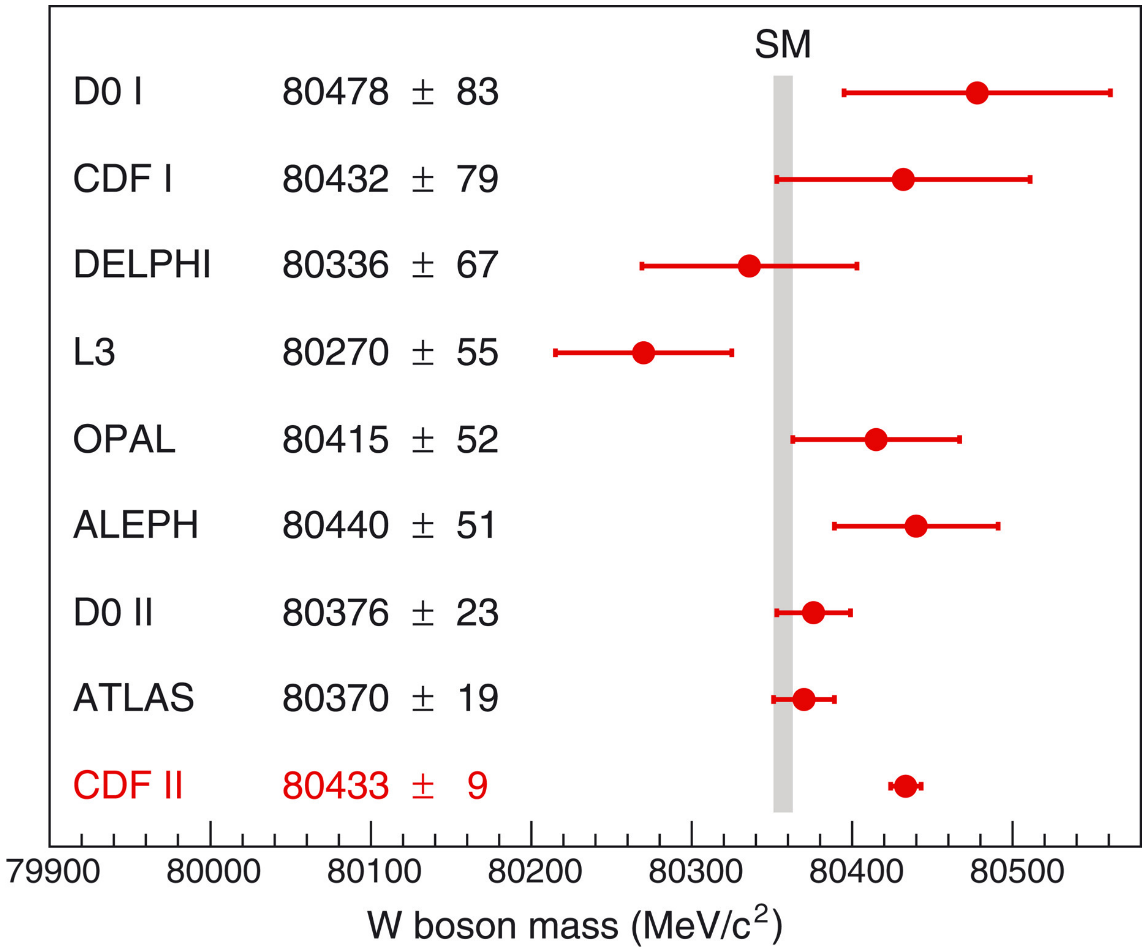
- These measurements are consistent with the relations between constants established in the EW model
- The model works! (we are missing a way of giving mass to the W and Z)
- The top quark discovered at the Tevatron completes the spectrum of quarks: $m_t = 173.5 \pm 1.0\,\text{GeV}$
- This value is as predicted by the quantum loop corrections of the EW model – impressive!

- Top mass world combination



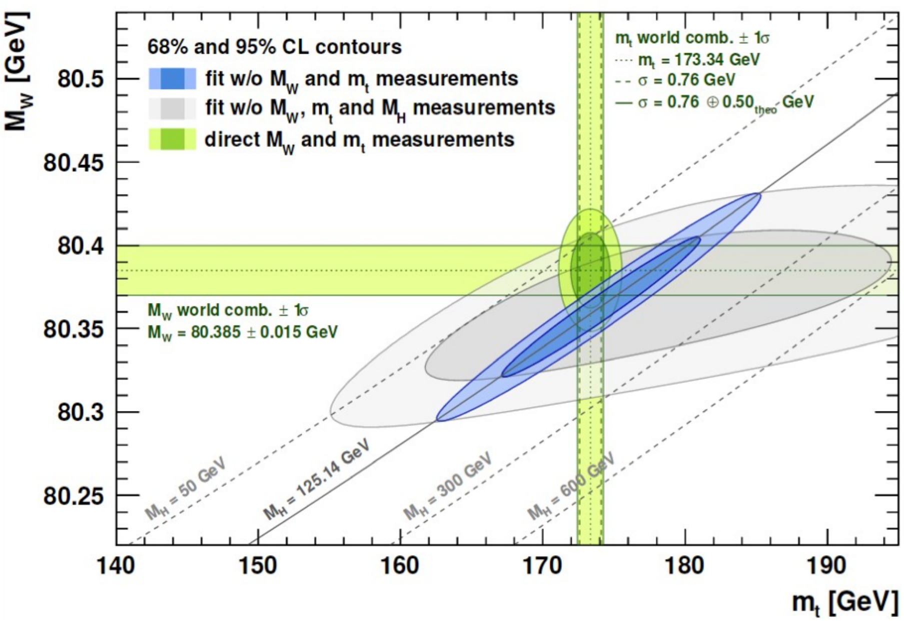
Summary

- W boson mass

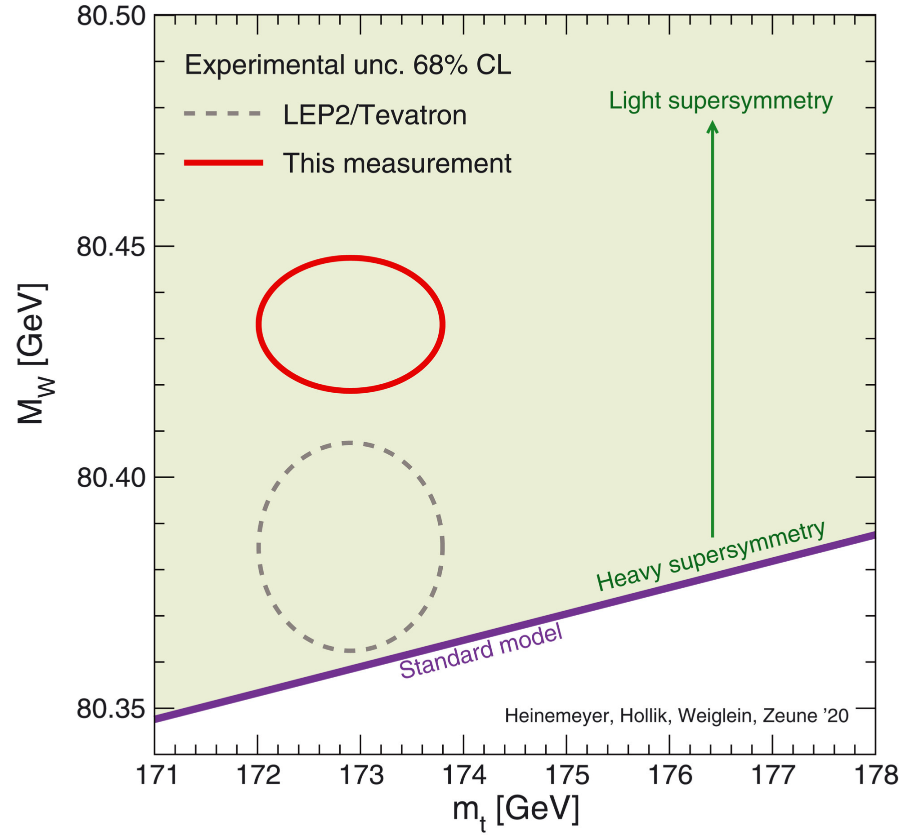


Summary

- Top mass vs. W boson mass



* arXiv:1407.3792



* https://doi.org/10.1126/science.abk1781