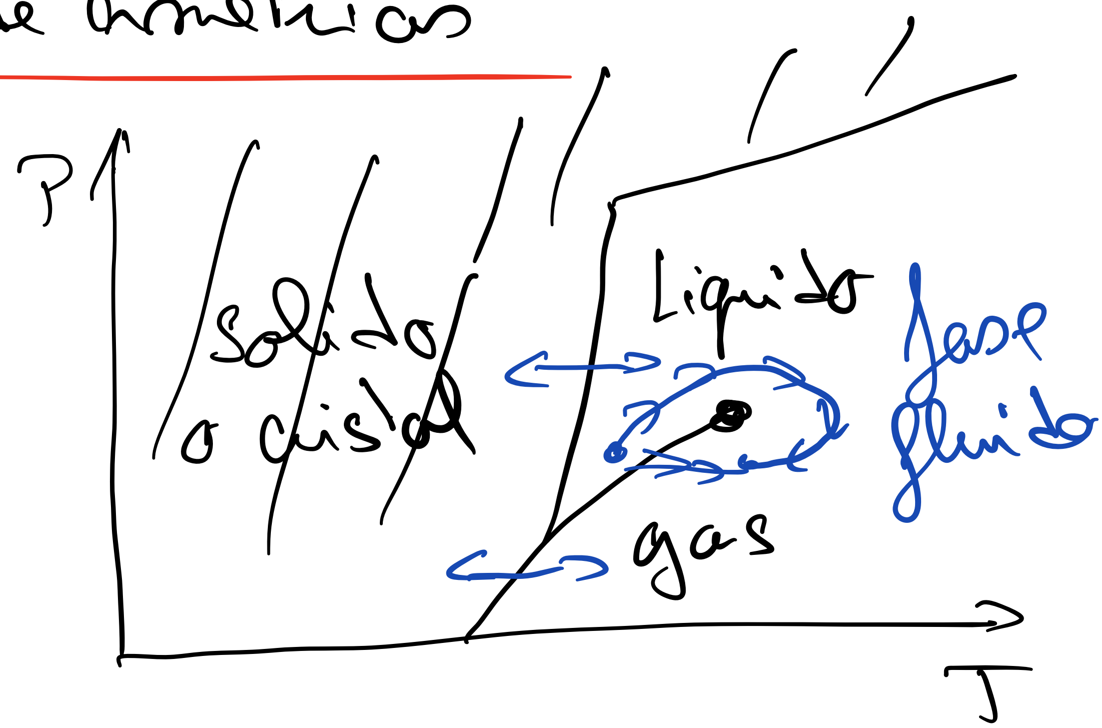
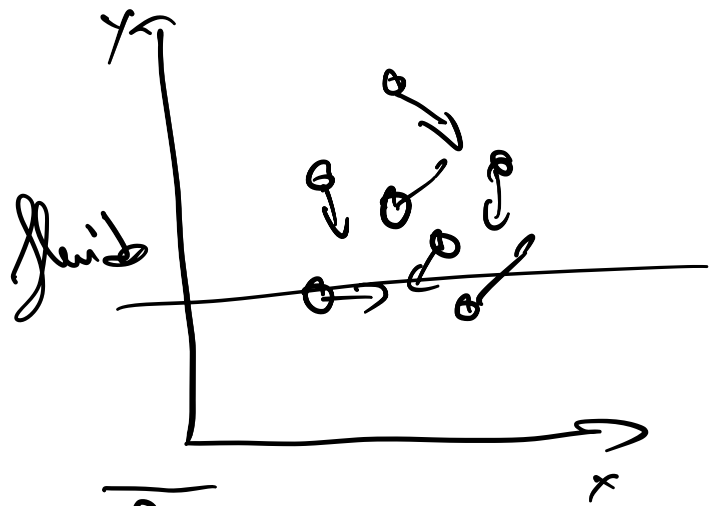
I Introducción a las Transiciones de fases y a la criticalidad

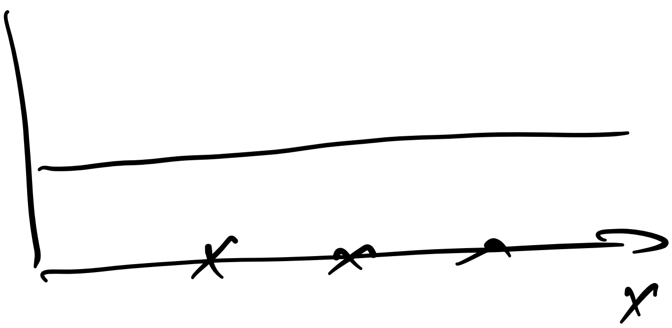
1) Rupturas espontáneas de simetrías



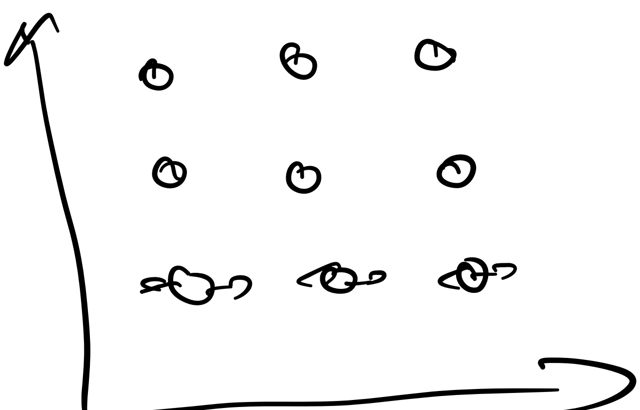
fluido

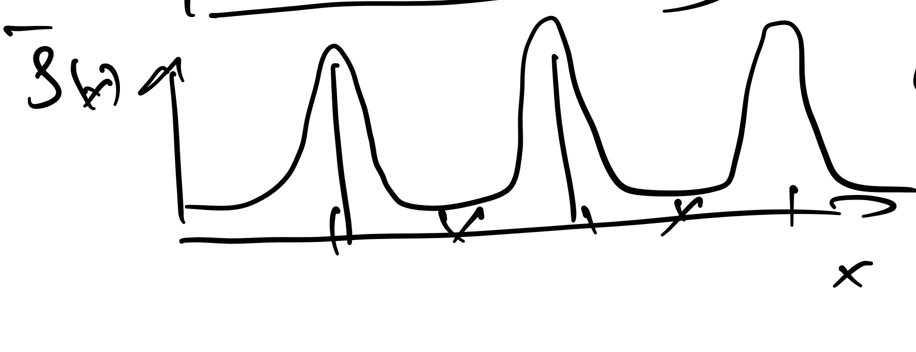


$\bar{\rho}(x)$



Cristal





→ ruptura espontánea de la simetría de traslación:

$$H = \sum_{i=1}^{N} \frac{\vec{p}_i^{\,2}}{2m} + \frac{1}{2} \sum_{i \neq j} V(\vec{r}_i - \vec{r}_j)$$

→ simetría de traslación

ej: $\vec{r}_i \to \vec{r}_i + \vec{b}$

→ clasificación de los estados de la materia

→ Noción de grupos de simetría

2) El modelo de Ising

$\sigma_i = \pm 1$

ejemplo 1 magnetismo de espines 1/2

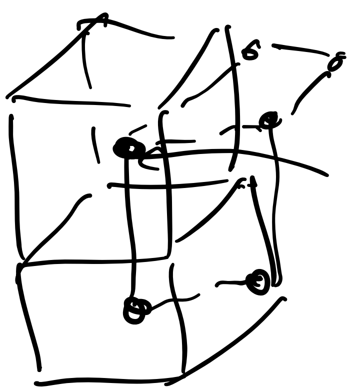
por razones energéticas,

$$\sigma_i = \frac{2}{\hbar} S^z_i = \pm 1$$

ejemplo 2 mezclas binarias

Zn + Cu

Body centred Cubic



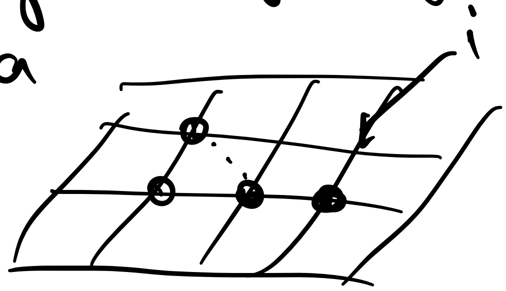
Zn $\rightarrow \sigma_i = +1$

Cu $\rightarrow \sigma_i = -1$

$$\boxed{H = -\sum_{i\neq j}{}' J_{ij}\, \sigma_i \sigma_j}$$

obs $\sigma_i^2 = 1$

el acople entre los sitios "i" y "j", en general → red regular, por ejemplo la red cuadrada



$J_{ij} \neq 0$ si "i" y "j" son primeros vecinos

Simetría

$$\left\{\begin{array}{l} \forall i,\ \sigma_i \to 1\,\sigma_i \\ \forall i,\ \boxed{\sigma_i \to -\sigma_i} \end{array}\right.$$

→ grupo $\mathbb{Z}_2 \leftrightarrow \{1, -1\}$

el caso de tres vecinos.

con $J_{ij} = \begin{cases} J & \text{si } i \text{ y } j \text{ son los vecinos} \\ 0 & \text{sino} \end{cases}$

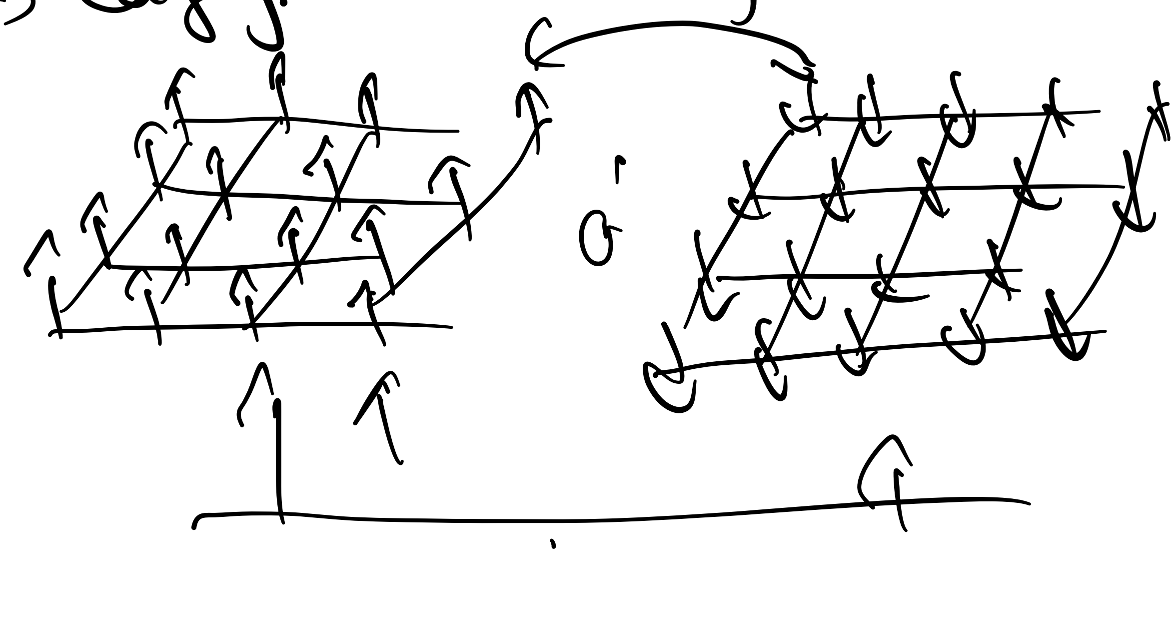
$$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j \quad , \quad J > 0$$

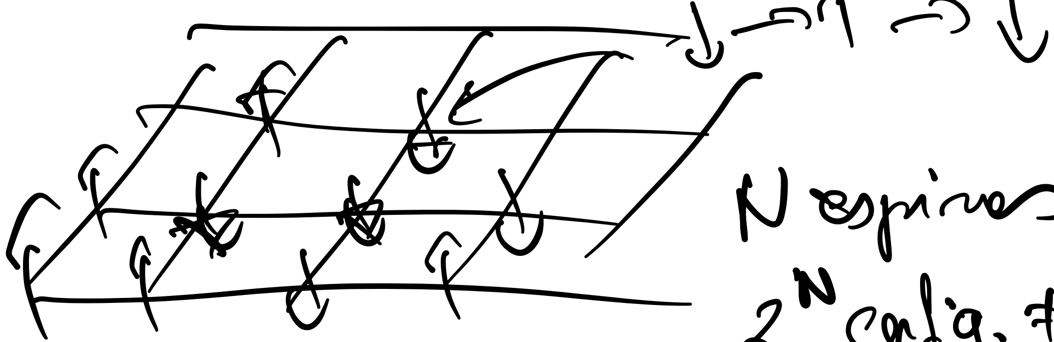
Configuración con más baja energía.

$\sigma_i = +1 \rightarrow \uparrow$

$\sigma_i = -1 \rightarrow \downarrow$

$\rightarrow$ config. de más baja energía





N espines

2^N config. #.

Función de partición:

$$Z(h) = \sum_{\{\sigma_i\}} e^{-\beta H(h)} \qquad \beta = \frac{1}{k_B T}$$

$$H(h) = -\sum_{i,j} J_{ij}\,\sigma_i \sigma_j - h \sum_i \sigma_i$$

rompe la simetría $\mathbb{Z}_2$

Valores medios

$$m_i = \langle \sigma_i \rangle = \frac{1}{Z}\left[\sum_{\{\sigma_j\}} e^{-\beta H} \sigma_i\right]$$

por ejemplo $\langle \sigma_i \sigma_j \rangle$

$$= \frac{1}{Z}\left[\sum_{\{\sigma\}} e^{-\beta H} \sigma_i \sigma_j\right]$$

Magnetización total

$$M = \sum_i m_i = \frac{1}{Z}\left[\sum_{\{\sigma\}} e^{-\beta H} \sum_i \sigma_i\right]$$

$$\sum_i \langle \sigma_i \rangle = \langle \sum_i \sigma_i \rangle$$

<u>obs</u> (ej)

$$\boxed{M = \frac{1}{\beta} \frac{\partial}{\partial h} \ln Z(h)}$$

energia libre: $Z = e^{-\beta F}$

$$F = -\frac{1}{\beta}\ln Z$$

$$\boxed{M = -\frac{\partial}{\partial h} F}$$

Susceptibilidad

$$\chi(h) = \frac{\partial}{\partial h} M(h) = \frac{1}{\beta}\frac{\partial^2}{\partial h^2}\ln Z$$

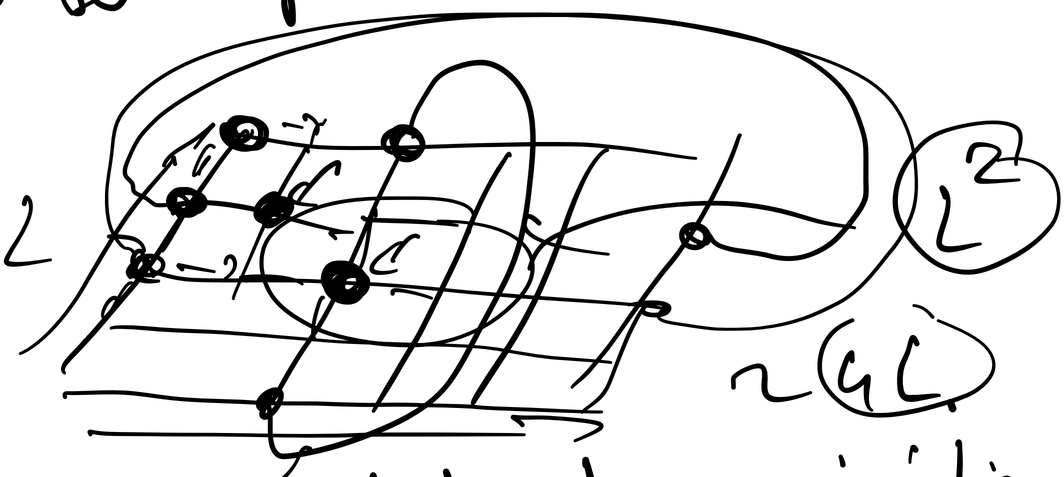
$$= -\frac{\partial^2}{\partial h^2} F$$

ej.

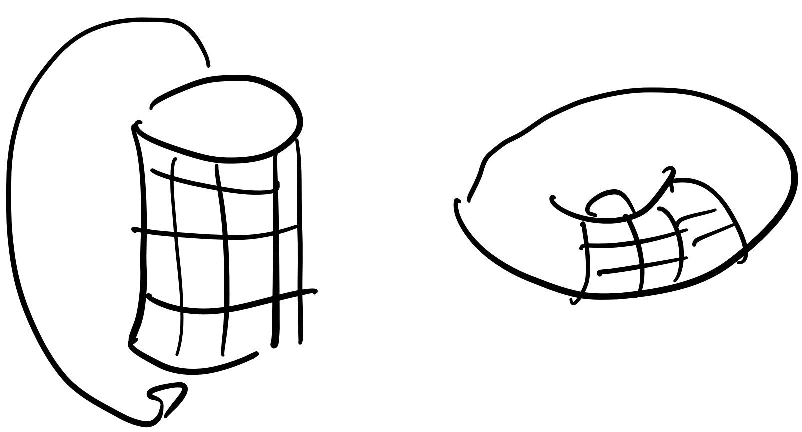
$$\boxed{= \frac{\beta}{Z(h)}\left[\sum_{\{\sigma\}} e^{-\beta H(h)} \sum_{i,j} \sigma_i \sigma_j\right] - \beta\left[\frac{1}{Z(h)}\sum_{\{\sigma\}} e^{-\beta H(h)} \sum_i \sigma_i\right]^2}$$

$$\boxed{\chi(h) = \beta \sum_{\forall i,j} \left[\langle \sigma_i \sigma_j \rangle - \langle \sigma_i \rangle \langle \sigma_j \rangle \right]}$$

El caso en el que todos los sitios son equivalentes



→ Condiciones de borde, periódicas



si todos los sitios son equ.

$$\langle \sigma_i \rangle = m_i = m$$

$$M = N m$$
$\uparrow$ # de espines

$$\boxed{\chi = \beta \sum_{ij} \langle \sigma_i \sigma_j \rangle - \beta N^2 m^2}$$

$$E = \frac{1}{Z(h)} \sum_{\{\sigma\}} e^{-\beta H(h)} H(h)$$

$$= -\frac{\partial}{\partial \beta} \ln Z(h)$$

Calor específico:

$$C = \frac{\partial E}{\partial T} = -k_B \beta^2 \frac{\partial E}{\partial \beta}$$

$$= k_B \beta^2 \frac{\partial^2}{\partial \beta^2} \ln Z$$

Funciones de correlación: (conexas)

$$\langle \sigma_i \sigma_j \rangle_c \underset{\substack{\uparrow \\ \text{def}}}{=} \underline{\langle \sigma_i \sigma_j \rangle - \langle \sigma_i \rangle \langle \sigma_j \rangle}$$

obs $\underline{\chi = \beta \sum_{ij} \langle \sigma_i \sigma_j \rangle_c}$

$$\boxed{\langle \sigma_i \sigma_j \rangle_c \underset{d_{ij} \to \infty}{\sim} e^{-\frac{d_{ij}}{\xi}} \leftarrow}$$

d_{ij} = distancia entre "i" y "j"

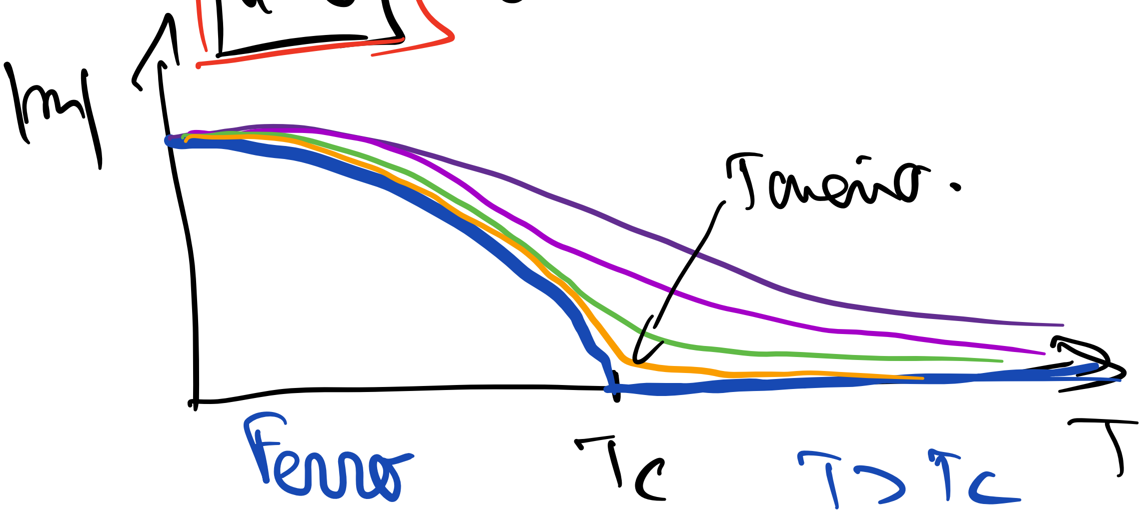
ξ longitud de correlación

Comportamiento con T:

d = dimensión espacial.

$\boxed{d \geq 2}$ (no para $d=1$)

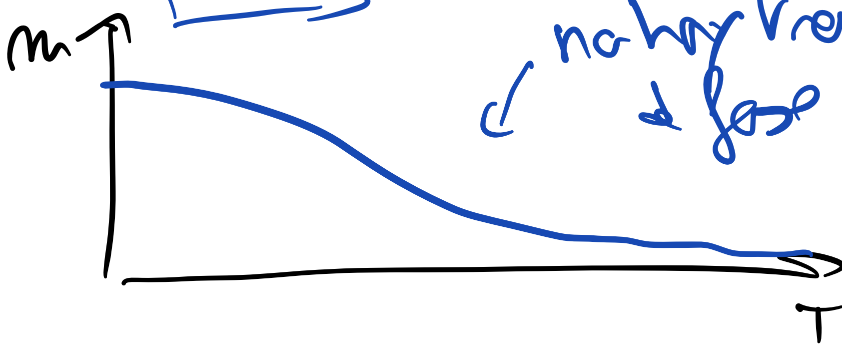
$\boxed{h=0}$ en el límite $N \to \infty$



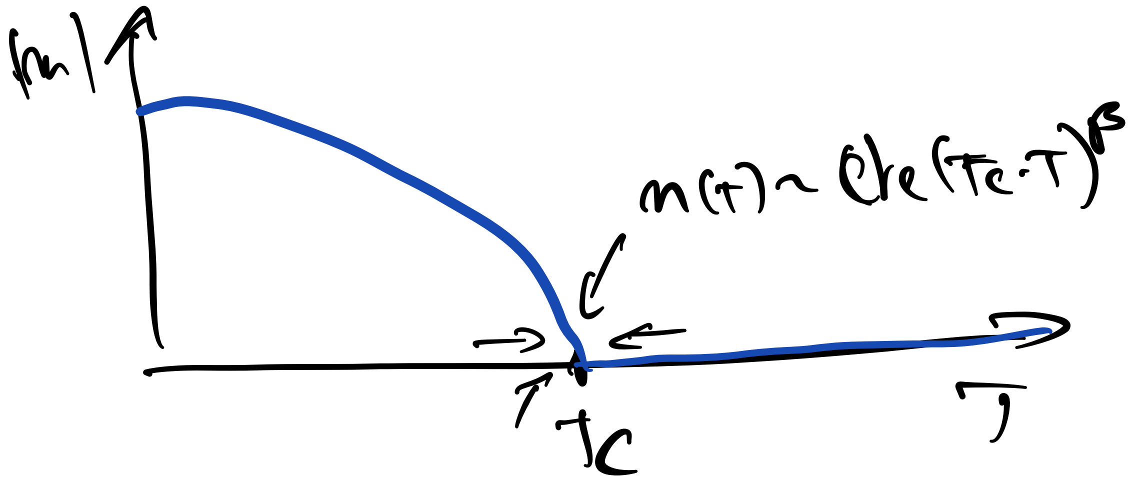
Ferro — sim. $\mathbb{Z}_2$ rota $T<T_c$

$T>T_c$ — fase PARA — sim $\mathbb{Z}_2$ no está rota.

obs si $\boxed{h \neq 0}$

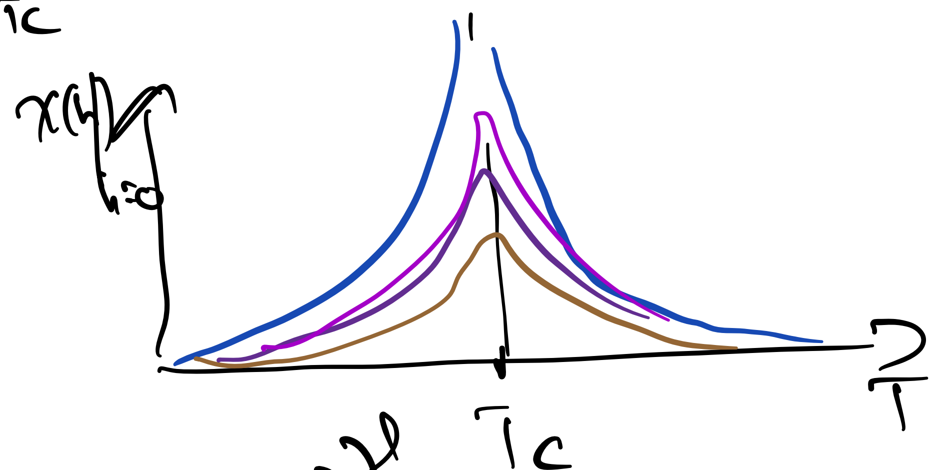


c/ no hay transición de fase.



$T \sim T_c \qquad T < T_c$

* $|m| \sim (T_c - T)^{\beta}$
* $\xi \to \infty$ At $T \to T_c$, $\xi \sim |T - T_c|^{-\nu}$ $\quad \nu > 0$
*



$$\frac{\chi(h)}{N}\Big|_{h=0} \sim |T - T_c|^{-\gamma}$$

$$\frac{C}{N} \sim |T-T_c|^{-\alpha}$$

si $T=T_c$ $(\xi \to \infty)$

$$\langle \sigma_i \sigma_j \rangle_c \underset{d_{ij}\to\infty}{\sim} \frac{1}{d_{ij}^{\,d-2+\eta}}$$

relacions de escalar: (scaling)

$$\boxed{\begin{aligned} \nu d &= 2-\alpha = 2\beta+\gamma \\ 2-\eta &= \frac{\gamma}{\nu} \end{aligned}}$$

$$\chi = \beta N \sum_i \langle \sigma_0 \sigma_i \rangle$$

$$\Rightarrow \frac{\chi}{N} \sim \int d\vec{r}\, \langle \sigma(0)\, \sigma(\vec{r}) \rangle_c$$

$T \neq T_c$ ξ es finito

$\langle \sigma(0)\, \sigma(\vec{r}) \rangle$

$$\frac{\chi}{N} \sim \int_{|\vec{r}|<\xi} \frac{1}{r^{d-2+\eta}}\, d^d\vec{r} \sim \xi^{2-\eta} = |T-T_c|^{-\nu(2-\eta)}$$

$$\frac{\chi}{N} \sim |T-T_c|^{-\gamma}$$