$$\langle \phi(\vec{r}_1)\,\phi(\vec{r}_2)\rangle = G(\vec{r}_1-\vec{r}_2)$$

$$G(\vec{r}) \simeq \tilde{c} \int d\vec{k}\, \frac{e^{i\vec{k}\cdot\vec{r}}}{\vec{k}^2+m^2}$$

$$\boxed{m^2 = \frac{a_2}{c}} \qquad m = \frac{1}{\xi}$$

1-D o 3-D

$\Rightarrow$ se calcula exactamente

(th. de residuos)

$G(\vec{r})$ se puede estimar $\forall$ D para

$|\vec{r}| \to \infty$

$$G(\vec{r}) = \tilde{c} \int d\vec{r}\, \frac{e^{i\vec{k}\cdot\vec{r}}}{\vec{k}^2+m^2}$$

$$= \tilde{c} \int d\vec{k} \int_0^\infty d\alpha\, e^{\vec{k}\cdot\vec{r} - \alpha(\vec{k}^2+m^2)}$$

Obs $\frac{1}{A} = \int_0^\infty d\alpha\, e^{-\alpha A}$

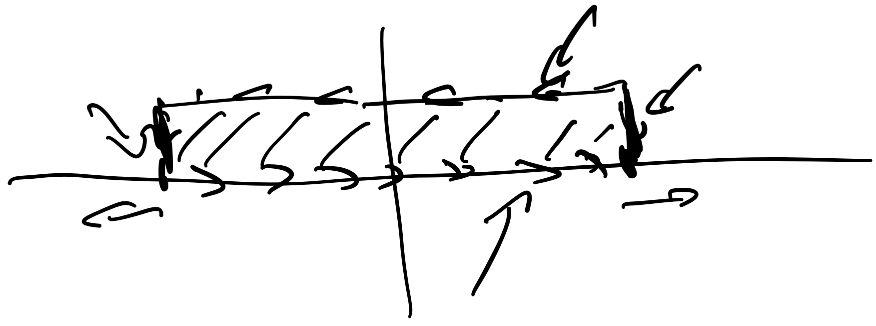
$A > 0$

la integral en $d\tilde{k}$ se puede hacer

$$\int_{-\infty}^{\infty} e^{ik_x x - \alpha k_x^2}\, dk_x = e^{-\frac{x^2}{4\alpha}} \int_{-\infty}^{\infty} e^{-\alpha\left(k_x - \frac{ix}{2\alpha}\right)^2} dk_x$$

$$= e^{-\frac{x^2}{4\alpha}} \sqrt{\frac{\pi}{\alpha}}$$

$$\int_{-\infty}^{\infty} e^{-\alpha\left(k_x - \frac{ix}{2\alpha}\right)^2} = \sqrt{\frac{\pi}{\alpha}}$$

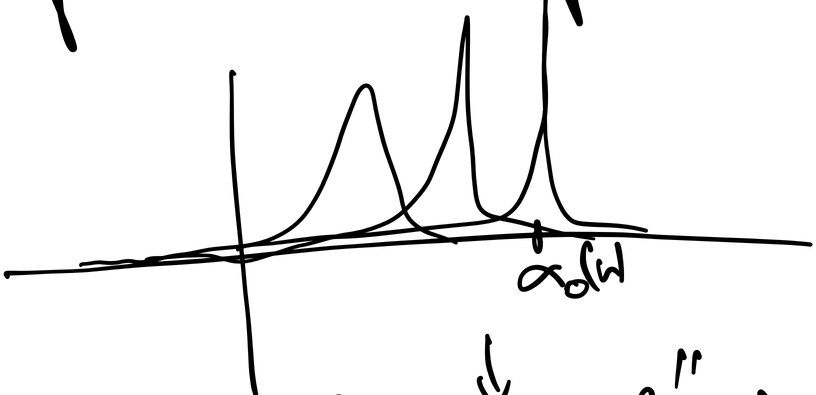


$$G(\bar{r}) = \tilde{c} \int_0^\infty \left(\frac{\pi}{\alpha}\right)^{d/2} e^{-\frac{r^2}{4\alpha} - m^2\alpha} \, d\alpha$$

$$G(\bar{r}) = \tilde{d} \int_0^\infty e^{-f(\alpha)} \, d\alpha$$

$$\boxed{f(\alpha) = \frac{r^2}{4\alpha} + \alpha m^2 + \frac{d}{2} \ln \alpha}$$

para calcular $G(r)$ en el límite $r \to \infty$, se usa el método de la aproximación del punto silla.

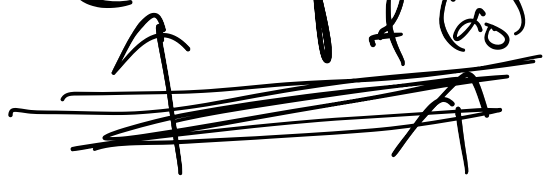


$$f(\alpha) = f(\alpha_0) + \frac{1}{2} f''(\alpha_0) (\alpha - \alpha_0)^2 + \dots$$

$$\int_0^\infty d\alpha\, e^{-f(\alpha)} \simeq \int_0^\infty e^{-f(\alpha_0) - \frac{1}{2} f''(\alpha_0)(\alpha-\alpha_0)^2 + \dots}\, d\alpha$$

$$= e^{-f(\alpha_0)} \int_0^\infty e^{-\frac{1}{2} f''(\alpha_0)(\alpha-\alpha_0)^2 + \dots}\, d\alpha$$

$$= e^{-f(\alpha_0)} \sqrt{\frac{2\pi}{f''(\alpha_0)}}\, (1 + \dots) \quad O\left(\frac{1}{n}\right)$$



$\hookrightarrow$ [illegible]

moeshna $\xrightarrow[n\to\infty]{} 0$

$$f'(\alpha) = \frac{-n^2}{4\alpha^2} + m^2 + \frac{d}{2\alpha}$$

$$\Rightarrow \alpha_0 = \frac{1}{2m^2}\left[-\frac{d}{2} + \sqrt{\frac{d^2}{4} + n^2 m^2}\right]$$

$$\xrightarrow[n\to\infty]{} \underline{\frac{n}{2m}}$$

$$f''(\alpha) = \frac{r^2}{2\alpha^3} - \frac{d}{2\alpha^2}$$

$$\begin{cases} f(\alpha_0) = m r + \frac{d}{2}\ln\left(\frac{r}{2m}\right) \\ f''(\alpha_0) = \frac{4m^3}{r} - \frac{4m^2 d}{r^2} \xrightarrow[r\to\infty]{} \frac{4m^3}{r} \end{cases}$$

$$\Rightarrow \boxed{G(\bar{r}) = \frac{c\,te}{r^{\frac{d-1}{2}}}\, \underbrace{e^{-mr}}_{e^{-\frac{r}{\xi}}}} \qquad m \leftrightarrow \frac{1}{\xi}$$

ξ: longitud de correlación.

Modelo gaussiano "crítico"

$a_2 = 0$ ($m = 0$)

$$G(\vec{r}) = c \int d\vec{k} \, \frac{e^{i\vec{k}\cdot\vec{r}}}{k^2} \propto$$ función de Green del operador Δ

$$G(\vec{r}) = \begin{cases} \tilde{c}\,|x| & d=1 \\ -\frac{\tilde{c}}{2\pi} \ln\left(\frac{|\vec{r}|}{r_0}\right) & d=2 \\ \frac{cte}{|\vec{r}|^{d-2}} & d \geq 3 \end{cases}$$

para el modelo

$$S = \int d\vec{r} \; c(\vec{\nabla}\phi)^2$$

→ invariante de escala:

$$\left| \begin{array}{l} \vec{r} \to \lambda \vec{r} \\ \vec{\nabla} \to \frac{1}{\lambda} \vec{\nabla} \\ c \to c \quad , \phi \to \lambda^{\frac{2-d}{2}} \phi \end{array} \right| \quad S \to S$$

No sirve si $a_2 \neq 0$