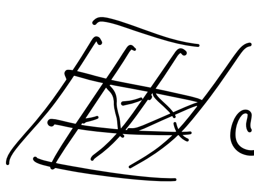
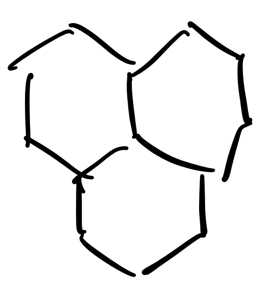
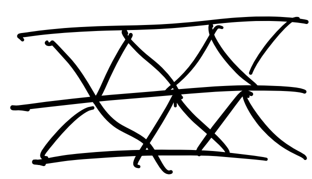
VII El grupo de renormalisación

1) En los puntos críticos los sistemas son invariantes de escala, y además hay Universalidad:

ejemplo:

Modelo de Ising Ferro

 cuadrada ó diagonal

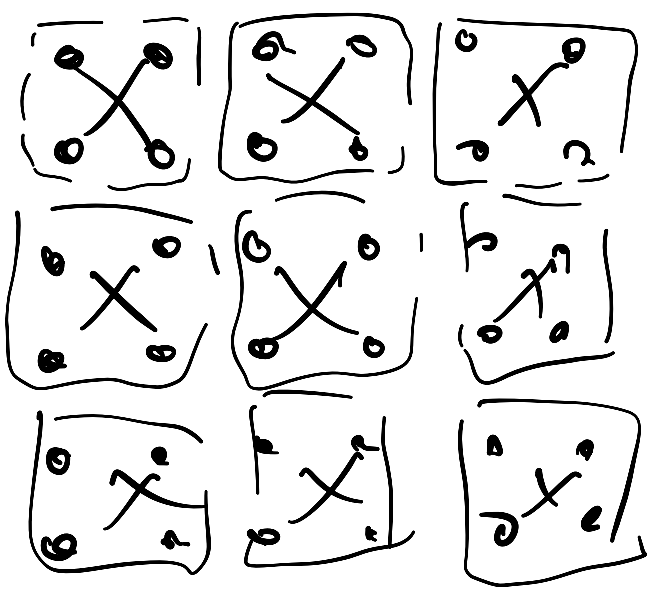
ó  

Tienen T_c que son diferentes, pero la clase de universalidad (los exp. críticos) son los mismos

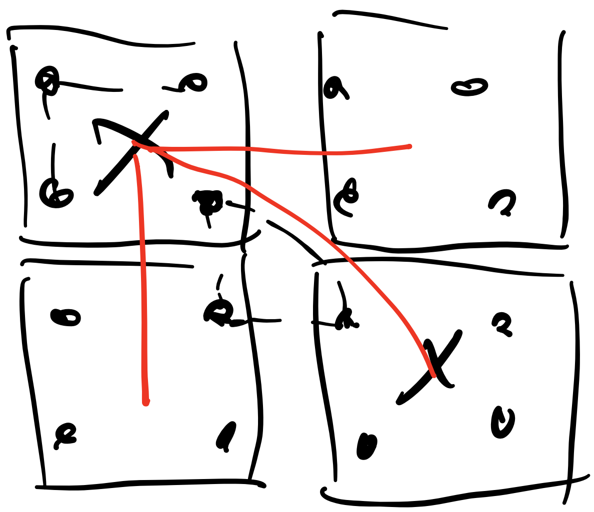
<u>Idea:</u> "Zoom out" del modelo

<u>2) R.G. en el espacio real</u>

(Kadanoff)



$$H(\sigma, J) \longrightarrow \tilde{H}(\tilde{\sigma}, \tilde{J})$$



3) El caso de D=1 (h=0)

$$Z = \sum_{\{\sigma_i\}} e^{K \sum_i \sigma_i \sigma_{i+1}}$$

$$K = \frac{J}{k_B T}$$

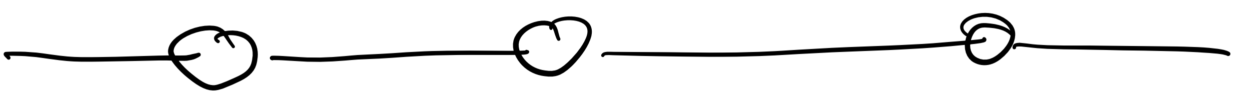
$$Z = \mathrm{Tr}\{T^N\} \qquad T = \begin{bmatrix} e^{K} & e^{-K} \\ e^{-K} & e^{K} \end{bmatrix}$$

en la base $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ y $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$

$$T = \begin{pmatrix} 2\mathrm{ch}K & 0 \\ 0 & 2\mathrm{sh}K \end{pmatrix}$$

N espines

b espins $\tilde{N} = \frac{N}{b}$ espines



$$Z = \mathrm{Tr}\{\tilde{T}^{\tilde{N}}\} \qquad \tilde{N} = \frac{N}{b}$$

$$T^N = (T^b)^{\frac{N}{b}}$$

$$\Rightarrow \tilde{T} = \underline{T^b}$$

$$T^b = \hat{T} = \text{cte} \begin{pmatrix} 2\,\mathrm{ch}\,\tilde{K} & 0 \\ 0 & 2\,\mathrm{sh}\,\tilde{K} \end{pmatrix}$$

$$\mathrm{th}(\tilde{K}) = (\mathrm{th}\,K)^b$$

$$\Rightarrow \boxed{\tilde{K} = \mathrm{th}^{-1}\left[\mathrm{th}(K)^b\right]}$$

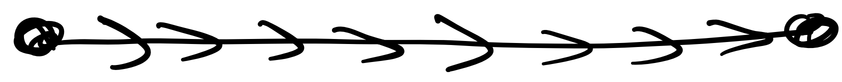
$$K = \frac{J}{k_B T} \Rightarrow \tilde{K} = \frac{\tilde{J}}{k_B T}$$

$$a' = \frac{J}{k_B \tilde{T}}$$

$$\boxed{\tilde{K} = \mathrm{th}^{-1}\left[\mathrm{th}(K)^b\right]}$$

esta recurrencia tiene 2 puntos fijos $\mathrm{th}(K) = 0$ $(T = \infty)$

$\mathrm{th}(K) = 1$ $(T = 0)$



$T=0$ $\quad\longrightarrow\quad$ $T=\infty$

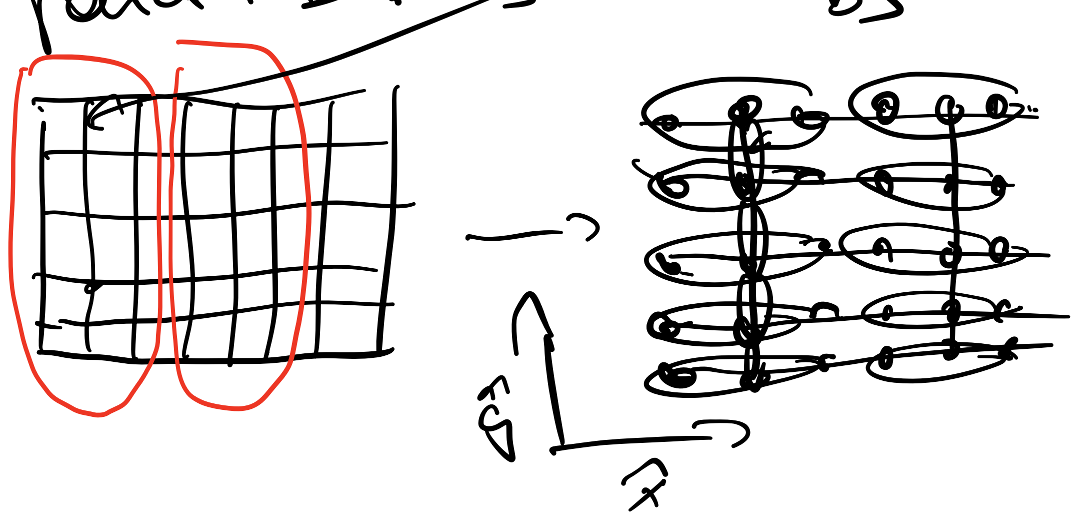
$\downarrow$ $\qquad\qquad\qquad$ $\downarrow$

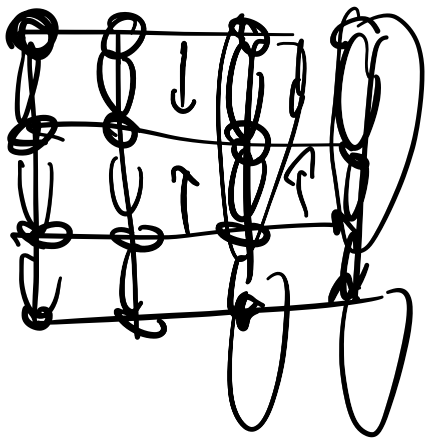
$\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow$ $\qquad\qquad$ $m = \langle\sigma\rangle = 0$

ó $\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow$

ej: $\quad \xi \rightarrow \tilde{\xi} = \dfrac{\xi}{b}$

4) La aproximación de Kadanoff para $\forall D$.





$\tilde{K}_y = b K_y \qquad \tilde{K}_x = \text{th}^{-1}\left[\left(\text{th}[K_x]\right)^b \right]$

al final

$$\boxed{\tilde{K} = b\,\text{th}^{-1}\left[\text{th}(bK)^b\right]}$$

$$b = e^{\delta\ell} \sim (1+\delta\ell)$$

$$\text{th}^{-1}\left[\text{th}[(1+\delta\ell)K]^{(1+\delta\ell)}\right]$$

$$\simeq (1+\delta\ell)K + \delta\ell\left[\tfrac{1}{2}\,\text{sh}\,2K \cdot \ln \text{th}(K)\right] + O(\delta\ell^2)$$

$\hat{K} = K + \delta K$

$$\boxed{\frac{dK}{dl} = K + \frac{1}{2}\left[Sh(2K)\ln(Th\,K)\right]}$$

para $D = 2$

Se muestra

$$\boxed{\frac{dK}{dl} = (D-1)K + \frac{1}{2}\left[Sh(2K)\ln(Th\,K)\right]}$$

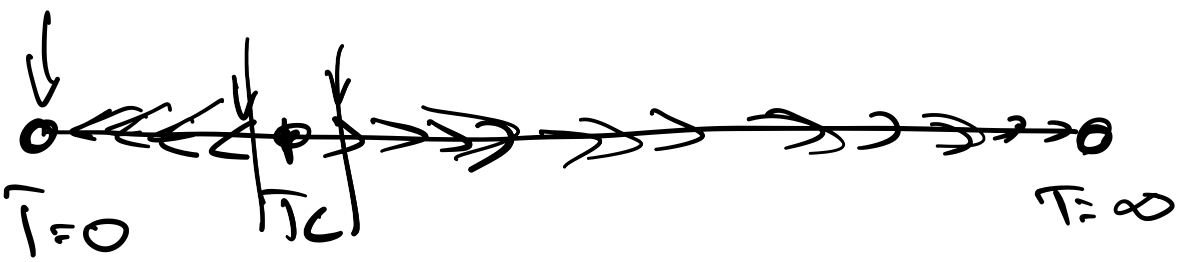
$T = \frac{J}{k_B K}$, para T pequeño.

$$\rightarrow \boxed{\frac{dT}{dl} = -\epsilon T + \frac{k_B}{2J}T^2 + O(T^3)}$$

$\epsilon = D-1$

tiene puntos fijos $\left(\frac{dT}{d\ell}=0\right)$

$T=0$ $\quad$ $T_c = \frac{2\epsilon J}{k_B}$ $\quad$ $\epsilon = D-1$



* $T = 0 + \delta T$

$$\Rightarrow \frac{d\,\delta T}{d\ell} = -\epsilon\,\delta T \qquad \epsilon > 0$$

* $T = T_c + \delta T$

$$\Rightarrow \boxed{\frac{d\,\delta T}{d\ell} = +\epsilon\,\delta T}$$

T_c es un punto fijo repulsivo.

$$\delta T(\ell) = \delta T(0)\, e^{\epsilon \ell}$$

$$\left.\begin{array}{l} \xi(\ell) = \xi(0)\, e^{-\ell} \\ \xi \sim \delta T^{-\nu} \end{array}\right] \Rightarrow \boxed{\nu = \frac{1}{e} = \frac{1}{D-1}}$$

5] La renormalización para la Teoría de campos.

$$S = \underbrace{\int d\vec{r}}_{D\,\dim} \left[\frac{K}{2} (\vec{\nabla}\phi)^2 + \frac{t}{2}\phi^2 + u\phi^4 + \mu_6\phi^6 + \cdots \right]$$

Sabemos que:

la teoría $S = \int d\vec{r}\, \frac{K}{2} (\vec{\nabla}\phi)^2$ es inv. de escala.

$$\vec{r} \to b\vec{r}\,', \quad K \to K$$

$$\phi \to b^{1-\frac{D}{2}} \phi'$$

si ahora $t \neq 0$, $u \neq 0$, $u_6 \neq 0$ etc..

$t \to b^{-2} t' \Rightarrow t' = b^2 t$

$u \to b^{-4+D} u' \Rightarrow \underline{u'} = \overline{b^{4-D} u}$

$u_6 \to b^{-6+2D} u_6' \quad u_6' = b^{-2D+6}$ $\quad u_6 \to b^{-2} u_6$ $\quad D=4$

$u_8 \to b^{-8+3D} u_8'$

<u>obs</u> si $D=4$ y $t=0$, $u_6 = u_8 = \cdots 0$

$u \neq 0$

$$S = \int d^4x \left[\frac{K}{2} (\nabla\phi)^2 + u\phi^4 \right]$$

en $D=4$ es invariante de escala!

para $D=3$, $u' > u$