Digital Signal Processing *DSP*

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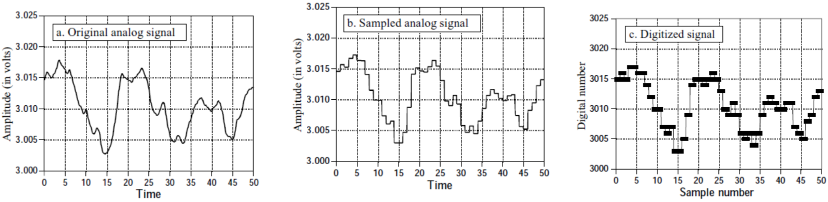
Contents

Analog to Digital Conversion

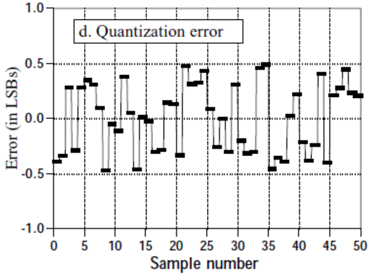
Digitization Process

- Digitization process implies two stages:
  - **Sampling:** A Sample and Hold (S/H) circuit retains an instantaneous value of the signal to allow ADC to have a stable value during conversion
  - **Quantization:** The ADC converts the voltage to the nearest integer number depending of the number of bits of the ADC



- We can see the effects of quantization, any one sample in the digitized signal can have a maximum error of $\pm 1/2$ LSB (Least Significant Bit)

- Subtracting the sampled analog signal to the digitized signal we obtain

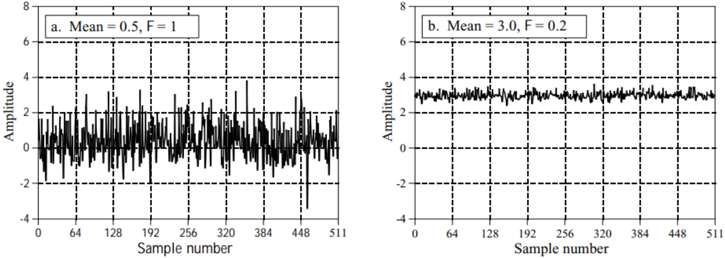


- **Quantization Error:** is a random noise added to the signal, it has
  - It is uniformly distributed between $\pm 1/2$ LSB
  - Mean value of zero
  - Standard deviation of $1/\sqrt{12}$ LSB

- Passing an analog signal to an 8 bit ADC adds an *rms noise* of: 0.29/256 or about 1/900 of the full scale value
- Passing an analog signal to an 12 bit ADC adds an *rms noise* of: 0.29/4096 or about 1/14000 of the full scale value
- Since quantization error is a random noise, the number of bits determines the precision of the data
- For the example you can make the statement : "We increased the precision of the measurement from 8 to 12 bits"
- This model is extremely powerful, because the random noise generated by quantization error will simply add to the noise present in the original signal

- **D**igital **S**ignal **P**rocessing is the mathematics, the algorithms and techniques used to manipulate signals after they have been converted into digital form.
- DSP applications are everywhere:
    - **Medical:** Diagnostic Imaging (CT, MRI, Ultrasound), ECG analysis
    - **Space:** Space photograph enhancement, Intelligent sensor analysis y remote probes
    - **Scientific:** Spectral analysis, simulation and modelling, Data acquisition
    - **Industrial:** Non-destructive testing, CAD and design tools, Process monitoring & control
    - **Commercial:** Image and sound compression, movie special effects, voice and data compression
- Common task for all these fields: Data Compression, Data acquisition, Spectral analysis, Echo reduction

- A **signal** is a description of how one parameter is related to another, for example $v(t)$ (analog signal) is converted with a 12bit 1KS/s ADC
- The converted signal has 4096 (2^{12} possible binary values), and time is defined only at one millisecond increments
- Let's define **N** as the total number of samples of a signal



- Mean and Standard deviation of the digitized signals can be calculated then

- **Mean** describes what is being measured
- **Standard deviation** represents *noise* and other interference
- **Signal-to-Noise Ratio SNR**

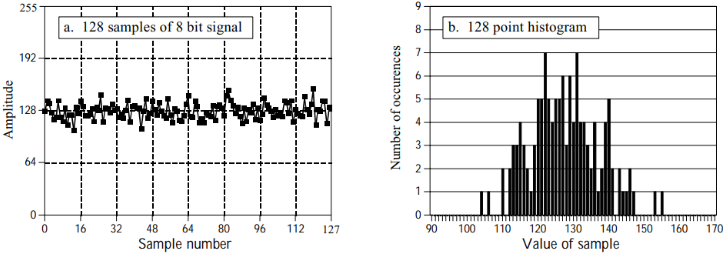
$$SNR = \frac{Mean}{Standard\ Deviation}$$

- **Coefficient of Variation CV**

$$CV = \frac{Standard\ Deviation}{Mean} \times 100$$

- A "good" signal means a high value of SNR and a low value of CV

- Suppose we have 128 samples of a 8bit ADC, the **histogram** displays the *number of samples* that have a *possible value*



- The histogram can be used to efficiently calculate the mean and standard deviation of very large data sets

- The sum of all of the values in the histogram must be equal to the number of points in the signal:

$$N = \sum_{i=0}^{M-1} H_i$$

- The histogram groups samples together that have the same value

$$\mu = \frac{1}{N} \sum_{i=0}^{M-1} H_i$$

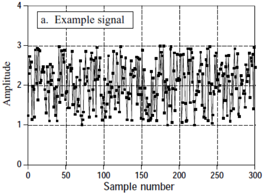
- This allows the statistics to be calculated by working with a few groups, rather than a large number of individual samples

$$\sigma^2 = \frac{1}{N-1} \sum_{i=0}^{M-1} (i-\mu)^2 H_i$$

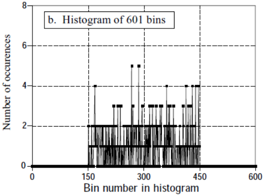
- A problem occurs in calculating the histogram when the number of levels each sample can take on is much larger than the number of samples in the signal
- This is always true for signals represented in floating point notation, where each sample is stored as a fractional value
- For example, integer representation might require the sample value to be 3 or 4, while floating point allows millions of possible fractional values between 3 and 4.
- For example, imagine a 10000 sample signal, with each sample having one billion possible values. The conventional histogram would consist of one billion data points, with all but about 10000 of them having a value of zero.

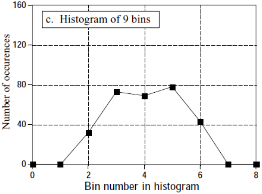
- **Binning** is a technique consisting in arbitrarily selecting the length of the histogram to be some convenient number, called **bin**
- The value of each bin represent the total number of samples in the signal that have a value within a certain range.
- For example, imagine a floating point signal that contains values from 0.0 to 10.0, and a histogram with 1000 bins.
- Bin 0 in the histogram is the number of samples in the signal with a value between 0 and 0.01, bin 1 is the number of samples with a value between 0.01 and 0.02, and so forth,
- **How many bins should be used?** This is a compromise between two problems: resolution along x and y axis.

- Original signal with 300 samples, each sample a floating point between 1 and 3



- Histograms with 601 bins (poor vertical resolution) and 9 bins (poor horizontal resolution)





Linear systems
Definitions

- Most DSP techniques are based on a divide-and-conquer strategy called superposition.
- The signal being processed is broken into simple components, each component is processed individually, and the results reunited.
- This approach has the tremendous power of breaking a single complicated problem into many easy ones.
- Superposition can only be used with linear systems, a term meaning that certain mathematical rules apply.
- Fortunately, most of the applications encountered in science and engineering fall into this category.

- **A signal** is a description of how one parameter varies with another parameter. For instance, voltage changing over time in an electronic circuit, or brightness varying with distance in an image.
- **A system** is any process that produces an output signal in response to an input signal.
- **Continuous systems** input $x(t)$ and output $y(t)$ continuous signals, such as in analog electronics.
- **Discrete systems** input $x[n]$ and output $y[n]$ discrete signals, such as computer programs that manipulate the values stored in arrays.
- You may want to design a system to remove noise in an electrocardiogram, sharpen an out-of-focus image.
- The system may represent some physical process that you want to study or analyze. Radar and sonar are good examples of this.

- A system is called linear if it has two mathematical properties: **homogeneity**

$$\textit{If } x[n] \rightarrow y[n] \textit{ then } kx[n] \rightarrow ky[n]$$

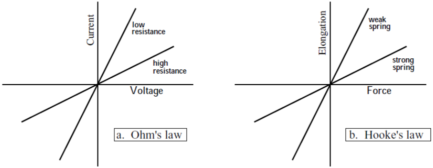
- and **additivity**

$$\textit{If } x_1[n] \rightarrow y_1[n] \textit{ and } x_2[n] \rightarrow y_2[n] \textit{ then } x_1[n] + x_2[n] \rightarrow y_1[n] + y_2[n]$$

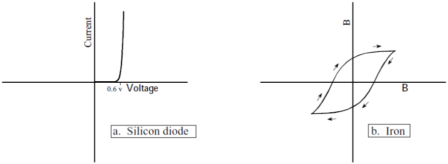
- If you can show that a system has both properties, then you have proven that the system is linear.
- A third property, **shift invariance**, is not a strict requirement for linearity, but it is a mandatory property for most DSP techniques.

$$\textit{If } x[n] \rightarrow y[n] \textit{ then } x[n+s] \rightarrow y[n+s]$$

- Linear Systems examples



- Non-linear system examples



Linear systems

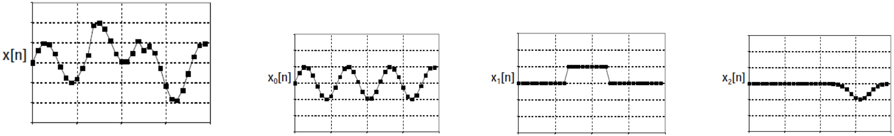
Applications

- Linear Systems applications
  - **Wave propagation:** mechanic and electromagnetic waves
  - **Electrical circuits:** resistors, capacitors, inductors
  - **Electronic circuits:** amplifiers and filters
  - **Mechanical motion:** interaction of masses, springs, dampeners
  - **Systems described by differential equations** RLC networks
- Nonlinear Systems applications
  - **Non static Linear Systems:** voltage and power in a circuit, radiant energy emission, intensity of light transmitted
  - **Non sinusoidal fidelity systems** peak detection, waveform conversion, frequency doubling
  - **Hysteresis and Saturation** magnetic flux, mechanical stress
  - **Systems with threshold** digital logic gates, seismic vibrations

- **Superposition principle** states that any complex signal can be represented as a linear combination of other simpler signals

$$xn = x_1[n] + x_2[n] + ... + x_m[n] + c$$

- **Decomposition** is the process where a single signal is broken into two or more additive components
- The process of combining signals through scaling and addition is called **synthesis**



References

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- Learn DSP
- A Beginner's guide to DSP