Física de Partículas

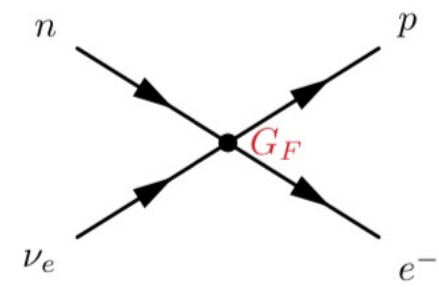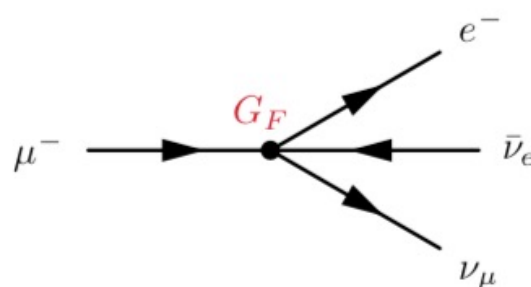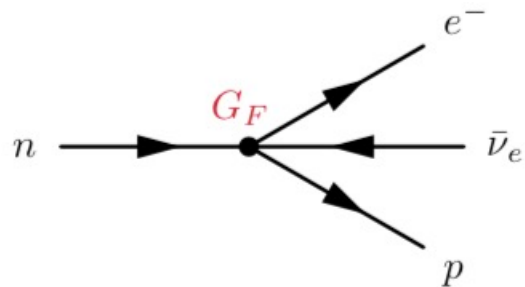## Interacción Débil

Carlos Sandoval

- By the 1930's it was clear there was a "weak" interaction responsible for beta decay and other decays involving Pauli's neutrino:

$$n \rightarrow p^{+} + e^{-} + \bar{v}$$
$$\pi \rightarrow \mu + v$$
$$\mu \rightarrow e + 2v$$

- Fermi's explanation was a 4-fermion contact interaction

- Where the coupling strength is the Fermi constant G_F

- Neutrino-neutron scattering in the Fermi theory
- The differential cross section (remember the Born approximation):

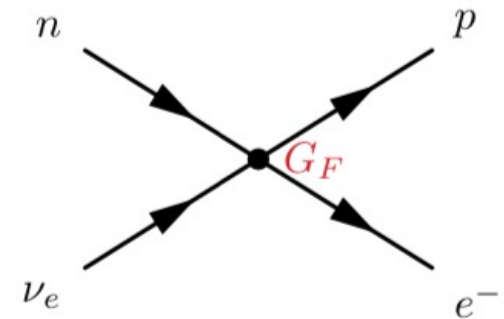$$\frac{d\sigma}{d\Omega} = \frac{E^2}{(2\pi)^2} |\mathcal{M}|^2$$

- And the matrix element:

$$|M_{fi}|^2 \approx 4G_F^2$$

- So the differential cross section reads:

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 E_e^2}{\pi^2}$$

- Fermi theory breaks down at high energies

- Remember that the parity operator for the Dirac spinors is the γ^0 matrix:

$$u \xrightarrow{\hat{P}} \hat{P}u = \gamma^0 u$$

$$\overline{u} \xrightarrow{\hat{P}} \overline{u}\gamma^0$$

- So the electron current in the $eq \rightarrow eq$ process, transforms as:

$$j_e^\mu = \overline{u}(p_3)\gamma^\mu u(p_1) \xrightarrow{\hat{P}} \overline{u}(p_3)\gamma^0\gamma^\mu\gamma^0 u(p_1)$$

- Remember that the parity operator for the Dirac spinors is the γ^0 matrix:

$$u \xrightarrow{\hat{P}} \hat{P}u = \gamma^0 u$$

$$\overline{u} \xrightarrow{\hat{P}} \overline{u}\gamma^0$$

- So the electron current in the $eq \rightarrow eq$ process, transforms as:

$$j_e^\mu = \overline{u}(p_3)\gamma^\mu u(p_1) \xrightarrow{\hat{P}} \overline{u}(p_3)\gamma^0\gamma^\mu\gamma^0 u(p_1)$$

- The time component: $\quad j_e^0 \xrightarrow{\hat{P}} \overline{u}\gamma^0\gamma^0\gamma^0 u = \overline{u}\gamma^0 u = j_e^0$

- The spatial component: $\quad j_e^k \xrightarrow{\hat{P}} \overline{u}\gamma^0\gamma^k\gamma^0 u = -\overline{u}\gamma^k\gamma^0\gamma^0 u = -\overline{u}\gamma^k u = -j_e^k$

- Remember that the parity operator for the Dirac spinors is the γ^0 matrix:

$$u \xrightarrow{\hat{P}} \hat{P}u = \gamma^0 u$$

$$\overline{u} \xrightarrow{\hat{P}} \overline{u}\gamma^0$$

- So the electron current in the $eq \rightarrow eq$ process, transforms as:

$$j_e^\mu = \overline{u}(p_3)\gamma^\mu u(p_1) \xrightarrow{\hat{P}} \overline{u}(p_3)\gamma^0\gamma^\mu\gamma^0 u(p_1)$$

- So the four-vector product of the QED matrix element:

$$j_e \cdot j_q = j_e^0 j_q^0 - j_e^k j_q^k \xrightarrow{\hat{P}} j_e^0 j_q^0 - (-j_e^k)(-j_q^k) = j_e \cdot j_q$$

remains unchanged

- So the four-vector product of the QED matrix element:

$$j_{\mathrm{e}} \cdot j_{\mathrm{q}} = j_{\mathrm{e}}^0 j_{\mathrm{q}}^0 - j_{\mathrm{e}}^k j_{\mathrm{q}}^k \xrightarrow{\hat{P}} j_{\mathrm{e}}^0 j_{\mathrm{q}}^0 - (-j_{\mathrm{e}}^k)(-j_{\mathrm{q}}^k) = j_{\mathrm{e}} \cdot j_{\mathrm{q}}$$

remains unchanged
- The QED matrix element is invariant under a Parity transformation
- The same applies to the Hamiltonian and therefore, Parity is conserved in QED

- So the four-vector product of the QED matrix element:

$$j_e \cdot j_q = j_e^0 j_q^0 - j_e^k j_q^k \xrightarrow{\hat{P}} j_e^0 j_q^0 - (-j_e^k)(-j_q^k) = j_e \cdot j_q$$

  remains unchanged
- The QED matrix element is invariant under a Parity transformation
- The same applies to the Hamiltonian and therefore, Parity is conserved in QED
- The form of the QCD vertex is the same except for colour factors: Parity is conserved in QCD

- In 1957 Wu and collaborators studied the beta decay of polarised cobalt-60:

$$^{60}\text{Co} \rightarrow {}^{60}\text{Ni}^* + e^- + \overline{\nu}_e$$

- For a fixed magnetic field, the nuclear magnetic moment of the cobalt was aligned
- Electrons were detected at different angles with respect to the direction of the field
- The rate at which electrons were emitted on either direction of the field, should be the same (if parity were conserved)

- In 1957 Wu and collaborators studied the beta decay of polarised cobalt-60:

$$^{60}\text{Co} \rightarrow {}^{60}\text{Ni}^* + e^- + \bar{\nu}_e$$

- For a fixed magnetic field, the nuclear magnetic moment of the cobalt was aligned
- Electrons were detected at different angles with respect to the direction of the field
- The rate at which electrons were emitted on either direction of the field, should be the same (if parity were conserved)
- However, more electrons were observed in the direction opposite the magnetic field – Parity Violation of Weak Interactions

- From the observation of parity violation, it is clear the weak interaction vertex must have a different form to the QED and QCD vertices:

$$j^\mu = \overline{u}(p')\gamma^\mu u(p)$$

- We need a Lorentz Invariant Matrix element, and there are only 5 such bilinear covariants :

Type	Form	Components	Boson spin
Scalar	$\overline{\psi}\phi$	1	0
Pseudoscalar	$\overline{\psi}\gamma^5\phi$	1	0
Vector	$\overline{\psi}\gamma^\mu\phi$	4	1
Axial vector	$\overline{\psi}\gamma^\mu\gamma^5\phi$	4	1
Tensor	$\overline{\psi}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\phi$	6	2

- The most general Lorentz Invariant form for the interaction between a fermion and a boson is a linear combination of the bilinear covariants
- If the boson has spin 1

$$j^{\mu} \propto \bar{u}(p')(g_V \gamma^{\mu} + g_A \gamma^{\mu} \gamma^5)u(p) = g_V j_V^{\mu} + g_A j_A^{\mu}$$

where the current has been split into vector and axial components

- The most general Lorentz Invariant form for the interaction between a fermion and a boson is a linear combination of the bilinear covariants
- If the boson has spin 1

$$j^\mu \propto \bar{u}(p')(g_V \gamma^\mu + g_A \gamma^\mu \gamma^5)u(p) = g_V j_V^\mu + g_A j_A^\mu$$

  where the current has been split into vector and axial components
- Parity transformation for axial current:

$$j_A^\mu = \bar{u}\gamma^\mu \gamma^5 u \xrightarrow{\hat{P}} \bar{u}\gamma^0 \gamma^\mu \gamma^5 \gamma^0 u = -\bar{u}\gamma^0 \gamma^\mu \gamma^0 \gamma^5 u$$

- Time component:

$$j_A^0 = \xrightarrow{\hat{P}} -\bar{u}\gamma^0 \gamma^0 \gamma^0 \gamma^5 u = -\bar{u}\gamma^0 \gamma^5 u = -j_A^0$$

- Spatial component:

$$j_A^k = \xrightarrow{\hat{P}} -\bar{u}\gamma^0 \gamma^k \gamma^0 \gamma^5 u = +\bar{u}\gamma^k \gamma^5 u = +j_A^k$$

- The vector and axial currents under a parity transformation:

$$j_V^0 \xrightarrow{\hat{P}} +j_V^0, \quad j_V^k \xrightarrow{\hat{P}} -j_V^k, \quad \text{and} \quad j_A^0 \xrightarrow{\hat{P}} -j_A^0, \quad j_A^k \xrightarrow{\hat{P}} +j_A^k$$

- The product of two vector or two axial currents is invariant under parity, but not the product of vector and axial currents
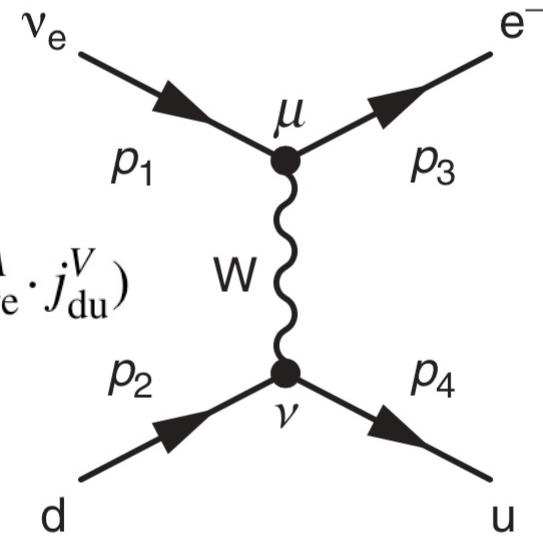- This structure could therefore explain the parity violation observed in the weak interaction

- Consider the inverse beta decay process:

$$j_{ve}^{\mu} = \bar{u}(p_3)(g_V\gamma^{\mu} + g_A\gamma^{\mu}\gamma^5)u(p_1) = g_V j_{ve}^V + g_A j_{ve}^A$$

$$j_{du}^{\nu} = \bar{u}(p_4)(g_V\gamma^{\nu} + g_A\gamma^{\nu}\gamma^5)u(p_2) = g_V j_{du}^V + g_A j_{du}^A$$

$$\mathcal{M}_{fi} \propto j_{ve} \cdot j_{du} = g_V^2 j_{ve}^V \cdot j_{du}^V + g_A^2 j_{ve}^A \cdot j_{du}^A + g_V g_A(j_{ve}^V \cdot j_{du}^A + j_{ve}^A \cdot j_{du}^V)$$

- Consider the inverse beta decay process:

$$j_{ve}^{\mu} = \bar{u}(p_3)(g_V\gamma^{\mu} + g_A\gamma^{\mu}\gamma^5)u(p_1) = g_V j_{ve}^{V} + g_A j_{ve}^{A}$$

$$j_{du}^{\nu} = \bar{u}(p_4)(g_V\gamma^{\nu} + g_A\gamma^{\nu}\gamma^5)u(p_2) = g_V j_{du}^{V} + g_A j_{du}^{A}$$

$$\mathcal{M}_{fi} \propto j_{ve}\cdot j_{du} = g_V^2 j_{ve}^{V}\cdot j_{du}^{V} + g_A^2 j_{ve}^{A}\cdot j_{du}^{A} + g_V g_A \left(j_{ve}^{V}\cdot j_{du}^{A} + j_{ve}^{A}\cdot j_{du}^{V}\right)$$

not invariant under Parity

- From experiments, we know the weak charged current due to the exchange of W bosons has the vertex factor:

$$\frac{-ig_W}{\sqrt{2}}\,\frac{1}{2}\gamma^{\mu}(1-\gamma^5)$$
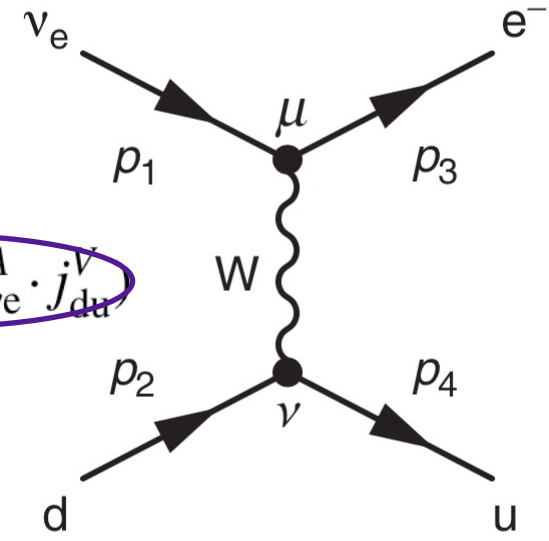
- Remember the Chiral projection operators:

  - Any Dirac spinor can be decomposed into left and right handed chiral components through projection operators:

  $$P_R = \tfrac{1}{2}(1 + \gamma^5)$$
  $$P_L = \tfrac{1}{2}(1 - \gamma^5)$$

  - P_R projects the right handed chiral particle states and the left handed chiral antiparticle states

    $$P_R u_R = u_R$$
    $$P_R u_L = 0$$
    $$P_R v_R = 0$$
    $$P_R v_L = v_L$$

  - P_L projects the left handed chiral particle states and the right handed chiral antiparticle states

    $$P_L u_R = 0$$
    $$P_L u_L = u_L$$
    $$P_L v_R = v_R$$
    $$P_L v_L = 0$$

- Remember the Chiral projection operators
- We also saw that in QED, only two combinations of spinors give non-zero currents (chiral nature of QED)
- For the weak interaction vertex:

$$j^{\mu}_{RR} = \frac{g_{\mathrm{W}}}{\sqrt{2}} \, \overline{u}_R(p')\gamma^{\mu}\frac{1}{2}(1 - \gamma^5)u_R(p)$$

$$= \frac{g_{\mathrm{W}}}{\sqrt{2}} \, \overline{u}_R(p')\gamma^{\mu}P_L u_R(p) = 0,$$

the only non-zero current for particle spinors involves only left-handed chiral states

- Remember the Chiral projection operators
- We also saw that in QED, only two combinations of spinors give non-zero currents (chiral nature of QED)
- For the weak interaction vertex:

$$j_{RR}^{\mu} = \frac{g_W}{\sqrt{2}} \bar{u}_R(p')\gamma^{\mu}\frac{1}{2}(1 - \gamma^5)u_R(p)$$

$$= \frac{g_W}{\sqrt{2}} \bar{u}_R(p')\gamma^{\mu}P_L u_R(p) = 0,$$

  the only non-zero current for particle spinors involves only left-handed chiral states
- For anti-particle spinors, PL projects the right-handed chiral states, so only right handed chiral anti-particle states interact with the charged weak interaction

- Example:

RH antiparticle   LH particle $\hat{P}$ RH particle   LH antiparticle

$\bar{\nu}_e$ $\mathbf{p}_\nu$ $\mathbf{p}_e$ e^- e^- $-\mathbf{p}_e$ $-\mathbf{p}_\nu$ $\bar{\nu}_e$

         allowed   not allowed
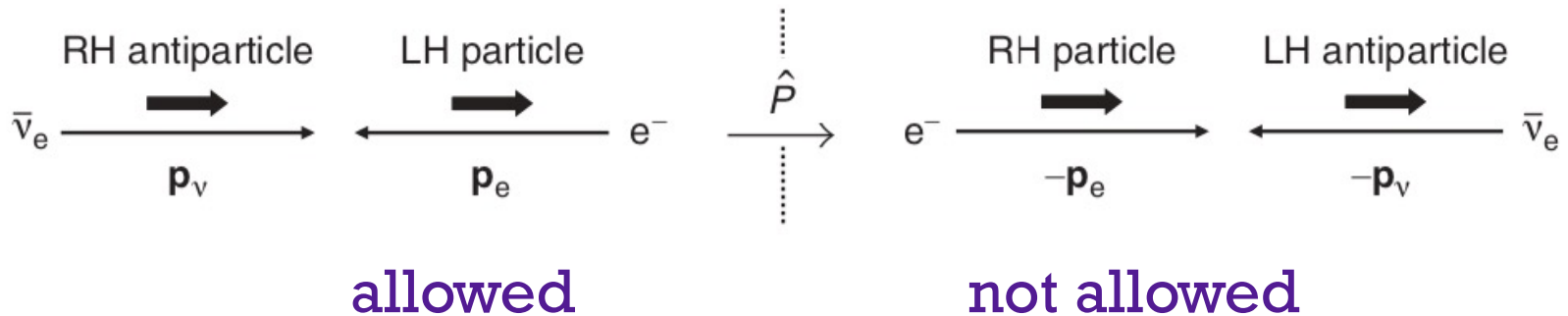
- So Parity violation implies we see more electrons in one direction than the opposite (Wu's experiment)

- Charge conjugation is also violated in the weak interactions
- However: Charge conjugation AND Parity could* be conserved
- This is called the combined CP symmetry
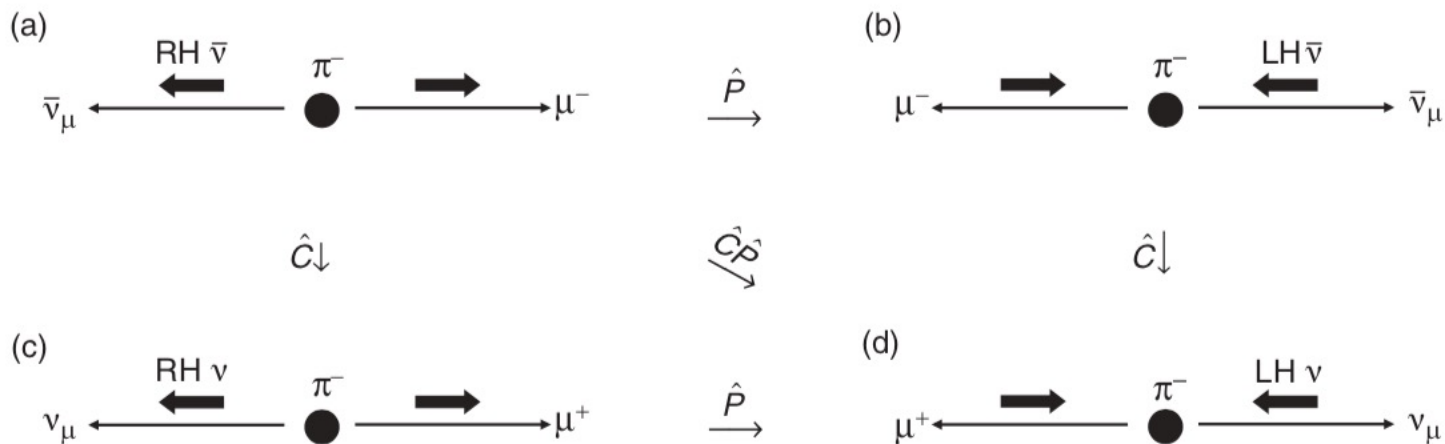
*CP violation is necessary to explain the difference in matter and anti-matter in the Universe. QED and QCD conserve CP, so the only place for CP violation in the Standard Model is the weak interaction. CP violation has been observed in meson systems, but this is not sufficient to account for the matter-antimatter asymmetry in the Universe

- For massive bosons have an additional degree of freedom of a longitudinal polarisation state:

$$\sum_{\lambda} \epsilon_\mu^{\lambda*} \epsilon_\nu^\lambda = -g_{\mu\nu} + \frac{q_\mu q_\nu}{m_{\mathrm{W}}^2}$$

so the Feynman rule for the propagator of the W boson:

$$\frac{-i}{q^2 - m_{\mathrm{W}}^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_{\mathrm{W}}^2}\right)$$

- In the limit $q^2 \ll m_W^2$:

$$\frac{-i g_{\mu\nu}}{q^2 - m_{\mathrm{W}}^2}$$

- At low energies the propagator can be approximated by:

$$i \cdot \frac{g_{\mu\nu}}{m_{\mathrm{W}}^2}$$

- This would correspond to an interaction that occurs at a single point (Fermi theory):

$$\mathcal{M}_{fi} = G_{\mathrm{F}} \, g_{\mu\nu} [\overline{\psi}_3 \gamma^\mu \psi_1][\overline{\psi}_4 \gamma^\nu \psi_2]$$

- Including the V-A structure to account for parity violation:

$$\mathcal{M}_{fi} = \frac{1}{\sqrt{2}} \, G_{\mathrm{F}} \, g_{\mu\nu} [\overline{\psi}_3 \gamma^\mu (1 - \gamma^5) \psi_1][\overline{\psi}_4 \gamma^\nu (1 - \gamma^5) \psi_2]$$

- At low energies the propagator can be approximated by:

$$i \frac{g_{\mu\nu}}{m_{\mathrm{W}}^2}$$

- This would correspond to an interaction that occurs at a single point (Fermi theory):

$$\mathcal{M}_{fi} = G_{\mathrm{F}}\, g_{\mu\nu}[\overline{\psi}_3 \gamma^\mu \psi_1][\overline{\psi}_4 \gamma^\nu \psi_2]$$

- Including the V-A structure to account for parity violation:

$$\mathcal{M}_{fi} = \frac{1}{\sqrt{2}}\, G_{\mathrm{F}}\, g_{\mu\nu}[\overline{\psi}_3 \gamma^\mu (1 - \gamma^5)\psi_1][\overline{\psi}_4 \gamma^\nu (1 - \gamma^5)\psi_2]$$

- And comparing to the full expression in the limit $q^2 \ll m_W^2$:

$$M_{fi} = \frac{g_{\mathrm{W}}^2}{8 m_{\mathrm{W}}^2}\, g_{\mu\nu}[\overline{\psi}_3 \gamma^\mu (1 - \gamma^5)\psi_1][\overline{\psi}_4 \gamma^\nu (1 - \gamma^5)\psi_2]$$

- At low energies the propagator can be approximated by:

$$i \frac{g_{\mu\nu}}{m_{\mathrm{W}}^2}$$

- This would correspond to an interaction that occurs at a single point (Fermi theory)

- We can relate both coupling constants:

$$\frac{G_{\mathrm{F}}}{\sqrt{2}} = \frac{g_W^2}{8 m_{\mathrm{W}}^2}$$

- The strength of the weak interaction is most precisely determined from low-energy measurements
- Muon lifetime ($m_\mu^2 \ll m_W^2$):

$$\Gamma(\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e) = \frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

with the measurements of the muon mass and lifetime:

$$m_\mu = 0.105\,658\,371\,5(35)\,\text{GeV} \qquad \tau_\mu = 2.196\,981\,1(22) \times 10^{-6}\,\text{s}$$

we can have a precise measurement of the Fermi constant:

$$G_F = 1.166\,38 \times 10^{-5}\,\text{GeV}^{-2}$$

- The strength of the weak interaction is related to the Fermi constant by the mass of the W boson:

$$m_W = 80.385 \pm 0.015\,\text{GeV}$$

$$\alpha_W = \frac{g_W^2}{4\pi} = \frac{8m_W^2 G_F}{4\sqrt{2}\pi} \approx \frac{1}{30}$$

- The coupling constant itself is larger than the QED constant
- However: the presence of the mass of the W boson in the propagator, makes the weak interaction weaker than QED

- Pions are the lightest mesons formed by the lightest quarks
- They can not decay via the strong interaction
- Pions decay via the weak interaction into leptons
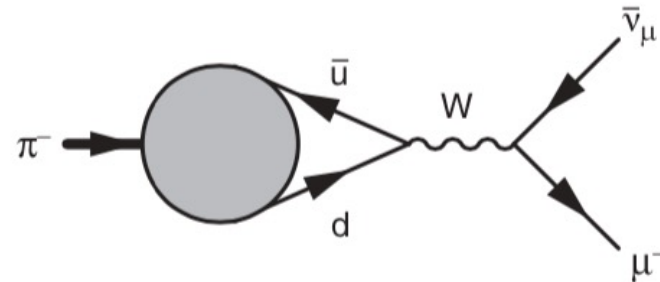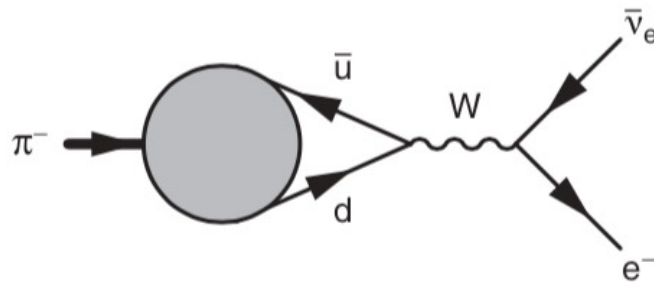
- Pions are found to decay much more frequently into muons than electrons:

$$\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} = 1.230(4) \times 10^{-4}$$

- Let us calculate the decay rate:
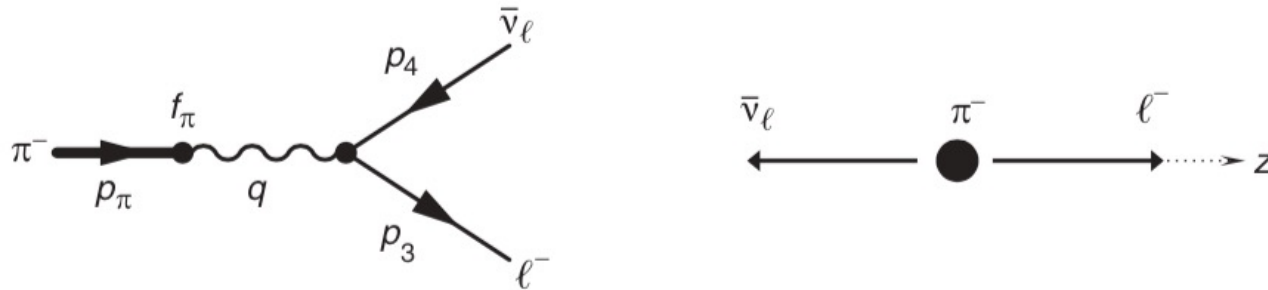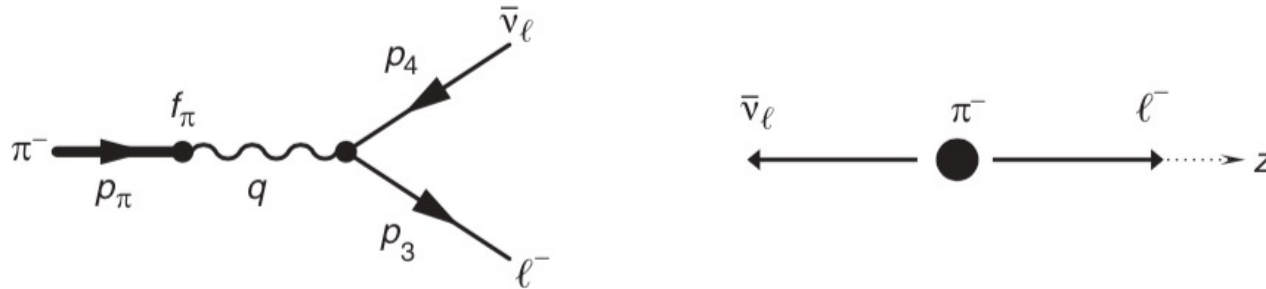
$$p_\pi = (m_\pi, 0, 0, 0), \quad p_\ell = p_3 = (E_\ell, 0, 0, \mathrm{p}) \quad \text{and} \quad p_{\bar{\nu}} = p_4 = (\mathrm{p}, 0, 0, -\mathrm{p})$$
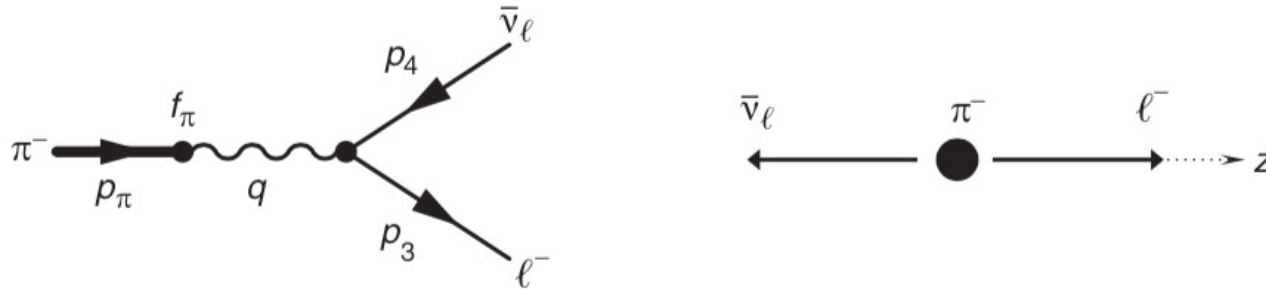
$$j_\ell^\nu = \frac{g_\mathrm{W}}{\sqrt{2}} \bar{u}(p_3) \tfrac{1}{2} \gamma^\nu (1 - \gamma^5) v(p_4)$$

- Let us calculate the decay rate:

$$p_\pi = (m_\pi, 0, 0, 0), \quad p_\ell = p_3 = (E_\ell, 0, 0, \mathrm{p}) \quad \text{and} \quad p_{\bar{v}} = p_4 = (\mathrm{p}, 0, 0, -\mathrm{p})$$

$$j_\ell^\nu = \frac{g_\mathrm{W}}{\sqrt{2}} \bar{u}(p_3) \tfrac{1}{2} \gamma^\nu (1 - \gamma^5) v(p_4)$$

- The pion current has to be a four-vector: pion four-momentum:

$$f_\pi p_\pi^\mu$$

where f is a constant associated with the decay

- Let us calculate the decay rate:
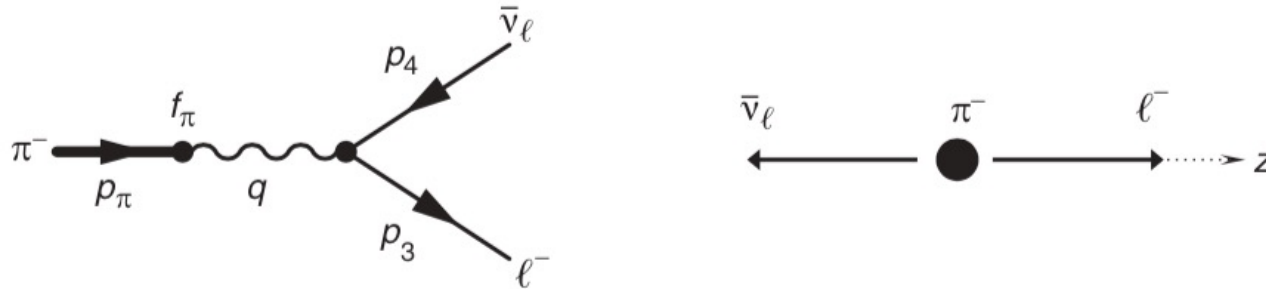
$$p_\pi = (m_\pi, 0, 0, 0), \quad p_\ell = p_3 = (E_\ell, 0, 0, \mathrm{p}) \quad \text{and} \quad p_{\overline{\nu}} = p_4 = (\mathrm{p}, 0, 0, -\mathrm{p})$$

- The matrix element:

$$\mathcal{M}_{fi} = \left[\frac{g_W}{\sqrt{2}} \frac{1}{2} f_\pi p_\pi^\mu \right] \times \left[\frac{g_{\mu\nu}}{m_W^2} \right] \times \left[\frac{g_W}{\sqrt{2}} \overline{u}(p_3) \gamma^\nu \frac{1}{2} (1 - \gamma^5) v(p_4) \right]$$

pion current   Propagator (approx. Fermi)   lepton current

- Let us calculate the decay rate:
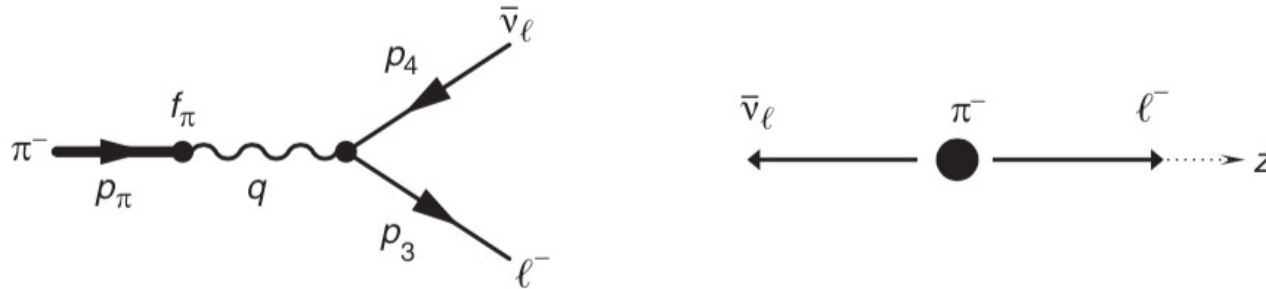
$$p_\pi = (m_\pi, 0, 0, 0), \quad p_\ell = p_3 = (E_\ell, 0, 0, \mathrm{p}) \quad \text{and} \quad p_{\bar{\nu}} = p_4 = (\mathrm{p}, 0, 0, -\mathrm{p})$$

- The matrix element:

$$\mathcal{M}_{fi} = \left[\frac{g_\mathrm{W}}{\sqrt{2}} \frac{1}{2} f_\pi p_\pi^\mu \right] \times \left[\frac{g_{\mu\nu}}{m_\mathrm{W}^2} \right] \times \left[\frac{g_\mathrm{W}}{\sqrt{2}} \bar{u}(p_3) \gamma^\nu \frac{1}{2} (1 - \gamma^5) v(p_4) \right]$$

$$= \frac{g_\mathrm{W}^2}{4 m_\mathrm{W}^2} g_{\mu\nu} f_\pi p_\pi^\mu \bar{u}(p_3) \gamma^\nu \frac{1}{2} (1 - \gamma^5) v(p_4),$$

where we will only have the time-like component

- Let us calculate the decay rate:

$$p_\pi = (m_\pi, 0, 0, 0), \quad p_\ell = p_3 = (E_\ell, 0, 0, p) \quad \text{and} \quad p_{\bar{v}} = p_4 = (p, 0, 0, -p)$$

- The matrix element:

$$\mathcal{M}_{fi} = \frac{g_W^2}{4m_W^2} f_\pi m_\pi \bar{u}(p_3) \gamma^0 \tfrac{1}{2}(1 - \gamma^5) v(p_4)$$

$$\mathcal{M}_{fi} = \frac{g_W^2}{4m_W^2} f_\pi m_\pi u^\dagger(p_3) \tfrac{1}{2}(1 - \gamma^5) v(p_4)$$

- Let us calculate the decay rate:
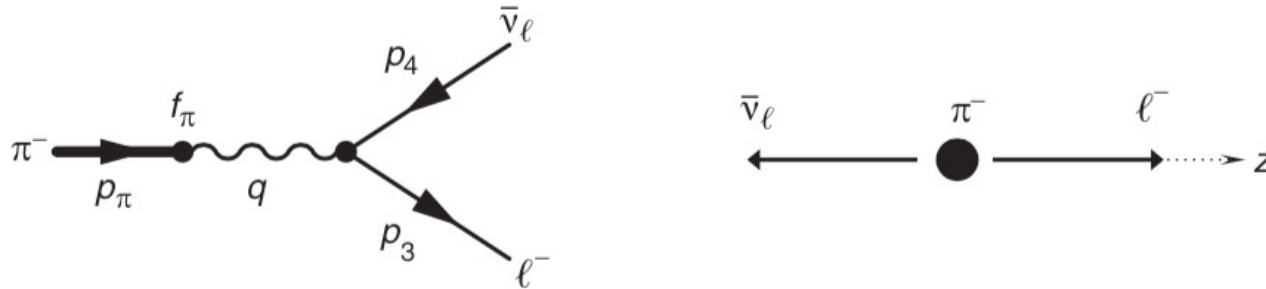
$$\mathcal{M}_{fi} = \frac{g_W^2}{4m_W^2} f_\pi m_\pi u^\dagger(p_3)\tfrac{1}{2}(1 - \gamma^5)v(p_4)$$

- For the neutrino the helicity eigenstates are the same as the chiral states ($m \ll E$):

$$\mathcal{M}_{fi} = \frac{g_W^2}{4m_W^2} f_\pi m_\pi u^\dagger(p_3)v_\uparrow(p_4)$$

- Let us calculate the decay rate:
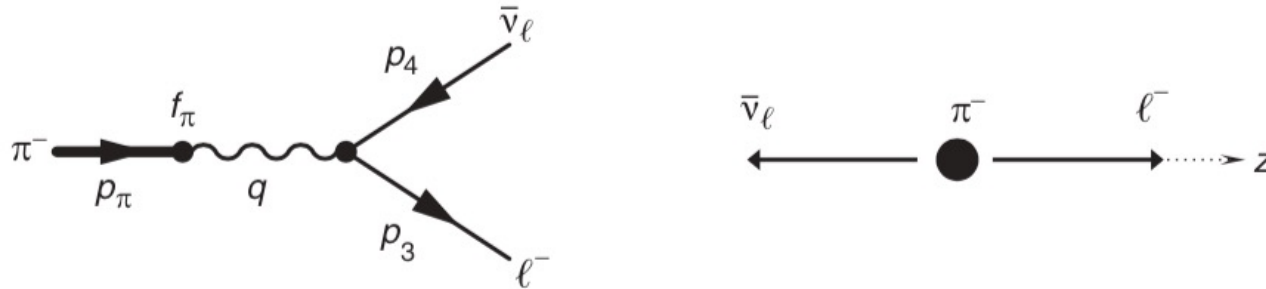
$$\mathcal{M}_{fi} = \frac{g_W^2}{4m_W^2} f_\pi m_\pi u^\dagger(p_3) v_\uparrow(p_4)$$

- The explicit form of the spinors for the lepton and neutrino:

$$u_\uparrow(p_3) = \sqrt{E_\ell + m_\ell} \begin{pmatrix} 1 \\ 0 \\ \frac{p}{E_\ell + m_\ell} \\ 0 \end{pmatrix} \qquad u_\downarrow(p_3) = \sqrt{E_\ell + m_\ell} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -\frac{p}{E_\ell + m_\ell} \end{pmatrix} \qquad v_\uparrow(p_4) = \sqrt{p} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

only one combination is not zero

- Let us calculate the decay rate:
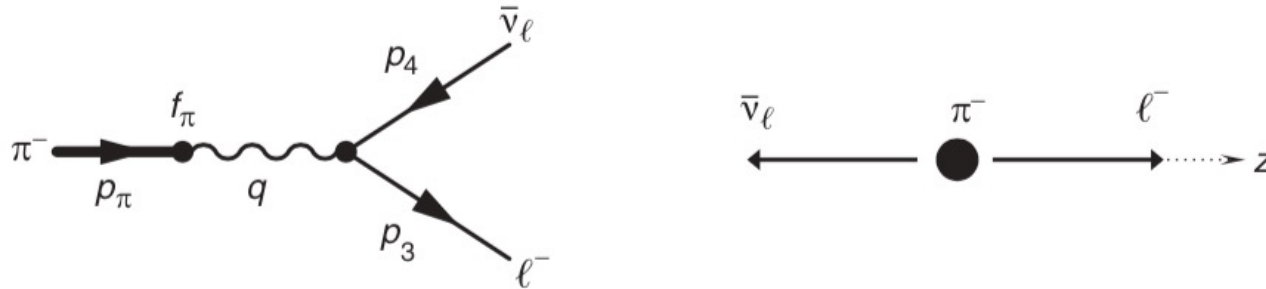
$$\mathcal{M}_{fi} = \frac{g_W^2}{4m_W^2} f_\pi m_\pi \sqrt{E_\ell + m_\ell} \sqrt{p}\left(1 - \frac{p}{E_\ell + m_\ell}\right)$$

- Which can be re-written using:

$$E_\ell = \frac{m_\pi^2 + m_\ell^2}{2m_\pi} \qquad p_\ell = \frac{m_\pi^2 - m_\ell^2}{2m_\pi}$$

to get:

$$\mathcal{M}_{fi} = \left(\frac{g_W}{2m_W}\right)^2 f_\pi m_\ell (m_\pi^2 - m_\ell^2)^{\frac{1}{2}}$$

- Let us calculate the decay rate:

- The matrix-element squared and corresponding decay rate:

$$\langle |\mathcal{M}_{fi}|^2 \rangle \equiv |\mathcal{M}_{fi}|^2 = 2G_F^2 f_\pi^2 m_\ell^2 (m_\pi^2 - m_\ell^2)$$

$$\Gamma = \frac{4\pi}{32\pi^2 m_\pi^2} p \langle |\mathcal{M}_{fi}|^2 \rangle = \frac{G_F^2}{8\pi m_\pi^3} f_\pi^2 \left[m_\ell (m_\pi^2 - m_\ell^2) \right]^2$$

- Let us calculate the decay rate:

- The matrix-element squared and corresponding decay rate:

$$\langle |\mathcal{M}_{fi}|^2 \rangle \equiv |\mathcal{M}_{fi}|^2 = 2 G_F^2 f_\pi^2 m_\ell^2 (m_\pi^2 - m_\ell^2)$$

$$\Gamma = \frac{4\pi}{32\pi^2 m_\pi^2} p \langle |\mathcal{M}_{fi}|^2 \rangle = \frac{G_F^2}{8\pi m_\pi^3} f_\pi^2 \left[m_\ell (m_\pi^2 - m_\ell^2) \right]^2$$

- Finally, the ratio:

$$\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} = \left[\frac{m_e (m_\pi^2 - m_e^2)}{m_\mu (m_\pi^2 - m_\mu^2)} \right]^2 = 1.26 \times 10^{-4}$$

- Let us calculate the decay rate:
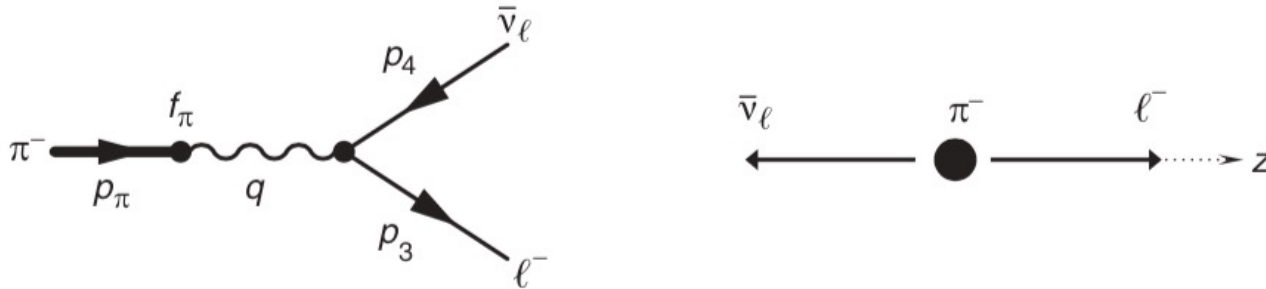
- Finally, the ratio:

$$\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} = \left[\frac{m_e(m_\pi^2 - m_e^2)}{m_\mu(m_\pi^2 - m_\mu^2)}\right]^2 = 1.26 \times 10^{-4}$$

this is evidence of the V-A structure of the weak interaction

- Muons do not decay via QED (the photon does not change flavour)
- Only the charged weak interaction changes lepton type

$$\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) \equiv \frac{1}{\tau_\mu} = \frac{G_F^{(e)} G_F^{(\mu)} m_\mu^5}{192\pi^3}$$
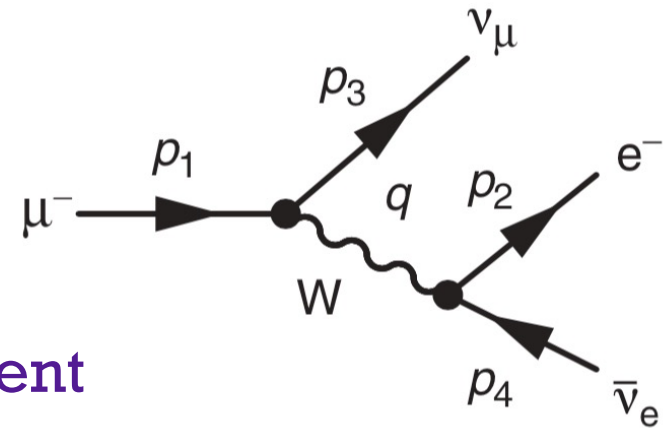
in principle the coupling to different leptons could vary

- Taus are sufficiently massive that they can decay into light quarks
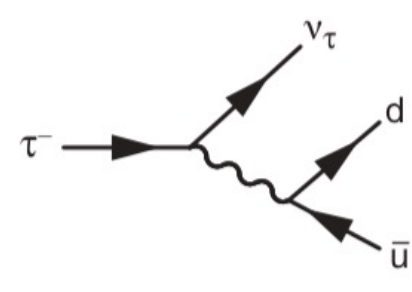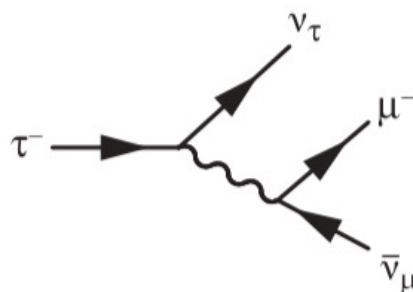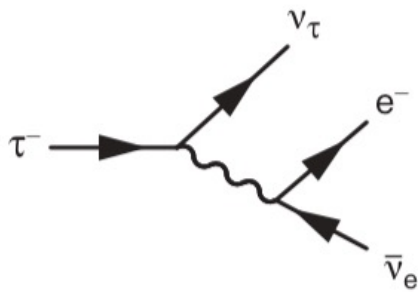
$$Br(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau) = \frac{\Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)}{\Gamma} = \Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau) \times \tau_\tau$$

$$\Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau) = \frac{G_F^{(e)} G_F^{(\tau)} m_\tau^5}{192\pi^3}$$

- We can compare the lifetimes to obtain the ratio of the couplings

$$\frac{G_F^{(\tau)}}{G_F^{(\mu)}} = \frac{m_\mu^5 \tau_\mu}{m_\tau^5 \tau_\tau} Br(\tau^- \to e^- \overline{\nu}_e \nu_\tau)$$

- Measuring masses, lifetimes, and decay fractions:

$$Br(\tau^- \to e^- \overline{\nu}_e \nu_\tau) = 0.1783(5)$$

$$m_\mu = 0.1056583715(35)\,\text{GeV}$$

$$m_\tau = 1.77682(16)\,\text{GeV}$$

$$Br(\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau) = 0.1741(4)$$

$$\tau_\mu = 2.1969811(22) \times 10^{-6}\,\text{s}$$

$$\tau_\tau = 0.2906(10) \times 10^{-12}\,\text{s}.$$

- We get a ratio for the couplings:

$$\frac{G_F^{(\tau)}}{G_F^{(\mu)}} = 1.0023 \pm 0.0033$$

$$\frac{G_F^{(e)}}{G_F^{(\mu)}} = 1.000 \pm 0.004$$

- Consider the underlying process for beta decay

$$|\mathcal{M}|^2 \propto G_F^{(e)} G_F^{(\beta)}$$

From observed decay rates, and comparing to the lepton coupling:

$$G_F^{(\mu)} = (1.166\,3787 \pm 0.000\,0006) \times 10^{-5}\ \mathrm{GeV}^{-2},$$
$$G_F^{(\beta)} = (1.1066 \pm 0.0011) \times 10^{-5}\ \mathrm{GeV}^{-2}.$$
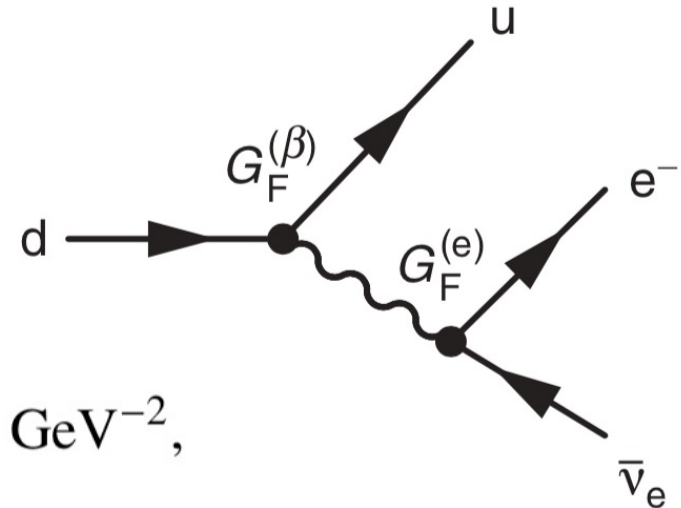
- The coupling to ud quarks is 5% smaller
- It is also found that the decay rate of $\mathrm{K}^-(\mathrm{u\bar{s}}) \rightarrow \mu^- \bar{\nu}_\mu$ is much smaller than for $\pi^-(\mathrm{u\bar{d}}) \rightarrow \mu^- \bar{\nu}_\mu$
- The coupling to quarks does not seem to be universal

- Cabibbo hypothesis: weak interactions of quarks have the same strength as the leptons, but the weak eigenstates of quarks differ from the mass eigenstates

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$
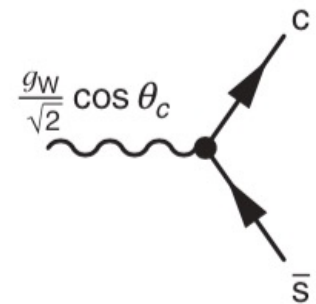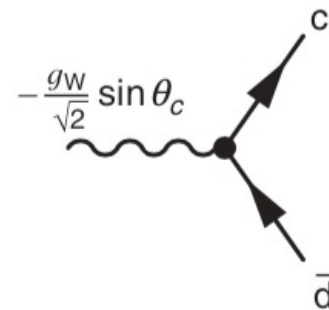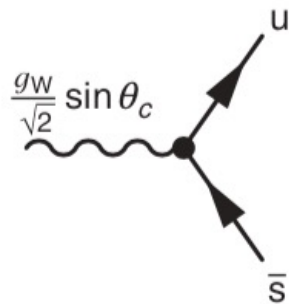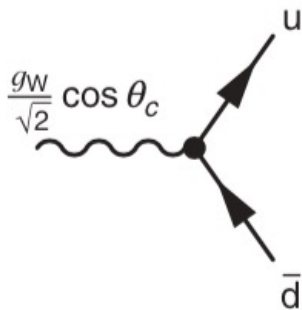
- The differences in the decay rates can be explained by a Cabibbo angle of 13º

- The Cabibbo mechanism can be extended to three generations (the charm had not been discovered when Cabibbo proposed his hypothesis):

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

which is known as the Cabibbo-Kobayashi-Maskawa (CKM) matrix

- The weak charged current vertices are then:

$$-i\frac{g_W}{\sqrt{2}} \left(\bar{u}, \bar{c}, \bar{t}\right) \gamma^{\mu} \tfrac{1}{2}(1 - \gamma^5) \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$
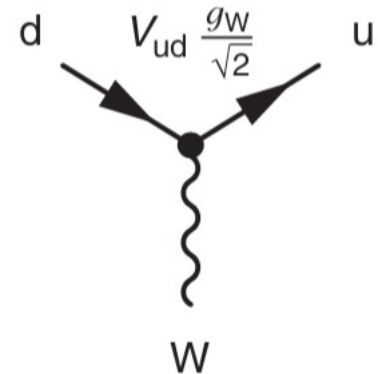
- If the down-type quark is the adjoint spinor in the matrix element, the CKM element is the complex conjugate
- The CKM matrix is unitary and can be written in terms of 3 rotation angles and a complex phase

$$V_{\text{CKM}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \times \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta'} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta'} & 0 & c_{13} \end{pmatrix} \times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

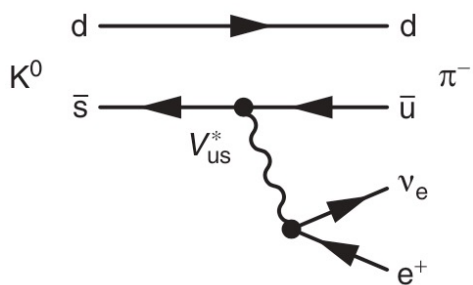$$s_{ij} = \sin \phi_{ij}$$
$$c_{ij} = \cos \phi_{ij}$$
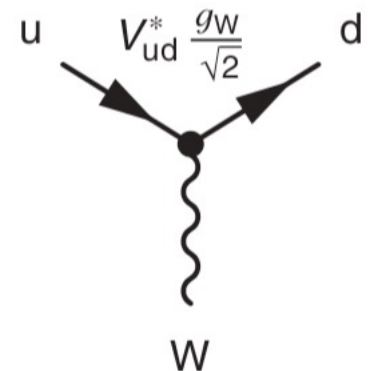
- From nuclear β decay:

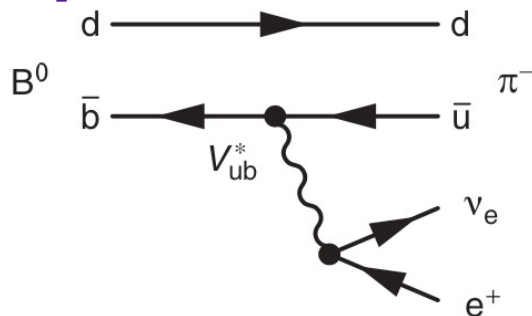$$|V_{\mathrm{ud}}| = \cos\theta_c = 0.974\ 25(22)$$

- From $K^0 \to \pi^- e^+ \nu_e$ decay:
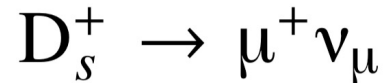
$$|V_{\mathrm{us}}| = 0.225\ 2(9)$$

- Similarly, from B meson decays:
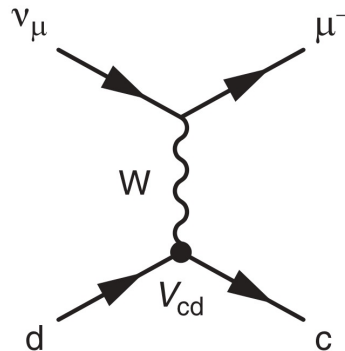
$$|V_{\mathrm{ub}}| = (4.15 \pm 0.49) \times 10^{-3}$$

- From D meson decays : $|V_{cs}| = 1.006 \pm 0.023$

  $$D_s^+ \rightarrow \mu^+ \nu_\mu$$

- From semi-leptonic B meson decays:

  $$|V_{cb}| = (40.9 \pm 1.1) \times 10^{-3}$$

- From neutrino-nucleon scattering:

  $$|V_{cd}| = 0.230(11)$$

- For the top:

  $$|V_{td}| = (8.4 \pm 0.6) \times 10^{-3} \qquad |V_{ts}| = (42.9 \pm 2.6) \times 10^{-3}$$

- The CKM matrix is unitary, so:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1$$

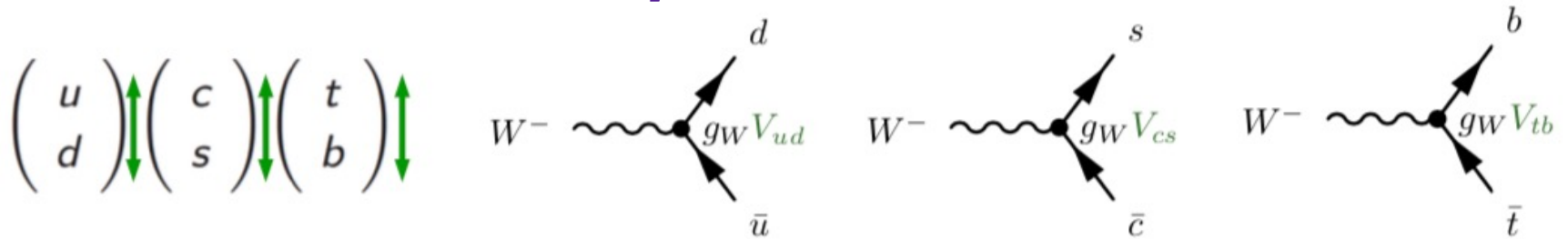$$|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1$$

- Summarising:

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.225 & 0.004 \\ 0.225 & 0.973 & 0.041 \\ 0.009 & 0.040 & 0.999 \end{pmatrix}$$

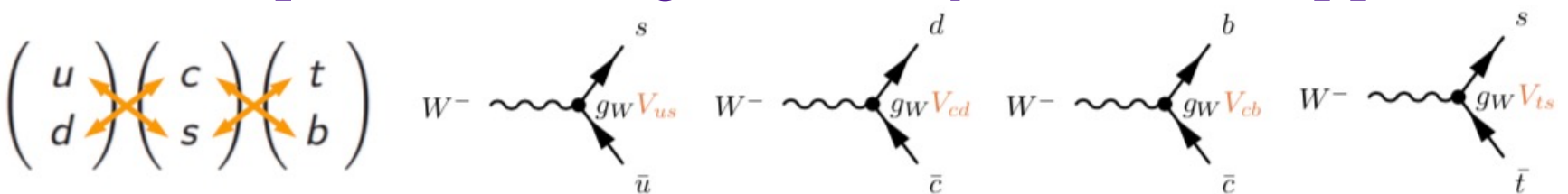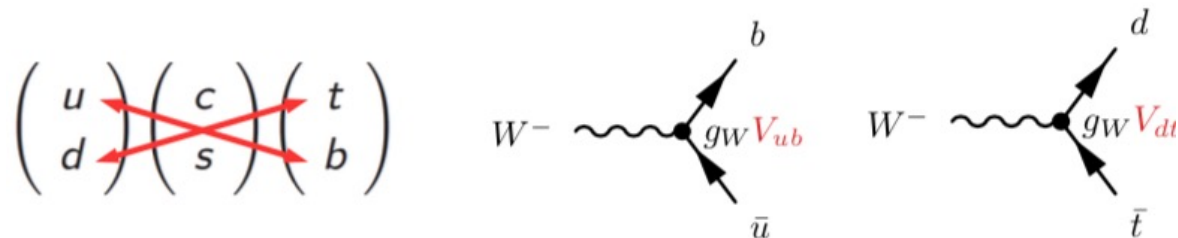$$\phi_{12} = 13° \qquad \phi_{23} = 2.3° \qquad \phi_{13} = 0.2°$$

- Between the same family: Cabibbo allowed

- Between quarks differing one family: Cabibbo suppressed
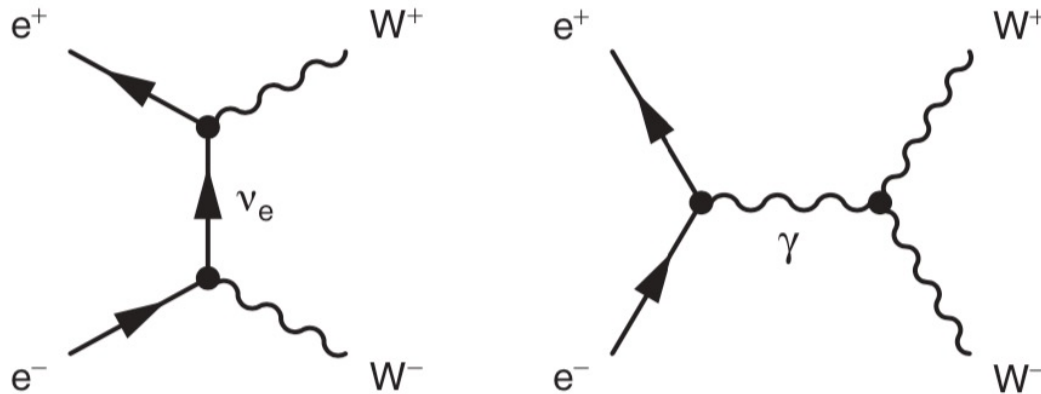

- Between quarks differing two families: Doubly Cabibbo suppressed

- Since W bosons are charged (EM charge), they couple to the photon
- W pairs can be produced in electron-positron colliders or hadron colliders

- However: the cross section diverges if only these diagrams are considered
- An additional gauge boson: the neutral Z boson

- Since W bosons are charged (EM charge), they couple to the photon
- W pairs can be produced in electron-positron colliders or hadron colliders

- An additional gauge boson: the neutral Z boson

$$|\mathcal{M}_\nu + \mathcal{M}_\gamma + \mathcal{M}_Z|^2 < |\mathcal{M}_\nu + \mathcal{M}_\gamma|^2$$

- Since W bosons are charged (EM charge), they couple to the photon
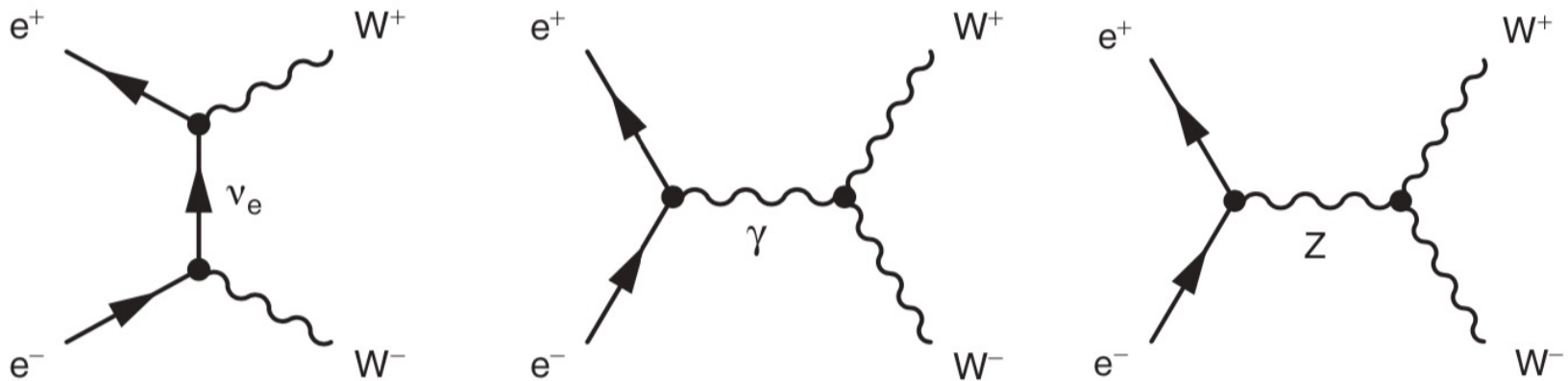- W pairs can be produced in electron-positron colliders or hadron colliders
- An additional gauge boson: the neutral Z boson

$$|\mathcal{M}_\nu + \mathcal{M}_\gamma + \mathcal{M}_Z|^2 < |\mathcal{M}_\nu + \mathcal{M}_\gamma|^2$$
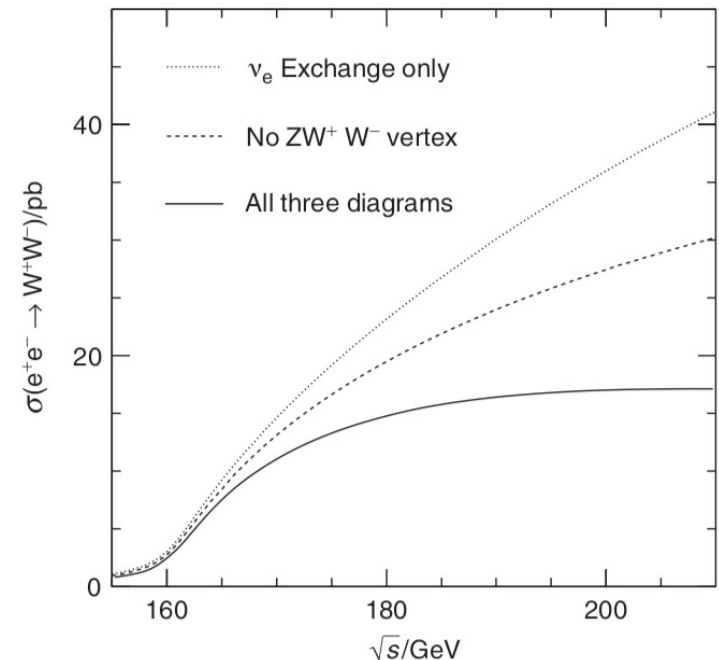
- This only works if the couplings are all related:
  Electroweak Unification

- QED and QCD are associated to U(1) and SU(3) local gauge symmetries
- The charged weak interaction can be associated with an SU(2) local gauge symmetry:

$$\varphi(x) \rightarrow \varphi'(x) = \exp\left[ig_W\, \boldsymbol{\alpha}(x) \cdot \mathbf{T}\right] \varphi(x) \qquad \mathbf{T} = \tfrac{1}{2}\boldsymbol{\sigma}$$

- 3 gauge fields: W_1, W_2, W_3
- The wave function must be a doublet: $\varphi(x) = \begin{pmatrix} \nu_e(x) \\ e^-(x) \end{pmatrix}$

- However: the weak interaction couples only to LH particles and RH anti-particles
- So RH particles and LH anti-particles are put in singlets (remain unaffected by local gauge transformation)

- QED and QCD are associated to U(1) and SU(3) local gauge symmetries
- The charged weak interaction can be associated with an SU(2)$_L$ local gauge symmetry

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L, \quad \begin{pmatrix} u \\ d' \end{pmatrix}_L, \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L, \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

$$e_R^-, \quad \mu_R^-, \quad \tau_R^-, \quad u_R, \quad c_R, \quad t_R, \quad d_R, \quad s_R, \quad b_R$$

- We get an interaction term:

$$ig_W T_k \gamma^\mu W_\mu^k \varphi_L = ig_W \frac{1}{2}\sigma_k \gamma^\mu W_\mu^k \varphi_L \qquad \varphi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

- The weak charged current corresponding to the exchange of the physical W bosons are:

$$j_{\pm}^{\mu} = \frac{1}{\sqrt{2}}\left(j_1^{\mu} \pm i j_2^{\mu}\right) = \frac{g_{\mathrm{W}}}{\sqrt{2}}\,\overline{\varphi}_L \gamma^{\mu} \tfrac{1}{2}(\sigma_1 \pm i\sigma_2)\varphi_L$$

$$= \frac{g_{\mathrm{W}}}{\sqrt{2}}\,\overline{\varphi}_L \gamma^{\mu} \sigma_{\pm} \varphi_L$$

- And the W bosons are linear combinations of W_1, W_2:

$$\mathrm{W}_{\mu}^{\pm} = \frac{1}{\sqrt{2}}\left(\mathrm{W}_{\mu}^{(1)} \mp i\mathrm{W}_{\mu}^{(2)}\right)$$

- Explicitly:

$$j_+^{\mu} = \frac{g_{\mathrm{W}}}{\sqrt{2}}\overline{\varphi}_L \gamma^{\mu}\sigma_+\varphi_L = \frac{g_{\mathrm{W}}}{\sqrt{2}}\begin{pmatrix}\overline{\mathrm{v}}_L & \overline{\mathrm{e}}_L\end{pmatrix}\gamma^{\mu}\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}\begin{pmatrix}\mathrm{v}_L\\ \mathrm{e}_L\end{pmatrix}$$

$$= \frac{g_{\mathrm{W}}}{\sqrt{2}}\overline{\mathrm{v}}_L \gamma^{\mu}\mathrm{e}_L \equiv \frac{g_{\mathrm{W}}}{\sqrt{2}}\overline{\mathrm{v}}\,\gamma^{\mu}\tfrac{1}{2}(1-\gamma^5)\mathrm{e}$$

$$j_-^{\mu} = \frac{g_{\mathrm{W}}}{\sqrt{2}}\overline{\varphi}_L \gamma^{\mu}\sigma_-\varphi_L = \frac{g_{\mathrm{W}}}{\sqrt{2}}\begin{pmatrix}\overline{\mathrm{v}}_L & \overline{\mathrm{e}}_L\end{pmatrix}\gamma^{\mu}\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}\begin{pmatrix}\mathrm{v}_L\\ \mathrm{e}_L\end{pmatrix}$$

$$= \frac{g_{\mathrm{W}}}{\sqrt{2}}\overline{\mathrm{e}}_L \gamma^{\mu}\mathrm{v}_L \equiv \frac{g_{\mathrm{W}}}{\sqrt{2}}\overline{\mathrm{e}}\,\gamma^{\mu}\tfrac{1}{2}(1-\gamma^5)\mathrm{v}$$

- The symmetry of the weak interaction results in the charged weak currents:

$$j_+^\mu = \frac{g_{\mathrm{W}}}{\sqrt{2}} \bar{\nu}\,\gamma^\mu \tfrac{1}{2}(1-\gamma^5)\mathrm{e} \qquad j_-^\mu = \frac{g_{\mathrm{W}}}{\sqrt{2}} \bar{\mathrm{e}}\,\gamma^\mu \tfrac{1}{2}(1-\gamma^5)\nu$$

  with the already familiar V-A structure

- The third gauge boson implies a neutral current:

$$j_3^\mu = g_{\mathrm{W}} \overline{\varphi}_L \gamma^\mu \tfrac{1}{2}\sigma_3 \varphi_L$$

$$j_3^\mu = g_{\mathrm{W}} \tfrac{1}{2} \begin{pmatrix} \bar{\nu}_L & \bar{\mathrm{e}}_L \end{pmatrix} \gamma^\mu \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \nu_L \\ \mathrm{e}_L \end{pmatrix}$$

$$= g_{\mathrm{W}} \tfrac{1}{2} \bar{\nu}_L \gamma^\mu \nu_L - g_{\mathrm{W}} \tfrac{1}{2} \bar{\mathrm{e}}_L \gamma^\mu \mathrm{e}_L$$

$$j_3^\mu = I_{\mathrm{W}}^{(3)} g_{\mathrm{W}} \bar{\mathrm{f}}\,\gamma^\mu \tfrac{1}{2}(1-\gamma^5)\mathrm{f} \qquad I_{\mathrm{W}}^{(3)} = \pm 1/2$$

- At this point it would be natural to assume the weak neutral current W_3 corresponds to the Z boson
- However: the Z boson does couple to RH particles and LH anti-particles
- Also, W and Z bosons would have the same coupling strength – not seen experimentally
- The solution: Unify QED and the weak force – Electroweak model

- The Glashow, Weinberg and Salam model: EM and weak interactions as different manifestations of a single unified electroweak force (Nobel Prize 1979)
- The local U(1) symmetry of QED is replaced by a local $U(1)_Y$ symmetry (weak hypercharge):

$$\psi(x) \rightarrow \psi'(x) = \hat{U}(x)\psi(x) = \exp\left[ig'\frac{Y}{2}\zeta(x)\right]\psi(x)$$

$$g'\frac{Y}{2}\gamma^{\mu}B_{\mu}\psi$$

- The photon and the Z boson can be written as combinations of the neutral gauge bosons:

$$A_{\mu} = +B_{\mu}\cos\theta_{W} + W_{\mu}^{(3)}\sin\theta_{W}$$

$$Z_{\mu} = -B_{\mu}\sin\theta_{W} + W_{\mu}^{(3)}\cos\theta_{W}$$

- Underlying symmetry: $SU(2)_L \times U(1)_Y$
- Photon properties must be the same we found for QED – couplings are related:

$$e = g_W \sin\theta_W$$

- Z boson coupling:

$$g_Z = \frac{g_W}{\cos\theta_W} \equiv \frac{e}{\sin\theta_W \cos\theta_W}$$

- And the Z boson current:

$$j_Z^\mu = g_Z \left(c_L \bar{u}_L \gamma^\mu u_L + c_R \bar{u}_R \gamma^\mu u_R \right)$$

the Z boson couples to both LH and RH particles, but differently

- **Underlying symmetry:** $SU(2)_L \times U(1)_Y$
- **Analogous to the charged weak current, the Z boson current can be written as:**

$$j_Z^\mu = g_Z \bar{u} \gamma^\mu \left[c_L \tfrac{1}{2}(1 - \gamma^5) + c_R \tfrac{1}{2}(1 + \gamma^5) \right] u$$

$$= g_Z \bar{u} \gamma^\mu \tfrac{1}{2} \left[(c_L + c_R) - (c_L - c_R)\gamma^5 \right] u.$$

$$j_Z^\mu = \tfrac{1}{2} g_Z \bar{u} \left(c_V \gamma^\mu - c_A \gamma^\mu \gamma^5 \right) u$$
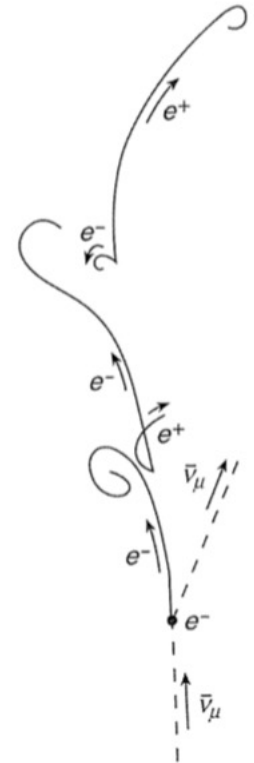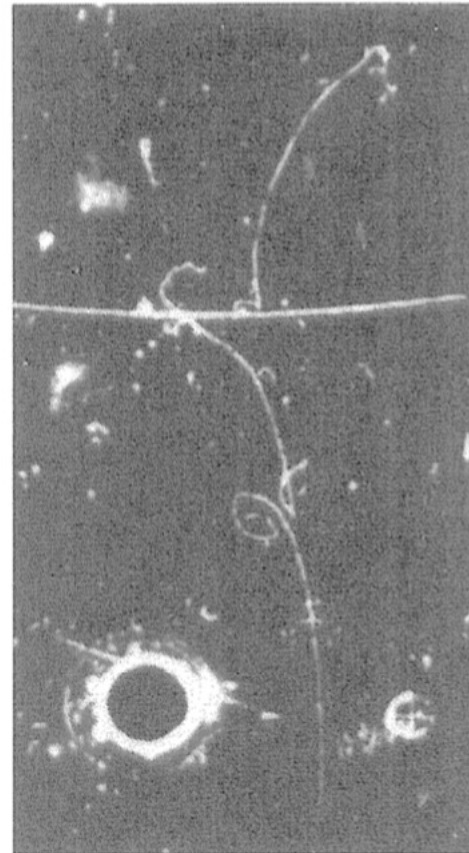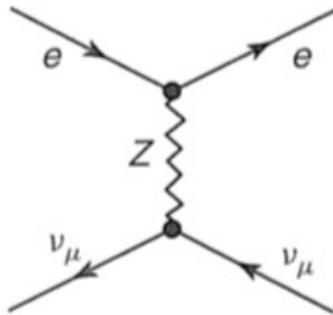
- **Z boson interaction vertex:**

$$-i\tfrac{1}{2} g_Z \gamma^\mu \left[c_V - c_A \gamma^5 \right]$$

f	c_V	c_A
ν_e, ν_μ, ν_τ	$\tfrac{1}{2}$	$\tfrac{1}{2}$
e^-, μ^-, τ^-	$-\tfrac{1}{2} + 2\sin^2\theta_w$	$-\tfrac{1}{2}$
u, c, t	$\tfrac{1}{2} - \tfrac{4}{3}\sin^2\theta_w$	$\tfrac{1}{2}$
d, s, b	$-\tfrac{1}{2} + \tfrac{2}{3}\sin^2\theta_w$	$-\tfrac{1}{2}$

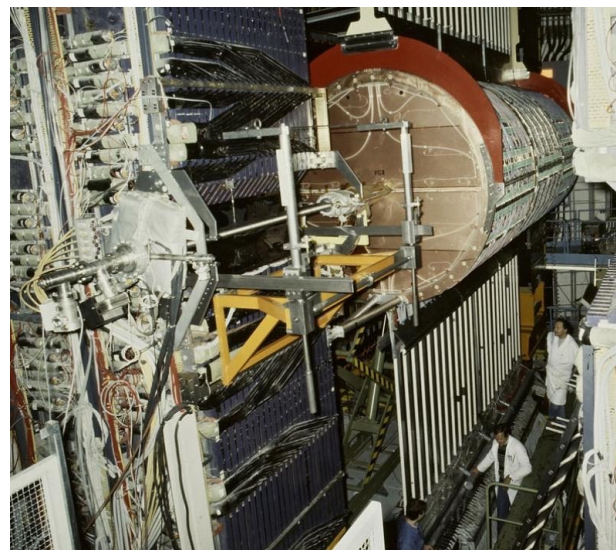- Discovery of Neutral Currents (1973): Only possible Feynman diagram (no $W^{\pm}$ diagram). Indirect evidence for Z.

- First direct observation in $p\bar{p}$ collisions at $\sqrt{s} = 540$ GeV via decays into $p\bar{p} \to Z + X$

- Precision measurements (LEP):

$$\sin^2 \theta_W = 0.23146 \pm 0.00012$$

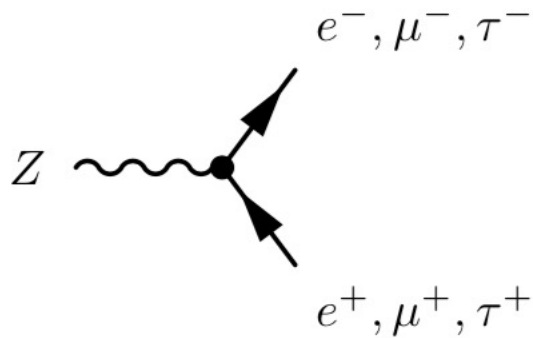$$\frac{\alpha}{\alpha_W} = \frac{e^2}{g_W^2} = \sin^2 \theta_W \sim 0.23$$

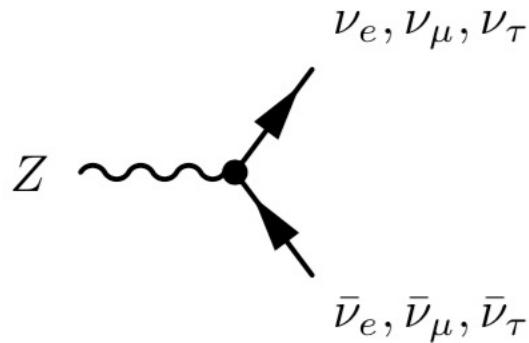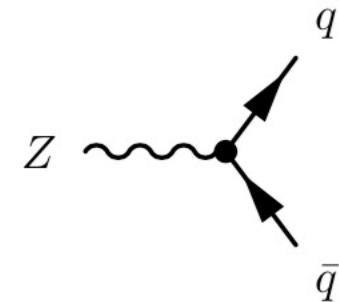UA1 Experiment at CERN Used Super Proton Synchrotron (now part of LHC)

$$Z \to e^+ e^-, \mu^+ \mu^-, \tau^+ \tau^- \qquad Z \to \nu_e \bar{\nu}_e, \nu_\mu \bar{\nu}_\mu, \nu_\tau \bar{\nu}_\tau \qquad Z \to q\bar{q}$$

- The Z boson does not change flavour
- The Z coupling is a mixture of EM and weak couplings – depends on $\sin \theta_W^2$

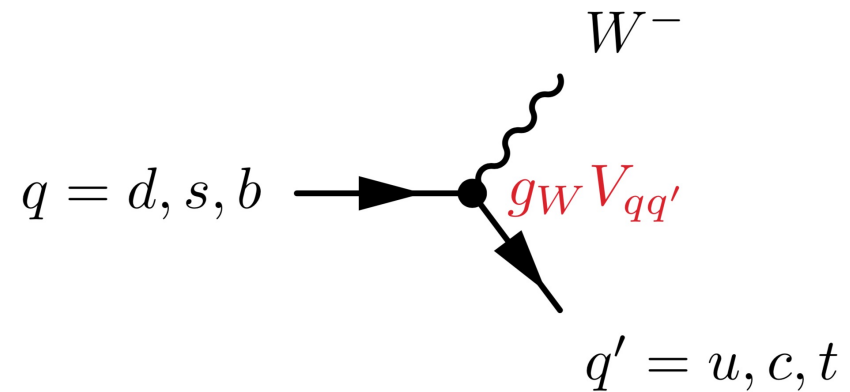So far <u>in the model</u> the gauge bosons remain massless

- The weak current has a V-A structure
- W boson coupling to leptons constant (lepton universality)

$$\frac{-ig_W}{\sqrt{2}} \frac{1}{2}\gamma^\mu(1-\gamma^5)$$

$$\alpha_W = \frac{g_W^2}{4\pi} = \frac{8m_W^2 G_F}{4\sqrt{2}\pi} \approx \frac{1}{30}$$

$$G_F = 1.166\,38 \times 10^{-5}\,\text{GeV}^{-2}$$

W^-

$q = d, s, b$ $g_W V_{qq'}$

$q' = u, c, t$

- W boson coupling to quarks depends on CKM matrix
- The W boson always changes quark flavour
- Mixing between different families can occur but less likely
- Parity and Charge conjugation is violated by the charged weak current: only couples to LH particles (RH anti-particles)
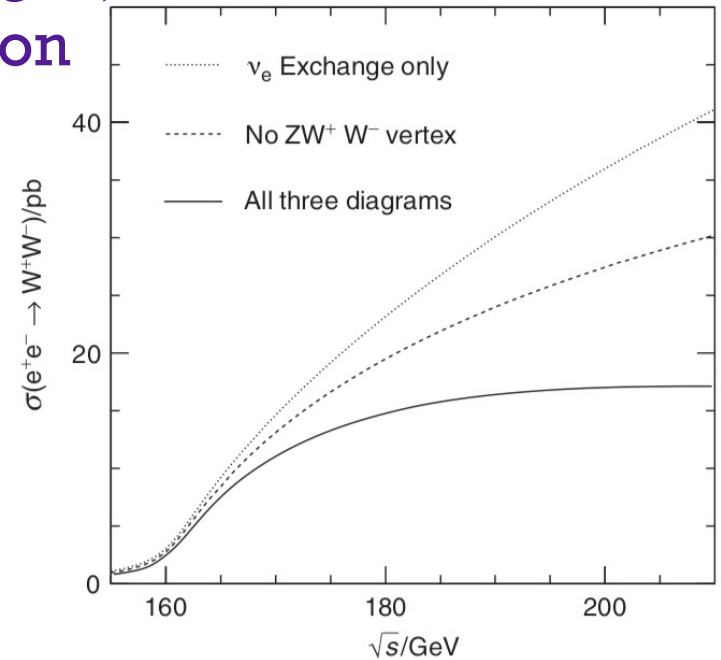
- The charged weak interaction alone does not fully explain W-pair production (cross section diverges)
- Neutral gauge boson needed: Z boson
- Unified $SU(2)_L \times U(1)_Y$ gauge theory
- One new parameter relates all couplings:
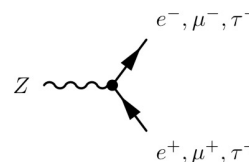
$$\sin^2 \theta_W = 0.23146 \pm 0.00012$$

$$\frac{\alpha}{\alpha_W} = \frac{e^2}{g_W^2} = \sin^2 \theta_W \sim 0.23$$

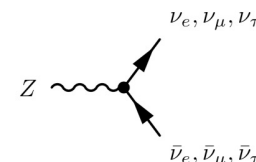$$g_Z = \frac{g_W}{\cos \theta_W} \equiv \frac{e}{\sin \theta_W \cos \theta_W}$$

$$-i\frac{1}{2} g_Z \gamma^\mu \left[c_V - c_A \gamma^5 \right]$$

ν_e Exchange only

No ZW^+W^- vertex

All three diagrams

$\sigma(e^+e^- \to W^+W^-)/\text{pb}$

$\sqrt{s}/\text{GeV}$

$Z \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-$

$Z \to \nu_e \bar{\nu}_e, \nu_\mu \bar{\nu}_\mu, \nu_\tau \bar{\nu}_\tau$

$Z \to q\bar{q}$