$$\boxed{S_{G.L}\{\phi\} = \int d\vec{r}\left[\frac{c}{2}(\vec{\nabla}\phi)^2 + \frac{a_2}{2}\phi^2 + \frac{a_4}{4}\phi^4 + \frac{a_6}{6}\phi^6 + \dots\right]}$$

$\to$ simetría $\mathbb{Z}_2$ $\quad \phi(\vec{r}) \to \begin{cases} \phi(\vec{r}) & \mathbb{I}_d \\ -\phi(\vec{r}) \end{cases}$

$$\boxed{a_2 = a\,t}\;; \quad t = \frac{T - T_c}{T_c} \begin{cases} t > 0 & T > T_c \\ t < 0 & T < T_c \end{cases}$$

el caso

$a_4 > 0$

$\to$ buscar el mínimo de $S(\phi)$:

$$Z = \int \mathcal{D}\{\phi\}\, e^{-S\{\phi\}}$$

$$I = \int_{-\infty}^{\infty} dx\, e^{-f(x)}$$

$f(x)$ tiene un mínimo en x_0

$\rightarrow e^{-f(x_0)}$ será el máximo.

$$\boxed{f(x) = f(x_0) + f'(x_0)\,\delta x + \frac{1}{2}\delta x^2 f''(x_0) + \dots}$$

$x = x_0 + \delta x$ $\qquad$ 0

$$I \simeq \int_{-\infty}^{\infty} d\delta x\, e^{-f(x_0) - \frac{1}{2}\delta x^2 f''(x_0)}$$

$$\simeq e^{-f(x_0)} \int_{-\infty}^{\infty} d\delta x\, e^{-\frac{1}{2}\delta x^2 f''(x_0)}$$

$$= \sqrt{\frac{2\pi}{f''(x_0)}}\, e^{-f(x_0)}$$

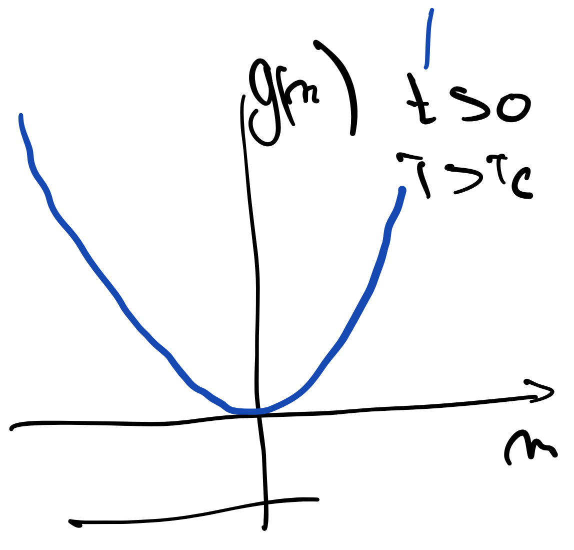
Sea $\phi_0(\vec{r})$ que minimiza $S[\phi]$

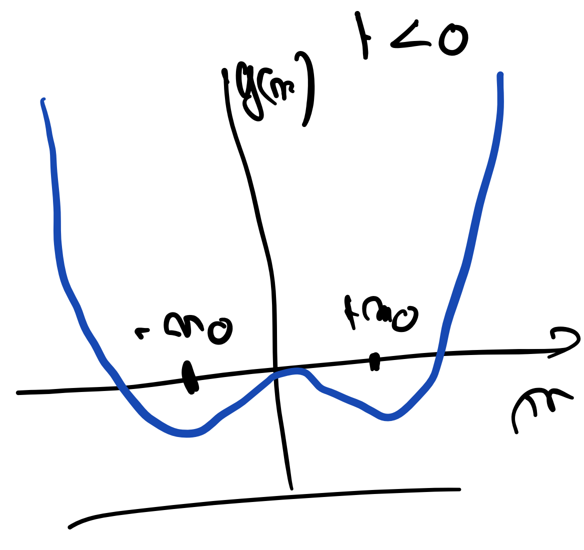
$\to$ para minimizar $(\vec{\nabla}\phi)^2$, $\phi = \text{cte} = m$

queda: $S[\phi_0] = V\left[\frac{a_2}{2}m^2 + \frac{a_4}{4}m^4 + \frac{a_6}{6}m^6 + \dots\right]$

$= V g(m)$

$\underline{\underline{a_4 > 0}}$, $a_6 > 0$, $a_2 = at \begin{cases} >0 \text{ si } t>0 \\ <0 \text{ si } t<0 \end{cases}$



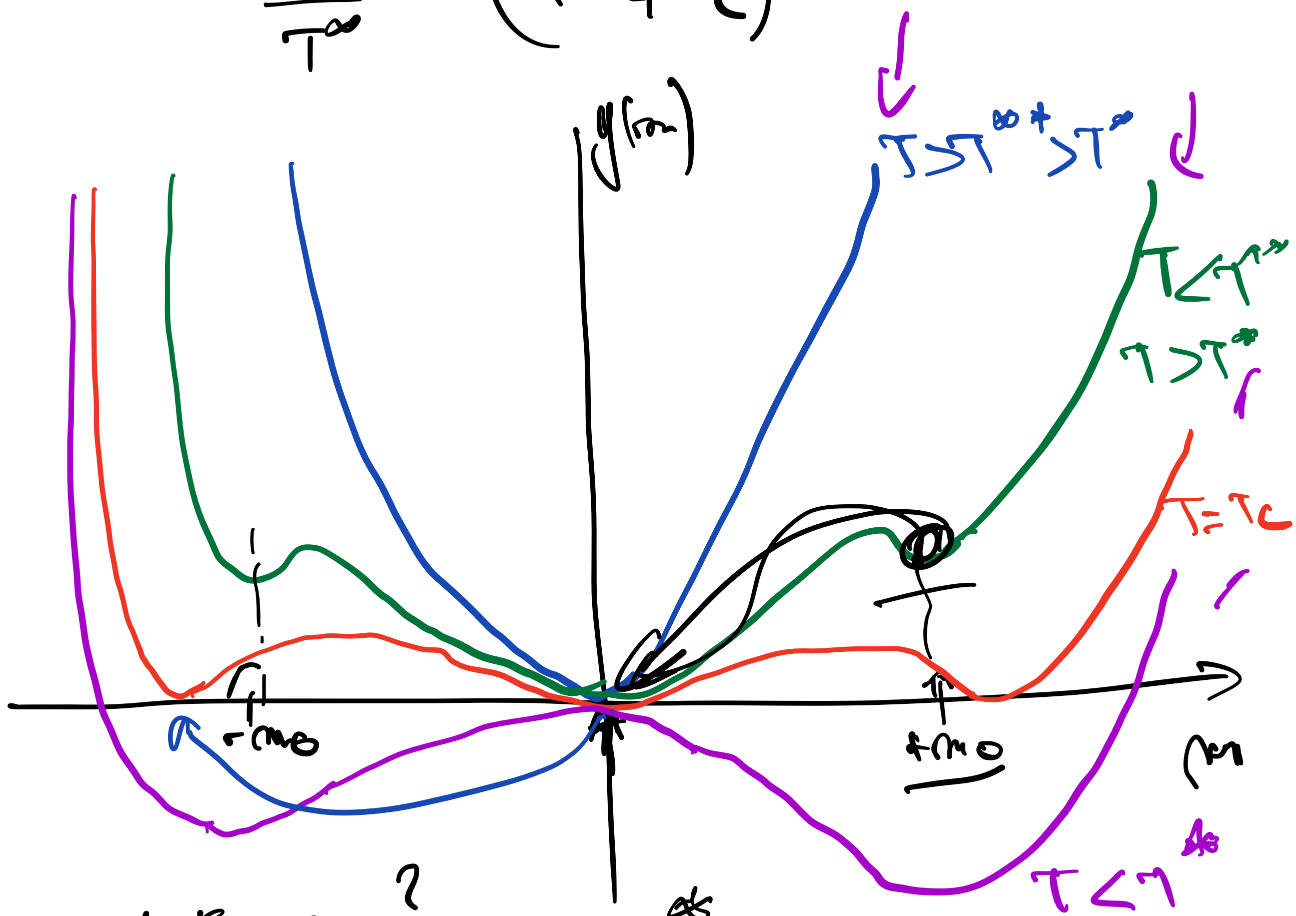


$$m_0 = \sqrt{\frac{-a_2}{a_4}} + \dots$$

ii: $a_4 < 0$, $a_6 > 0$

$$S\{\phi_0\} = V\, g(m) = V\left[\frac{at}{2} m^2 + \frac{a_4 m^4}{4} + \frac{a_6 m^6}{6}\right]$$

$$t = \frac{T - T^*}{T^*} \quad (T^* \neq T_c)$$



$$T^{**} = \frac{3 a_4^2}{4 a a_6} > T^*$$

$$T_c = T^* + \frac{3 a_4^2}{16 a a_6}$$

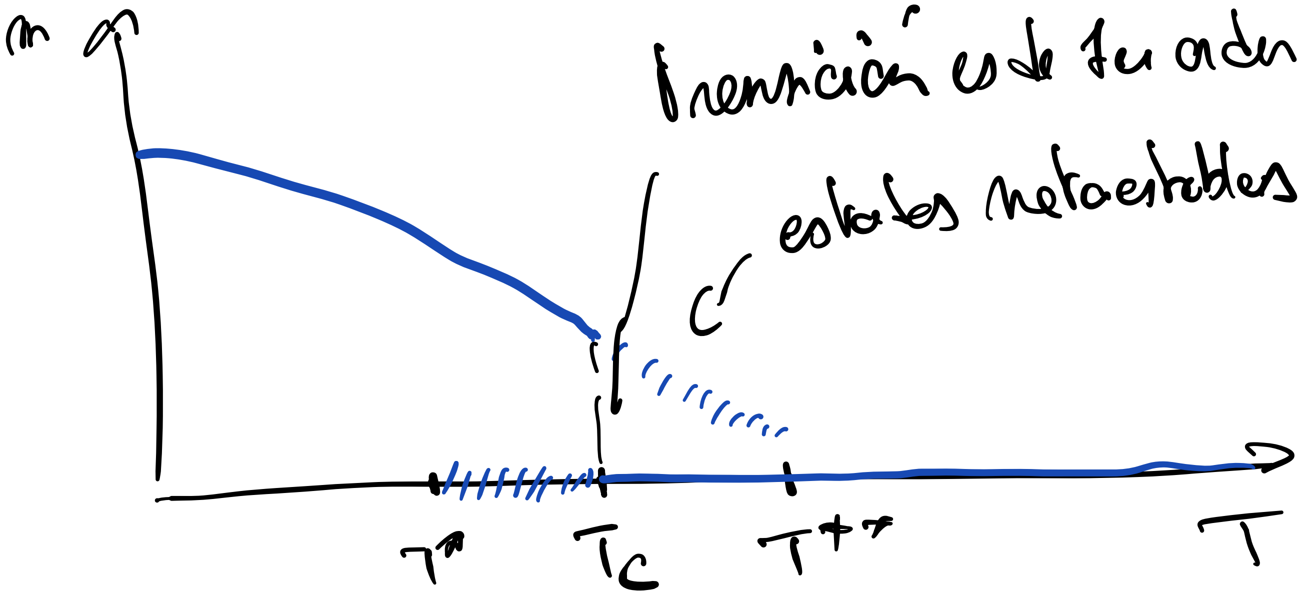
si $T > T^{**}$, el único mínimo está en $m=0$

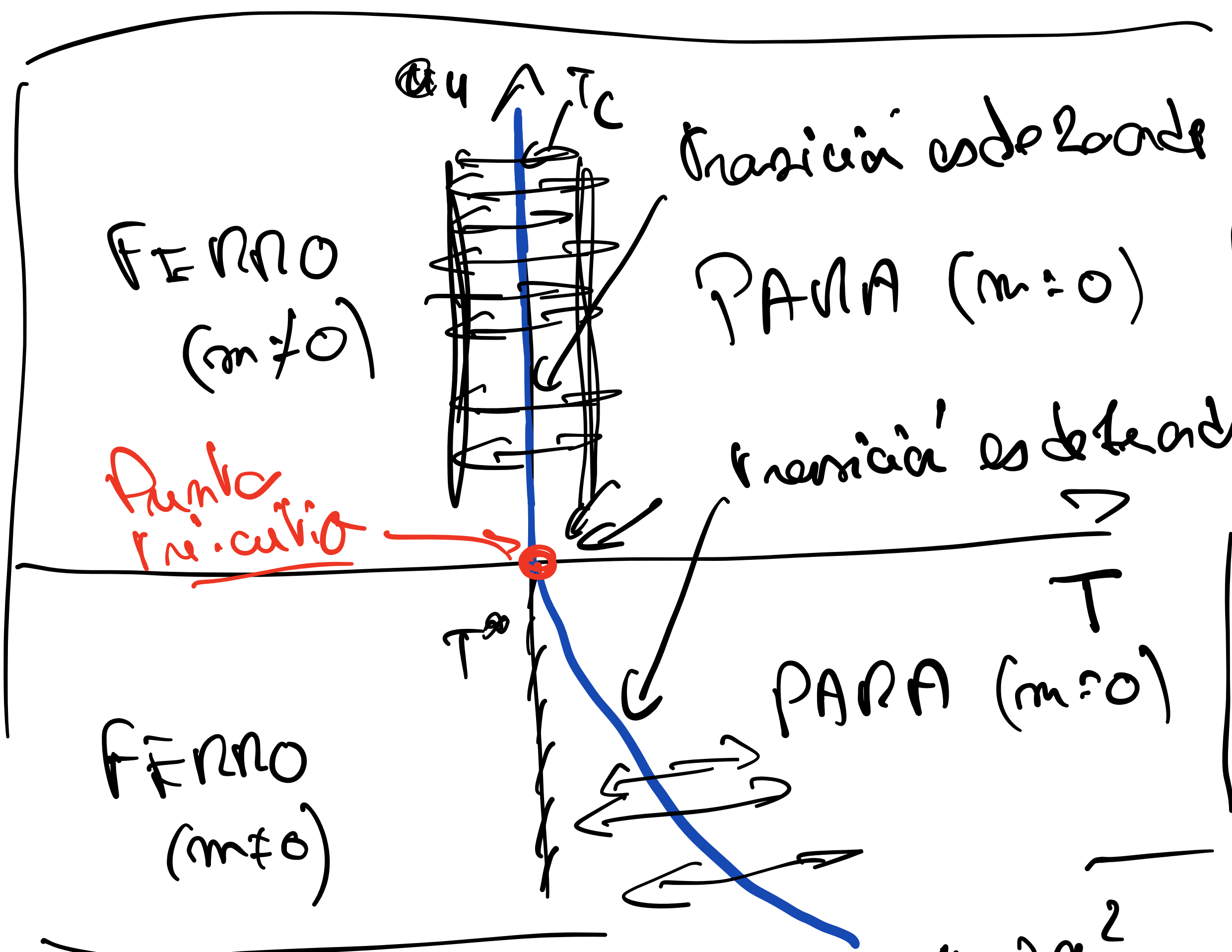
si $T^{**} > T > T_c$
→ el mínimo absoluto está en $m=0$, pero hay mínimos en $\pm m_0$ → estados metaestables

si $T_c > T_c > T^*$
los mínimos absolutos son para $\pm m_0$, y $m=0$ estado metaestable.

si $T < T^*$, solo hay mínimos en $\pm m_0$

Transición es de 1er orden.





$$T_c = T^* + \frac{3a_4^2}{16 a a_6}$$

Aproximación gaussiana:

$a_4 > 0$

el mínimo era para $\phi = m = \begin{cases} 0 & \text{si } t > 0 \\ \pm\sqrt{\frac{-a}{a_4}} & \text{si } t < 0 \end{cases}$

$\phi(\vec{r}) = m + \delta\phi$

$a_2 > 0 \; (t > 0, T > T_c)$

$$S[\delta\phi] \doteq \int d\vec{r} \left[\frac{c}{2} (\nabla \delta\phi)^2 + \frac{a_2}{2} \delta\phi^2 + \dots \right]$$

$\frac{a_2}{c}$ tiene dimensiones $\frac{1}{\text{distancia}^2}$

$\frac{a_2}{c} = \frac{1}{\xi^2}$ ξ: longitud de correlación

obs $\xi \to \infty$ si $t \to 0$ (o $T \to T_c$)

$a_2 = at = \frac{a}{T_c}(T - T_c)$

$\xi \sim |T - T_c|^{-1/2}$; $\boxed{\nu = \frac{1}{2}}$

si $T < T_c$ $(a_2 < 0)$ $m = \pm m_0 = \pm\sqrt{\frac{-a_2}{a_4}}$

$\phi(\vec{r}) = m_0 + \delta\phi(\vec{r})$

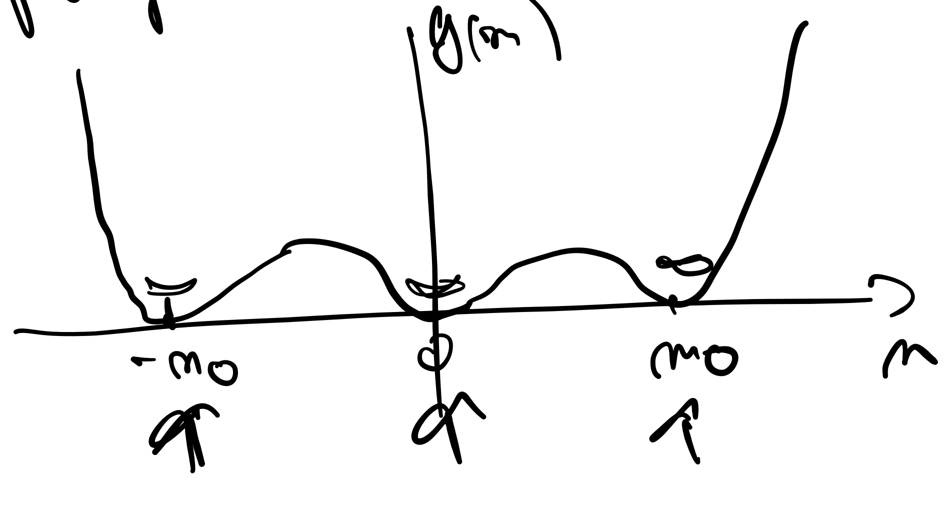
$$S\{\delta\phi\} = \int d\vec{r} \left[\frac{c}{2} (\vec{\nabla}\delta\phi)^2 \underbrace{-a_2}_{>0} \delta\phi^2 + \dots \right]$$

$$\frac{-a_2}{c} = \frac{1}{\xi^2} \quad , \xi \sim |t|^{-\frac{1}{2}} \quad \boxed{\nu = \frac{1}{2}}$$

$$m_0 \sim \sqrt{\frac{-a_2}{a}} \sim \sqrt{|t|} \Rightarrow \boxed{\beta = \frac{1}{2}}$$

si $a_4 < 0$

Pongámonos en $T = T_c$



$$\mu_0 = \frac{d^2 g}{dm^2}\Big|_{m=0} \quad , \quad \mu_m = \frac{d^2 g}{dm^2}\Big|_{m=\pm m_0}$$

$$\phi(\bar{r}) = \begin{cases} 0 + \delta\phi(\bar{r}) \\ \pm m_0 + \delta\phi(\bar{r}) \end{cases}$$

$$S[\delta\phi] = \int d\bar{r} \left[\frac{c}{2}(\bar{\nabla}\delta\phi)^2 + \frac{\mu_{0/m}}{2}\delta\phi^2 \right]$$

μ_0 y $\mu_m \neq 0$ en $T = T_c$

dos longitudes de correlación

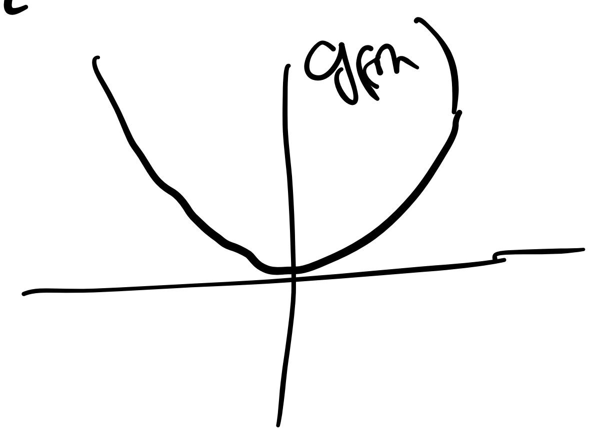
$$\xi_0 = \sqrt{\frac{c}{\mu_0}} \qquad \xi_m = \sqrt{\frac{c}{\mu_m}}$$

finitas en T_c

Por último: el punto tri-crítico $a_4 = 0$

$$S\{\phi\} = \int d\vec{x} \left[\frac{c}{2} (\vec{\nabla}\phi)^2 + \underbrace{\frac{a_2}{2}\phi^2 + \frac{a_6}{6}\phi^6 + \dots}_{} \right]$$

$a_2 = at$ si $t > 0$ $(T > T_c)$



$m = 0$ mínimo

si $t < 0$

dos mínimos, $m = \pm \left(\frac{-at}{a_6} \right)^{1/4}$

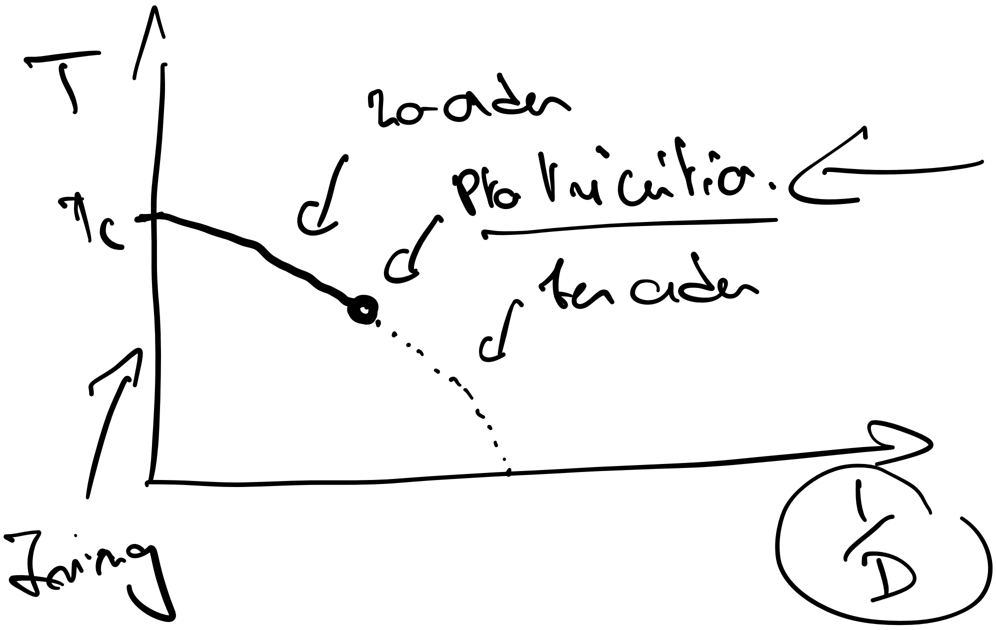
$\Rightarrow \beta = \frac{1}{4}$!

Ejemplo: modelo de Blume-Capel (1966)

$S_i = \pm 1, 0$ 2-D

$$H = -J \sum_{\langle i,j \rangle} S_i S_j - D \sum_i S_i^2$$

(obs $D \to \infty \to$ Ising)


T
T_c
2o orden
Pto tricritico.
1er orden
Ising
1/D