Módulo de Teoría: Teoría Cuántica de Campos

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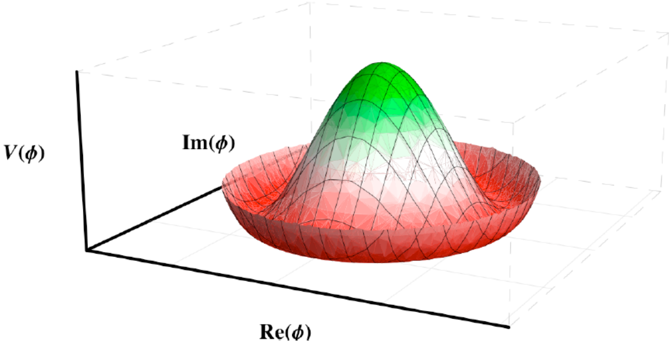
Latin American alliance for Capacity buildiNG in Advanced physics
LA-CoNGA physics

Cofinanciado por el programa Erasmus+ de la Unión Europea

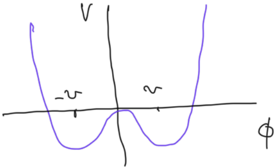
Ruptura espontánea de simetría

Mecanismo Higgs



Recap Clase 14

* ruptura espontánea de simetría : $\mathcal{L}$ invariante pero el estado de mínima energía no.



Ej. $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi)$, $V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4!}\phi^4$, $\mu^2 > 0$

simetría $\mathbb{Z}_2$: $\phi' = -\phi$, $\mathcal{L}' = \mathcal{L}$ rota por $\langle\phi\rangle = v = \sqrt{\frac{6}{\lambda}}\,\mu$

$\langle\phi\rangle$: valor de expectación de vacío (vev) de ϕ

$\phi(x) = \sigma(x) + v$, $\langle\sigma\rangle = 0$

ejercicio $m_\sigma = \sqrt{2}\,\mu$

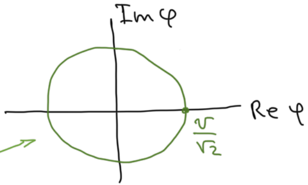
* ruptura de simetría global continua $\Rightarrow$ bosones de Goldstone
campos escalares de masa nula

Ej. $\mathcal{L} = \partial_\mu \varphi^* \partial^\mu \varphi - V(\varphi^*, \varphi)$, simetría $U(1)$ global : $\varphi' = e^{i\alpha}\varphi$

$$V(\varphi^*, \varphi) = -\mu^2 \varphi^* \varphi + \lambda(\varphi^* \varphi)^2 \quad , \mu^2 > 0$$

potencial del "sombrero mexicano"

mínimo en $|\varphi|^2 = \frac{\mu^2}{2\lambda} = \frac{v^2}{2}$



$$\varphi = e^{i\xi/v}\left(\frac{\eta + v}{\sqrt{2}}\right)$$

$\langle\varphi\rangle = \frac{v}{\sqrt{2}}$ rompe la simetría

$$\langle \xi \rangle = 0 \,, \quad \langle \eta \rangle = 0$$

ejercicio $\quad m_\eta = \sqrt{2}\,\mu \quad , \quad m_\xi = 0$

ξ es el bosón de Goldstone

* Teo. de Goldstone: por cada simetría interna global continua rota existe un bosón de Goldstone

* simetría local $U(1)$

$$\varphi' = e^{ig\chi(x)}\varphi$$

derivada covariante $\quad D_\mu\varphi = \partial_\mu\varphi + igB_\mu\varphi$

$$(D_\mu\varphi)' = e^{ig\chi(x)}D_\mu\varphi$$

$$B'_\mu = B_\mu - \partial_\mu\chi$$

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu = F'_{\mu\nu} \quad , \quad \mathcal{L}_B = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Grupos de simetría en el Modelo Estándar

$U(1) = \{ e^{i\alpha} \} \quad (e^{i\alpha})^{\dagger} e^{i\alpha} = 1$

1 parámetro

1 vector de calibre B_μ

grupo Abeliano: $e^{i\alpha} e^{i\beta} = e^{i\beta} e^{i\alpha}$

$SU(2) = \{ \text{matrices } U_{2\times 2}, U^{\dagger} U = \mathbb{1}, \det U = 1 \}$

3 parámetros

3 vectores de calibre

$W_\mu^1, W_\mu^2, W_\mu^3$

$$U = \begin{pmatrix} c_0 + i c_3 & c_2 + i c_1 \\ -c_2 + i c_1 & c_0 - i c_3 \end{pmatrix}, \quad c_0^2 + c_1^2 + c_2^2 + c_3^2 = 1$$

$$c_0, c_1, c_2, c_3 \in \mathbb{R}$$

no Abeliano

$SU(3) = \{ \text{matrices } U_{3\times 3}, U^{\dagger} U = \mathbb{1}, \det U = 1 \}$

8 parámetros

8 vectores de calibre

$G_\mu^1, G_\mu^2, \ldots, G_\mu^8$

no Abeliano

ruptura de simetría local $SU(2) \times U(1)$ en el Modelo Estándar

$W_\mu^1, W_\mu^2, W_\mu^3, B_\mu \longrightarrow W_\mu^\pm, Z_\mu$ masivos , A_μ masa nula

$$W_\mu^\pm = \frac{1}{\sqrt{2}} \left(W_\mu^1 \mp i W_\mu^2 \right)$$

$$Z_\mu = \cos\theta_W W_\mu^3 - \operatorname{sen}\theta_W B_\mu$$

$$A_\mu = \operatorname{sen}\theta_W W_\mu^3 + \cos\theta_W B_\mu$$

SU(2)

$$U = e^{i\alpha_a T_a} = e^{i(\alpha_1 T_1 + \alpha_2 T_2 + \alpha_3 T_3)} \, , \quad T_a = \frac{\sigma_a}{2}$$

matrices de Pauli

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \cos \frac{|\vec{\alpha}|}{2} \sigma_0 + i \,\mathrm{sen} \frac{|\vec{\alpha}|}{2} \hat{n} \cdot \vec{\sigma}$$

$$\vec{\alpha} = (\alpha_1, \alpha_2, \alpha_3) = |\vec{\alpha}| \, \hat{n}$$

T_a : generadores

$$[T_1, T_2] = iT_3 \, , \quad [T_2, T_3] = iT_1 \, , \quad [T_3, T_1] = iT_2$$

$$\boxed{[T_a, T_b] = i\epsilon_{abc} T_c}$$

$T_a = \frac{\sigma_a}{2}$ son los generadores en la representación doblete ó **2**

Existen otras representaciones

Ej. rep. adjunta $(T_a)_{bc} = -i\epsilon_{abc}$

$$T_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, T_2 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, T_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

transformaciones SU(2) locales

$$\Phi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} \text{ doblete} \quad , \quad \Phi' = e^{ig\chi_a(x) T_a} \Phi \quad , \quad T_a = \frac{\sigma_a}{2}$$

$$D_\mu \Phi = \partial_\mu \Phi + i g T_a W_\mu^a \Phi = \begin{pmatrix} \partial_\mu \varphi_1 \\ \partial_\mu \varphi_2 \end{pmatrix} + ig \begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & W_\mu^3 \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$$

$$(D_\mu \Phi)' = e^{ig\chi_a T_a} D_\mu \Phi$$

$$W'^a_\mu = W_\mu^a - \partial_\mu \chi^a + g\,\epsilon_{abc} W_\mu^b \chi^c \quad , \quad \chi^c \text{ infinitesimal}$$

$$T_a W'^a_\mu = U \left(T_b W_\mu^b + \frac{i}{g} U^{-1} \partial_\mu U \right) U^{-1} \quad , \quad U = e^{ig\chi^a T_a}$$

* para cualquier rep. con $[T_a, T_b] = i\epsilon_{abc}T_c$, no solo $T_a = \frac{\sigma_a}{2}$

$$\Phi' = e^{ig\chi^a T_a}\,\Phi \quad , \quad D_\mu \Phi = \partial_\mu \Phi + ig\,W_\mu^a T_a \Phi$$

$$(D_\mu \Phi)' = e^{ig\chi^a T_a} D_\mu \Phi \quad , \quad W_\mu'^{a} = W_\mu^a - \partial_\mu \chi^a + g\,\epsilon_{abc} W_\mu^b \chi^c$$

χ^c infinitesimal

* tensor de campo $\quad F_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\,\epsilon_{abc} W_\mu^b W_\nu^c$

Lagrangiano invariante: $\quad \mathcal{L}_W = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu\, a}$ $\quad$ Yang-Mills SU(2)

* se generaliza a SU(3), etc, $\quad \epsilon_{abc} \rightarrow f_{abc}$

$$[T_a, T_b] = i f_{abc} T_c$$

Mecanismo Higgs

P.W. Higgs (1964)

F. Englert, R. Brout (1964)

G.S. Guralnik, C.R. Hagen, T.W. Kibble (1964)

Mecanismo Higgs U(1)

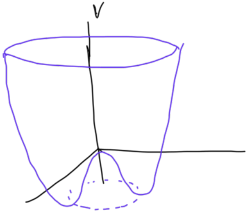
$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \varphi)^* (D^\mu \varphi) + \mu^2 (\varphi^* \varphi) - \lambda (\varphi^* \varphi)^2$$

$$D_\mu \varphi = \partial_\mu \varphi + i g B_\mu \varphi \quad , \quad F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$\mathcal{L}$ es invariante bajo transformaciones U(1) locales

$$\varphi' = e^{ig\chi(x)} \varphi \quad , \quad B'_\mu = B_\mu - \partial_\mu \chi$$

$$V = -\mu^2 \varphi^* \varphi + \lambda (\varphi^* \varphi)^2 \quad , \quad \mu^2 > 0$$



mínimo en $|\varphi|^2 = \frac{v^2}{2} = \frac{\mu^2}{2\lambda}$

$$\langle \varphi \rangle = \frac{v}{\sqrt{2}} \quad , \quad \varphi = e^{i\xi/v} \frac{(v + \eta)}{\sqrt{2}}$$

$$V = \underbrace{\lambda v^2 \eta^2}_{\frac{1}{2} m_\eta^2 \eta^2} + \lambda v \eta^3 + \frac{\lambda}{4}\eta^4 + \text{const}$$

$$m_\eta = \sqrt{2\lambda}\, v = \sqrt{2}\,\mu$$

$$D_\mu \varphi = \frac{1}{\sqrt{2}} e^{i\xi/v}\left[\partial_\mu \eta + i\left(g B_\mu + \frac{1}{v}\partial_\mu \xi\right)(\eta + v)\right]$$

$$(D_\mu \varphi)^* D^\mu \varphi = \frac{1}{2} g^2 (\eta + v)^2 \underbrace{\left(B_\mu + \frac{1}{gv}\partial_\mu \xi\right)\left(B^\mu + \frac{1}{gv}\partial^\mu \xi\right)}_{\text{incluye } B_\mu \partial^\mu \xi} + \frac{1}{2}\partial_\mu \eta\, \partial^\mu \eta$$

¡Pero se puede hacer una transformación de calibre!

$$B'_\mu = B_\mu - \partial_\mu \chi = B_\mu + \frac{1}{gv}\partial_\mu \xi \,, \quad \text{escogiendo} \quad \chi = -\frac{1}{gv}\xi$$

$$F'_{\mu\nu} = F_{\mu\nu}$$

$$(D_\mu \varphi)^* D^\mu \varphi = \frac{1}{2} g^2 (\eta + v)^2 B'_\mu B'^\mu + \frac{1}{2} \partial_\mu \eta \partial^\mu \eta$$

$$= \underbrace{\frac{1}{2} g^2 v^2 B'_\mu B'_\mu}_{} + \dots$$

B'_μ es masivo, tiene $m_B = g v$

* $$\mathcal{L} = -\frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} + \frac{1}{2} g^2 v^2 B'_\mu B'^\mu + \frac{1}{2} \partial_\mu \eta \partial^\mu \eta - \frac{1}{2} (2\lambda v^2) \eta^2$$
$$+ v g^2 \eta B'_\mu B'^\mu + \frac{1}{2} g^2 \eta^2 B'_\mu B'^\mu - \lambda v \eta^3 - \frac{1}{4} \lambda \eta^4 + \text{const.}$$

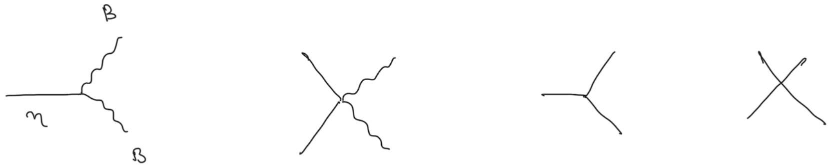
* bosón de Goldstone ξ desaparece de $\mathcal{L}$, en vez aparece B'_μ masivo

B'_μ se "come" a ξ.

ξ proporciona la polarización longitudinal de B'_μ

$$B'_\mu = B_\mu + \frac{1}{g v} \partial_\mu \xi$$

* interacciones



* Al hacer la transformación $B'_\mu = B_\mu - \partial_\mu \chi$, $\chi = -\frac{1}{gv}\xi$.
Se escoge el llamado calibre unitario, en el cual los campos en $\mathcal{L}$ corresponden a partículas físicas. En calibre unitario

$$\varphi' = e^{ig\left(-\frac{1}{gv}\xi\right)}\varphi = e^{-i\frac{\xi}{v}}\, e^{i\frac{\xi}{v}}\, \frac{(\eta+v)}{\sqrt{2}}$$

$$\varphi' = \frac{(\eta+v)}{\sqrt{2}}$$

Mecanismo Higgs en el Modelo Estándar

doblete Higgs $\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$, φ^+, φ^0 campos escalares complejos

Φ transforma bajo $SU(2) \times U(1)_Y$ $\quad$ Y : hipercarga

$$\Phi' = e^{i \frac{g}{2} \chi^a \sigma_a + \frac{i}{2} g' \chi} \Phi \, , \quad (D_\mu \Phi)' = e^{i \frac{g}{2} \chi^a \sigma_a + \frac{i}{2} g' \chi} D_\mu \Phi$$

$$D_\mu \Phi = \partial_\mu \Phi + i g \frac{\sigma_a}{2} W_\mu^a \Phi + \frac{i}{2} g' B_\mu \Phi$$

en las convenciones usadas, Φ tiene $Y=1$

$SU(2)$: constante de acoplamiento g , campos de calibre $W_\mu^1, W_\mu^2, W_\mu^3$

$U(1)_Y$: constante de acoplamiento g' , campo de calibre B_μ

$$\mathcal{L}_{Higgs} = (D_\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi^\dagger, \Phi)$$

$$V(\Phi^\dagger, \Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 , \qquad \mu^2 > 0$$

ejercicio mínimo en $\Phi^\dagger \Phi = |\varphi^+|^2 + |\varphi^0|^2 = \frac{v^2}{2}$, $v^2 = \frac{\mu^2}{\lambda}$

$$\langle \Phi \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$ rompe la simetría

calibre unitario $\Phi(x) = \begin{pmatrix} 0 \\ \frac{v + h(x)}{\sqrt{2}} \end{pmatrix}$ $h(x)$ campo escalar real

$$\Phi^\dagger \Phi = \frac{1}{2}(v+h)^2 \quad \Rightarrow \quad V = \lambda v^2 h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4 + \text{const.}$$

$$D_\mu \Phi = \partial_\mu \Phi + i g \frac{\sigma_a}{2} W_\mu^a \Phi + \frac{i g'}{2} B_\mu \Phi$$

$$D_\mu \Phi = \begin{pmatrix} \partial_\mu + \frac{i g'}{2} B_\mu + \frac{i g}{2} W_\mu^3 & \frac{i g}{2}(W_\mu^1 - i W_\mu^2) \\ \frac{i g}{2}(W_\mu^1 + i W_\mu^2) & \partial_\mu + \frac{i g'}{2} B_\mu - \frac{i g}{2} W_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$D_\mu \Phi = \begin{pmatrix} \frac{ig}{2\sqrt{2}}\left(W_\mu^1 - iW_\mu^2\right)(v+h) \\ \frac{1}{\sqrt{2}}\partial_\mu h + \frac{i}{2\sqrt{2}}\left(g' B_\mu - g W_\mu^3\right)(v+h) \end{pmatrix}$$

$$(D_\mu \Phi)^\dagger (D_\mu \Phi) = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{8} g^2 (v+h)^2 \left(W_\mu^1 + iW_\mu^2\right)\left(W^{1\mu} - iW^{2\mu}\right)$$

$$+ \frac{1}{8}(v+h)^2 \left(g W_\mu^3 - g' B_\mu\right)\left(g W^{3\mu} - g' B^\mu\right)$$

$$\mathcal{L}_{Higgs} = \frac{1}{2}\partial_\mu h\, \partial^\mu h - \overbrace{\lambda v^2 h^2}^{-\frac{1}{2} m_h^2 h^2} - \lambda v h^3 - \frac{\lambda}{4} h^4$$

$$+ \frac{1}{8} g^2 (v+h)^2 (W_\mu^1 + i W_\mu^2)(W^{1\mu} - i W^{2\mu})$$

$$+ \frac{1}{8} (v+h)^2 (g W_\mu^3 - g' B_\mu)(g W^{3\mu} - g' B^\mu)$$

$$\boxed{m_h = \sqrt{2\lambda}\, v}$$

$h(x)$ es el bosón Higgs

Masas para vectores de calibre

$$\mathcal{L}_{Higgs} \supset \frac{1}{8} g^2 v^2 \left(W_\mu^1 + i W_\mu^2\right)\left(W^{1\mu} - i W^{2\mu}\right)$$

$$= \frac{1}{2} m_W^2 \left(W_\mu^1 W^{1\mu} + W_\mu^2 W^{2\mu}\right) = m_W^2 W_\mu^+ W^{-\mu}$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right)$$

$$\boxed{m_W = \frac{1}{2} g v}$$

$$\mathcal{L}_{Higgs} \supset \frac{1}{8} v^2 (g W^3_\mu - g' B_\mu)(g W^{3\mu} - g' B^\mu)$$

$$= \frac{1}{8} \frac{v^2 g^2}{\cos^2\theta_W} Z_\mu Z^\mu = \frac{1}{2} m_Z^2 Z_\mu Z^\mu$$

$$\boxed{\frac{g'}{g} = \tan\theta_W}$$

$$Z_\mu = \cos\theta_W W^3_\mu - \operatorname{sen}\theta_W B_\mu$$

$$\boxed{m_Z = \frac{v g}{2\cos\theta_W}}$$

$$A_\mu = \operatorname{sen}\theta_W W^3_\mu + \cos\theta_W B_\mu$$

$$\boxed{m_A = 0}$$

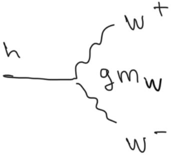
A_μ es el fotón

Acoplamientos h, $W^{\pm}_\mu$, Z_μ

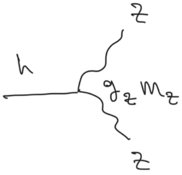
$$\mathcal{L}_{Higgs} \supset \frac{g^2}{8\cos^2\theta_W}(2vh+h^2)Z_\mu Z^\mu$$

$$+\frac{1}{4}g^2(2vh+h^2)W^+_\mu W^{-\mu}$$

$\Rightarrow$ acoplamientos cúbicos proporcionales a masas



$h \to W^+W^-$



$h \to ZZ$

$$g_Z = \frac{g}{\cos\theta_W}$$

Derivada covariante redux

Con hipercarga Y genérica

$$D_\mu = \partial_\mu + ig\frac{\sigma_a}{2}W_\mu^a + \frac{ig'}{2}Y B_\mu$$

Ejercicio

$$D_\mu = \partial_\mu + \frac{ig}{\sqrt{2}}\left(T^+W_\mu^+ + T^-W_\mu^-\right) + \frac{ig}{\cos\theta_W}\left(T_3 - \operatorname{sen}^2\theta_W Q\right)Z_\mu + ieQA_\mu$$

$$T^+ = \frac{1}{2}(\sigma_1 + i\sigma_2) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$T^- = \frac{1}{2}(\sigma_1 - i\sigma_2) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad T_3 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

$$Q = T_3 + \frac{Y}{2}$$

$$e = g \operatorname{sen}\theta_W$$

Resumen 1

grados de libertad

Antes	gdl	Después	gdl
φ^+, φ^-	$2+2$	h	1
$W_\mu^1, W_\mu^2, W_\mu^3$	$2+2+2$	W_μ^+, W_μ^-	$3+3$
B_μ	2	Z_μ	3
		A_μ	2
	12		12
$SU(2)\times U(1)_Y$		$U(1)_{EM}$	

Resumen 2

$$m_W = \frac{1}{2} g v, \qquad m_Z = \frac{g v}{2\cos\theta_W}, \qquad m_h = \sqrt{2\lambda}\, v, \qquad e = g \,\mathrm{sen}\,\theta_W$$

dependen de g, g', λ, μ en $\mathcal{L}_{Higgs}$ $\qquad g' = g\,\mathrm{tg}\,\theta_W, \quad \mu^2 = \lambda v^2$

W, Z descubiertos en 1983, $\qquad h$ descubierto en 2012

$$m_W = 80.379 \text{ GeV}$$
$$m_Z = 91.1876 \text{ GeV}$$
$$\frac{g^2}{m_W^2} = 4\sqrt{2}\, G_F, \quad G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$$
$$m_h = 125.10 \text{ GeV}$$

$$\Longrightarrow \quad v = 246 \text{ GeV}, \quad \lambda = 0.13$$

Fin

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LA-CoNGA **physics**

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