Mecanismo de Higgs-Brout-Englert en materia condensada

* Ginzburg-Landau 1951
* Anderson 1963

S.C. descubierta en 1911
por K. Onnes

Superconductividad | Efecto Meissner

IRM | 1933

\# Teoría de G.L. para las transiciones de fase

→ la transición de fase a la S.C.

→ ruptura espontánea de una simetría

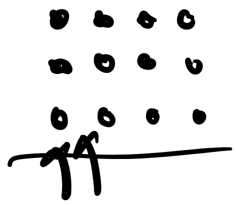
Efecto Meissner:

Los S.C. "repelen" las líneas de campo magnético. ($\vec{B} = \vec{0}$ en un S.C.)

→ levitación magnética.

ruptura espontanea de una sim.

hielo | agua fluida

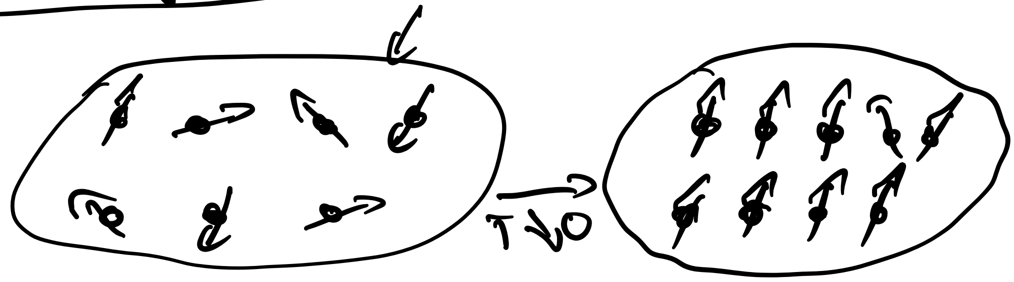


simetría de traslación

se rompe

Transiciones de fases:

Ferromagnetismo:

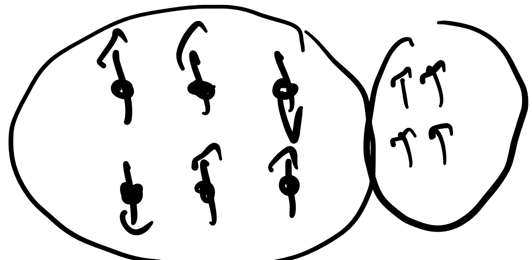


$\vec{M} = \vec{0}$ Paramagnética $\qquad$ $\vec{M} \neq \vec{0}$ Ferromagnética

$$F\{\vec{M}\} = \alpha \underbrace{(\vec{M}\cdot\vec{M})}_{\vec{M}^2} + \frac{\beta}{2}(\vec{M}\cdot\vec{M})^2 \qquad \vec{M} = \begin{pmatrix} M_1 \\ \vdots \\ M_n \end{pmatrix}$$

Simetría $O(n)$ $n \geq 2$

imaginemos el caso $n=1$



$\vec{M}(\vec{r})$

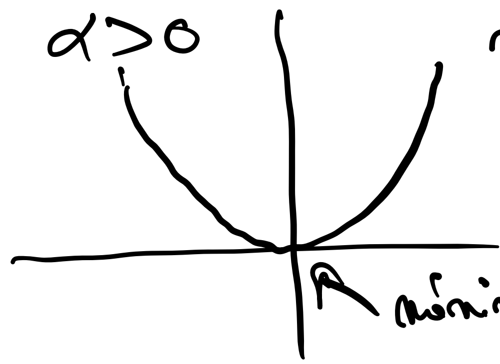
$M \in \mathbb{R}$

$$\boxed{F(M) = \alpha M^2 + \frac{\beta}{2} M^4}$$

Simetría $\begin{cases} M \to M \\ M \to -M \end{cases}$ $\mathbb{Z}_2$

obs $\beta > 0$!

$f(M)$ $\alpha > 0$ Paramag



mínimo $M = 0$

$\alpha < 0$ Ferro

mínimos $M = \pm\sqrt{\frac{-\alpha}{\beta}}$

Si M es inhomogénea

$\vec\nabla M \cdot \vec\nabla M$

$$F = \int_V d\vec r \left[c(\vec\nabla M)^2 + \alpha M^2 + \frac{\beta}{2} M^4 \right]$$

volumen del material

$c > 0$

teoría de G.L.

obs $M(\vec r) \longrightarrow \phi(\vec r)$

$\beta \longrightarrow \lambda$

Y ahora para los SC.?

$$F = \int d\vec{r} \left[\quad ? \quad \right]$$

para acoplar un campo a $\vec{B}$

→ campo complejo.

$$\vec{M}(\vec{r}) \longrightarrow \underline{\psi(\vec{r})}$$

$$F = \int_V d\vec{r} \left[c \underset{\uparrow}{|\vec{\nabla}\psi|^2} + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 \right]$$

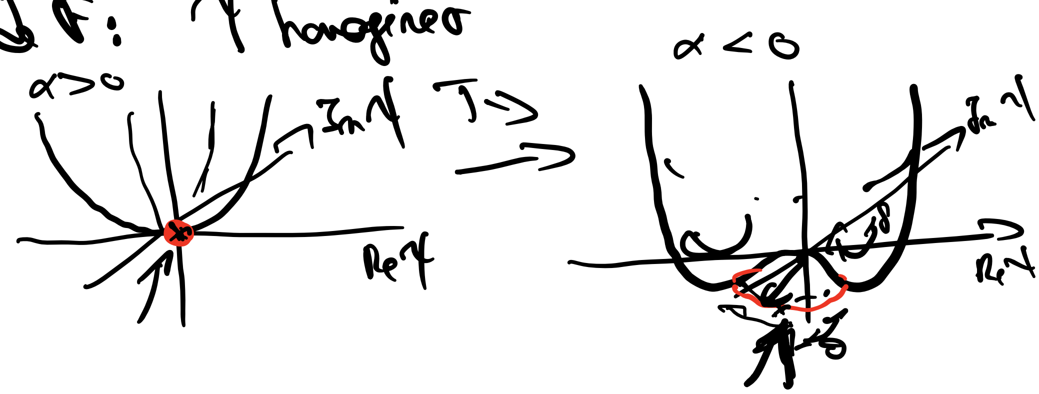
$$\vec{\nabla}\psi^* \cdot \vec{\nabla}\psi$$

Simetría : $\psi \to e^{i\varphi x}\psi$

→ grupo $U(1)$

$$F = \int d\vec{r} \left[\frac{\hbar^2}{2m} |\vec{\nabla}\psi|^2 + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 \right]$$

Mínimo de F: ψ homogéneo



si $\alpha<0$ ψ_{min} est.q: $|\psi_{min}| = \sqrt{\frac{-\alpha}{\beta}}$

$\alpha = \alpha_0(T-T_c)$ T_c temperatura crítica.

Approx cuadrática

$$\boxed{\psi(\vec r) = e^{i\theta(\vec r)}\left[\rho_0 + \delta(\vec r)\right]}$$

con $\rho_0 = \begin{cases} 0 & \text{si } \alpha>0 \\ \sqrt{\frac{-\alpha}{\beta}} & \text{si } \alpha<0 \end{cases}$

$$F \simeq \int d\vec r \left[\frac{\hbar^2 \rho_0^2}{2m}|\nabla\theta|^2 + \frac{\hbar^2}{2m}|\nabla\delta(\vec r)|^2 + \gamma\,\delta(\vec r)^2 + \dots\right]$$

$\gamma>0 \begin{cases} \alpha>0 \\ \alpha<0 \end{cases}$ | $\gamma = \begin{cases} \alpha & \text{si } \alpha>0 \\ -2\alpha & \text{si } \alpha<0 \end{cases}$

$$F = \int d\vec q \left[|\hat\theta(\vec q)|^2 \underline{\frac{\hbar^2\rho_0^2}{2m}\vec q^{\,2}} + |\hat\delta(\vec q)|^2 \left[\frac{\hbar^2}{2m}\vec q^{\,2} + \gamma\right] + \dots\right]$$

función de correlación:

$$\langle \delta(\vec 0)\,\delta(\vec r)\rangle \sim e^{-\frac{r}{\xi}} \qquad \xi = \sqrt{\frac{\hbar^2}{2m\gamma}}$$

longitud de coherencia del S.C.

el campo θ no tiene escala asociada.

$\Rightarrow$ el campo/modo de Goldstone:

$$\mathcal{E}_\delta(\vec q \to \vec 0) \neq 0$$

$$E_0(\vec{\Psi} \to \vec{0}) \to 0$$

Efecto Meissner y ecuaciones de London-Landau (oxford)

→ acople a un $\vec{B}$

$$F_{\vec{B}} = \frac{1}{8\pi}\int d\vec{r}\, \vec{B}(\vec{r})^2 \qquad \left(= \frac{1}{8\pi}\int F_{\mu\nu}F^{\mu\nu}\right)$$

$$\vec{\nabla}\cdot\vec{B} = \vec{0} \qquad \Rightarrow \vec{B} = \vec{\nabla}\times\vec{A} \quad \text{inv}$$

$$\vec{A} \to \vec{A} + \vec{\nabla}\Omega$$

$$F = \int d\vec{r} \left\{ \frac{\hbar^2}{2m}\left|\left[\vec{\nabla} - \frac{iq}{\hbar}\vec{A}(\vec{r})\right]\Psi(\vec{r})\right|^2 + \alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 + \frac{1}{8\pi}\vec{B}^2 \right\}$$

$$\begin{cases} \vec{A} \to \vec{A} + \vec{\nabla}\Omega(\vec{r}) \\ \vec{B} \to \vec{B} \\ \Psi \to e^{\frac{iq}{\hbar}\Omega(\vec{r})}\Psi \end{cases}$$

$$\frac{\delta F}{\delta \Psi^*}$$

$$\to \boxed{-\frac{\hbar^2}{2m}\left[\vec{\nabla} - \frac{iq}{\hbar}\vec{A}\right]^2\Psi(\vec{r}) + \alpha\Psi(\vec{r}) + \beta|\Psi|^2\Psi(\vec{r}) = 0} \quad (1)$$

$$\frac{\delta F}{\delta \bar{A}} = \bar{0}$$

$$\bar{\nabla} \times \bar{B} = 4\pi \bar{J}(\bar{r}) \quad (2)$$

$$\boxed{\bar{J}(\bar{r}) = \frac{-iq\hbar}{2m}\left[\psi^* \bar{\nabla}\psi - \psi \bar{\nabla}\psi^*\right] - \frac{q^2}{m}|\psi|^2 \bar{A}(\bar{r})}$$

$$\psi(\bar{r}) = e^{i\theta(\bar{r})}\left(f_0 + f(\bar{r})\right)$$

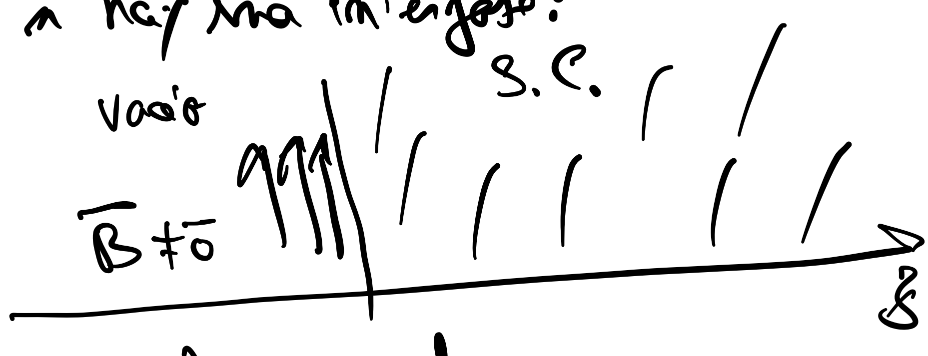
para que satisfaga a (1)

$$f(\bar{r}) = 0 \quad y \quad \bar{A} = \frac{\hbar}{q}\bar{\nabla}\theta(\bar{r})$$

gauge puro

$\rightarrow \bar{B} = \bar{0}$

Pero si hay una interfase:



en el s.c., tomamos el gauge

$$\psi(\bar{r}) = f_0$$

$$\Rightarrow \boxed{\vec{j}(\vec{r}) = -\frac{q^2}{m}\rho_0^2\, \vec{A}(\vec{r})}$$

$$\vec{\nabla}\cdot\vec{j} = 0 \Rightarrow \vec{\nabla}\cdot\vec{A} = 0 \quad \text{(gauge de London)}$$

ecuación de London.

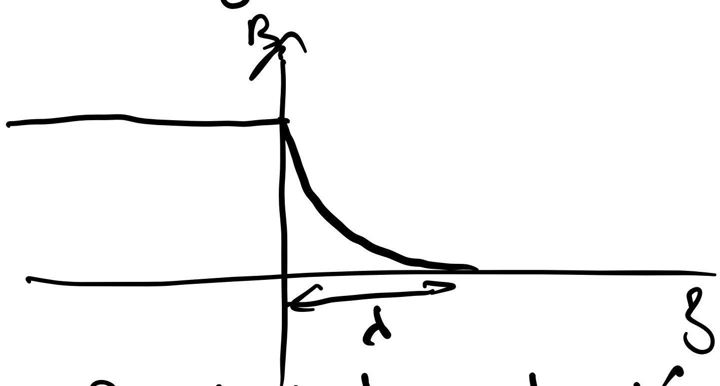
$$\vec{\nabla}\times\vec{B} = 4\pi\vec{j}$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = -\frac{4\pi q^2\rho_0^2}{m}\vec{A}(\vec{r})$$

$$\underbrace{\vec{\nabla}(\vec{\nabla}\cdot\vec{A})}_{0} - \Delta\vec{A} = -\frac{4\pi q^2\rho_0^2}{m}\vec{A}$$

$$\Rightarrow \left(\Delta - \frac{1}{\lambda}\right)\vec{A}(\vec{r}) = 0 \qquad A$$

$$\left(\frac{d^2}{dz^2} - \frac{1}{\lambda}\right)\vec{A}(\vec{r}) = 0 \Rightarrow \vec{A} = \vec{A}_0\, e^{-\frac{z}{\lambda}}$$



λ : longitud de penetración

L. longitudes :

$$\xi = \sqrt{\frac{2m\alpha}{\hbar^2}}$$

$$\lambda = \sqrt{\frac{m}{4\pi q^2 \psi_0^2}}$$

$$\alpha = \alpha_0 (T - T_c)$$

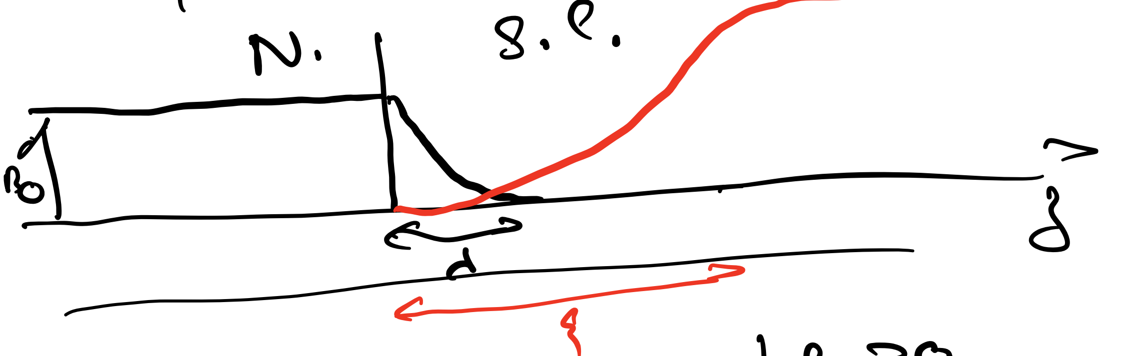
$\xi(T)$ y $\lambda(T) \sim \sqrt{(T_c - T)}$

$$\rightarrow \boxed{\kappa = \frac{\lambda}{\xi} = \frac{m}{q\hbar}\sqrt{\left(\frac{\beta}{2\pi}\right)}}$$

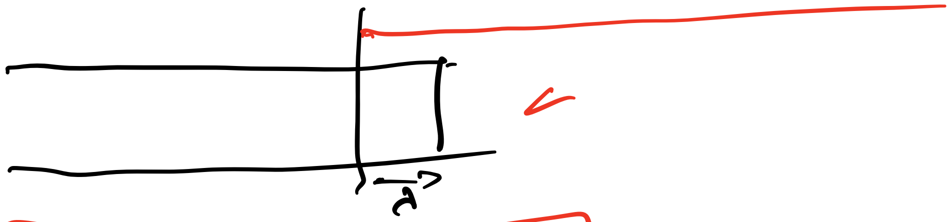
$\kappa > 1 \rightarrow$ Tipo II $\leftarrow$

$\kappa < 1 \rightarrow$ Tipo I $\leftarrow$

IV Termodinámica de los S.C. de tipo I y II.

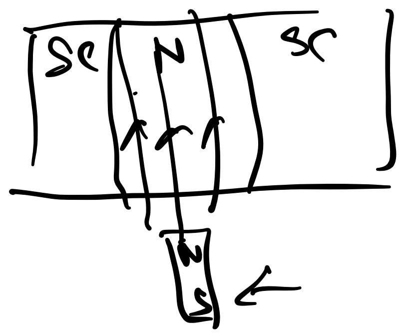


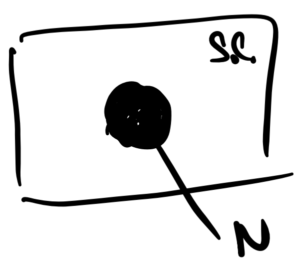
$z = 0 \;\Big|\; B = B_0,\; \psi = 0$ en $z \to \infty \;\Big|\; B \to 0,\; \psi \to \psi_0$



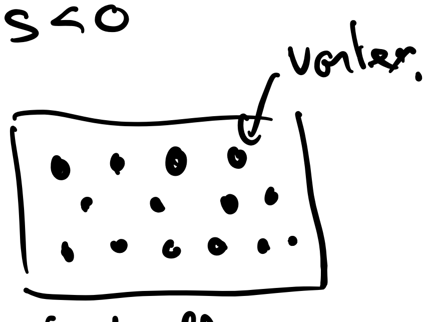
$$\boxed{E_s \simeq A \frac{B_0^2}{8\pi} \underline{(\xi - \lambda)}}$$

Tipo I $\underline{E_s > 0}$

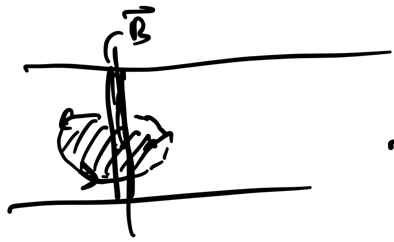




Tipo II $E_s < 0$



⇒ Cuantificación del flujo magnético.



$$\Phi_B = \iint_S d\vec{S}\cdot\vec{B} = \oint_C \vec{A}\cdot d\vec{\ell}$$

pero, en el Bulk $\vec{A} = \frac{\hbar}{q}\vec{\nabla}\theta$ $\qquad \psi = e^{i\theta}\rho_0$

$$\Phi_B = \frac{\hbar}{q}\oint_C \vec{\nabla}\theta\cdot d\vec{\ell} = \frac{\hbar}{q}\underbrace{\left(\theta(2\pi)-\theta(0)\right)}_{2\pi n}$$

$$\Rightarrow \Phi_B = \frac{h}{q}\,n$$ cuantificación del flujo

$-2e$

red de Abrikosov.