Buenas Tardes!

Módulo: Datos: Montecarlo / Dinámica Mol

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Montecarlo?

Clase general de algoritmos que dependen de 'muestreo' de #'s aleatorios → para resolver problemas deterministas

- Optimización, integración, Fluidos/gases difusión, sistemas con desorden

- Medidas de riesgo en la bolsa
- Descripción del clima y desastres naturales

Monte Carlo → Sala de juegos roullete

Ulan + Metropolis

Teorema del limite Central

Variable aleatoria X_i

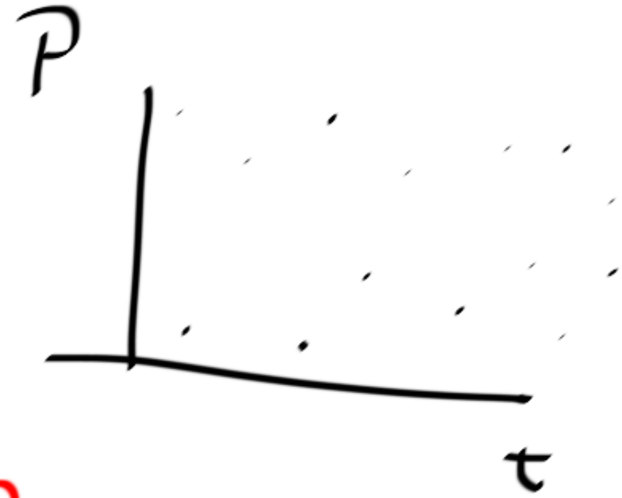
$$X = \sum_{i}^{N} X_i$$

$$? \; P(X) \qquad P(x_i)$$

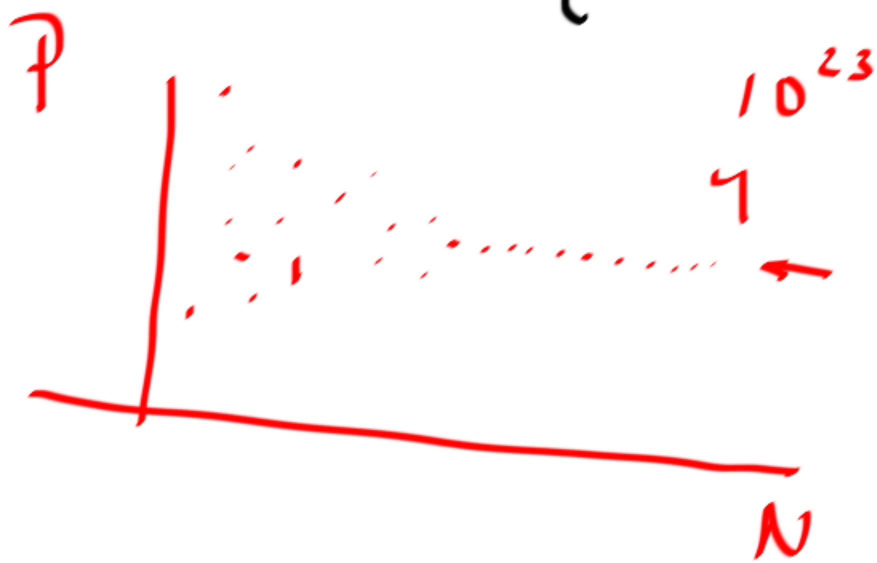
dist limite

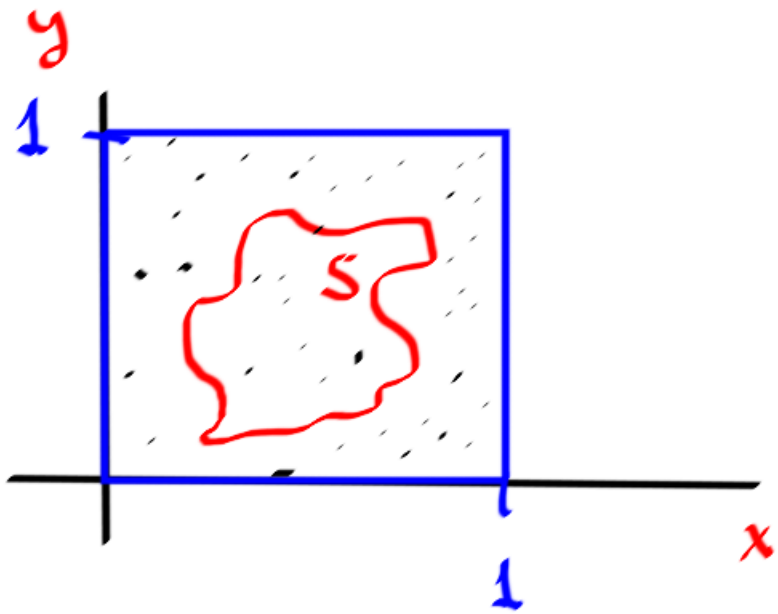
Gaussiana

$$\frac{Var(X)}{\langle X \rangle^2} \propto \frac{1}{N} \leftarrow$$



$$\frac{\mathrm{Var}(X)}{\langle X \rangle^2} \propto \frac{1}{N}$$





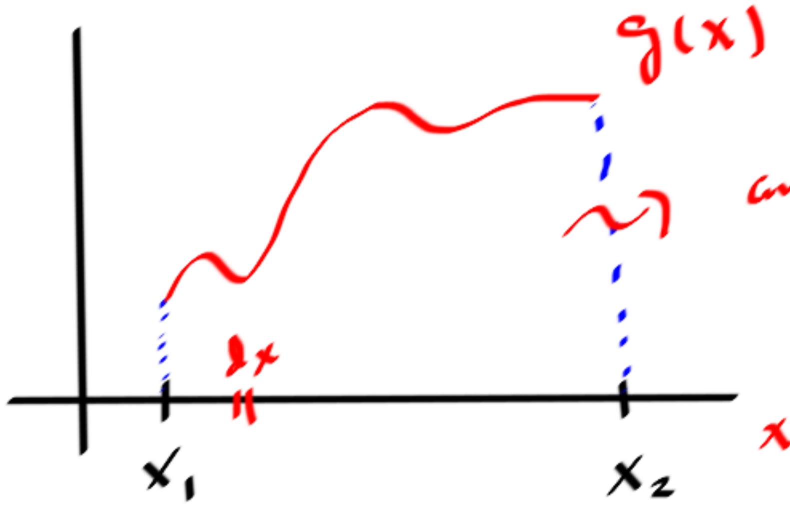
$A_s \leq 1 \cdot 1$

$A_s = \lim_{N_{TOT}\to\infty} \frac{N'}{N_{TOT}}$

$=$ area de S

importancia de dist. de #'s aleatorios

para de #'s aleatorios $\Rightarrow$ se encuentran uniformemente dist. en el int $[0,1]$



área bajo la curva

$$I = \int_{x_1}^{x_2} g(x)\,dx =$$

Supongamos una variable aleatoria distribuida como $P_\xi(x)$

ξ → variable aleatoria

$$\eta = \frac{g(x)}{P_\xi(x)}$$

$$\langle \eta \rangle = \int_a^b \frac{g(x)}{P_\xi(x)} \cdot P_\xi(x)\, dx = I$$

$$\underbrace{\qquad}_{=}$$

$$P\left\{ \left| \frac{1}{N} \sum_{j}^{N} \eta_j - I \right| < 3\sqrt{\frac{D_\eta}{N}} \right\} \xrightarrow[N \to \infty]{} 1$$

$$\frac{1}{\sqrt{N}}$$

$$\boxed{\frac{1}{N} \sum_{j=1}^{N} \frac{g(\xi_j)}{P_\xi(\xi_j)} \simeq I}$$

$$I = \int_0^{\pi/2} \operatorname{Sen} x \, dx = 1$$

$$= -\cos x \Big|_0^{\pi/2} =$$

$$= -(0-1) = 1$$

Evaluemos las bondades de

1) $P_\xi(x) =$ uniforme

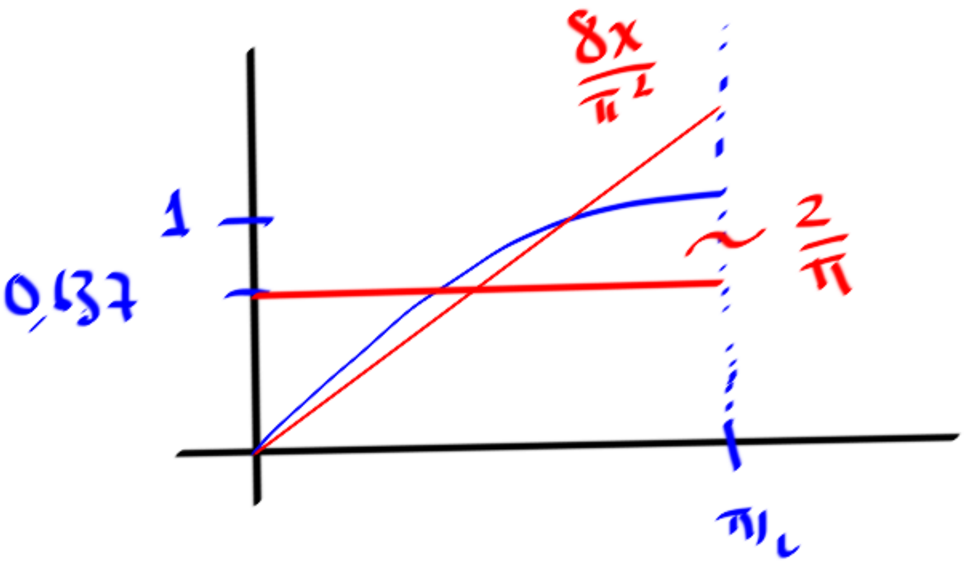
2) $P_\xi(x) =$ lineal

1) $P_\xi(x) = \dfrac{1}{\pi/2} = \dfrac{2}{\pi}$ $\Big\|$ $\displaystyle\int_0^{\pi/2} \frac{2}{\pi} dx = \frac{2}{\pi}\frac{\pi}{2} = 1$

2) $P_{r'}(x) = \frac{8x}{\pi^2}$

$$\int_0^{\pi/2} P_{r'}(x)\,dx = \frac{8}{\pi^2}\int_0^{\pi/2} x\,dx = \frac{8}{\pi^2}\left.\frac{x^2}{2}\right|_0^{\pi/2}$$

$$= \frac{8}{\pi^2}\left(\frac{1}{2}\left(\frac{\pi}{2}\right)^2 - 0\right) = \frac{8}{\pi^2}\cdot\frac{1}{2}\cdot\frac{\pi^2}{4} = 1$$



Approx elect...

1) $$I = \int_0^{\pi/2} \operatorname{sen} x \, dx \to \frac{1}{N} \sum_{j=1}^{N} \frac{g(\xi_j)}{p(\xi_j)}$$ unif.

$$I = \frac{1}{N} \frac{\pi}{2} \sum_{j=1}^{N} \operatorname{sen} \xi_j$$

uniform $\xi = \frac{2}{\pi} x$

$$\int_0^{\pi/2} \frac{\sin x}{P_\xi(x)} P_\xi \, dx = \int_0^{\pi/2} \frac{\sin x}{P_\xi(x)} \underbrace{\frac{2}{\pi} dx}_{d\left(\frac{2}{\pi}x\right)}$$

$\xi = \frac{2}{\pi} x$

$x = \frac{\pi}{2} \xi$

$$= \int_0^1 \frac{\sin\left(\frac{\pi}{2}\xi\right)}{2/\pi} \, d\xi \qquad \underbrace{}_{\xi}$$

$$I = \frac{1}{N} \sum_j \frac{\sin \xi_j}{P_{\xi_j}} =$$

$$I = \frac{1}{N}\frac{\pi}{2}\sum_{j}^{N} \sin \frac{\pi}{2} x_j$$ uniform 0 − 1

N = 10 $\quad I = 0.952$

2) $P_j = 8x/\pi^2$

$$\Rightarrow I = \frac{\pi^2}{8N}\sum_{j=1}^{N} \frac{\sin \frac{\pi}{2}\sqrt{x_j}}{\pi/2 \sqrt{x_j}}$$ uniform

N = 10 $\quad I = 1.016$

$$\frac{8x}{\pi^2}dx = dy \quad \Rightarrow y = \frac{4}{\pi^2}x^2$$

$$\frac{\pi^2}{4}y = x^2$$

$$\boxed{x = \frac{\pi}{2}\sqrt{y}} \Leftarrow$$

Tarea para el Martes

1) Estimacion de π y la relacion de

$$\frac{Var(X)}{\langle X \rangle^2} \propto \frac{C}{N}$$

estimar

2) Estimar el error en la integral I con dos dist. pero como función de N