Latin-American alliance for capacity building in advanced physics
LA-CoNGA physics

Módulo de Instrumentación

Introducción a los Sistemas de Medida

Dennis Cazar Ramírez

Universidad San Francisco de Quito USFQ

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Introducción

Bienvenidos al Modulo de Instrumentación

- Toda la información del modulo se encuentra en la LA-CoNGA GitmiLab
- Ahora empezamos con el submódulo de Introducción a los sistemas de medida
- Este submódulo durará 4 semanas
- Tendremos 4 clases y 4 laboratorios

Temas de la clase

- Caracterización de circuitos AC
- Visualización de señales
- Análisis de señales
- Errores en medidas

AC circuits

- Currents and voltages that vary in time are called *AC* quantities
- AC circuits involves Resistances, Capacitors and Inductors
- Let´s consider the response of an RC circuit to an AC signal V_{in}, KVL gives:

$$V_i n = \frac{Q}{C} + IR \tag{1}$$

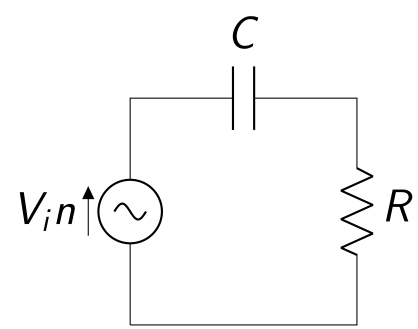


Figure 1: Simple RC circuit driven by a sinusoidal voltage..

Response to a step

- Let´s assume that V_in is an step function ($V_{in}(t=0)=0$, $V_{in}(t=t_1)=V$
- Taking the derivate of eq. (1)

$$0=\frac{I}{C}+R\frac{dI}{dt} \tag{2}$$

- Solving for I we obtain:

$$I=I_0\,exp\left(-\frac{t}{RC}\right) \tag{3}$$

- Voltage drop in the capacitor is equal to:

$$V_c=V_{in}-IR \;\rightarrow\; V_c=V_{in}-I_0R\,exp\left(-\frac{t}{RC}\right) \tag{4}$$

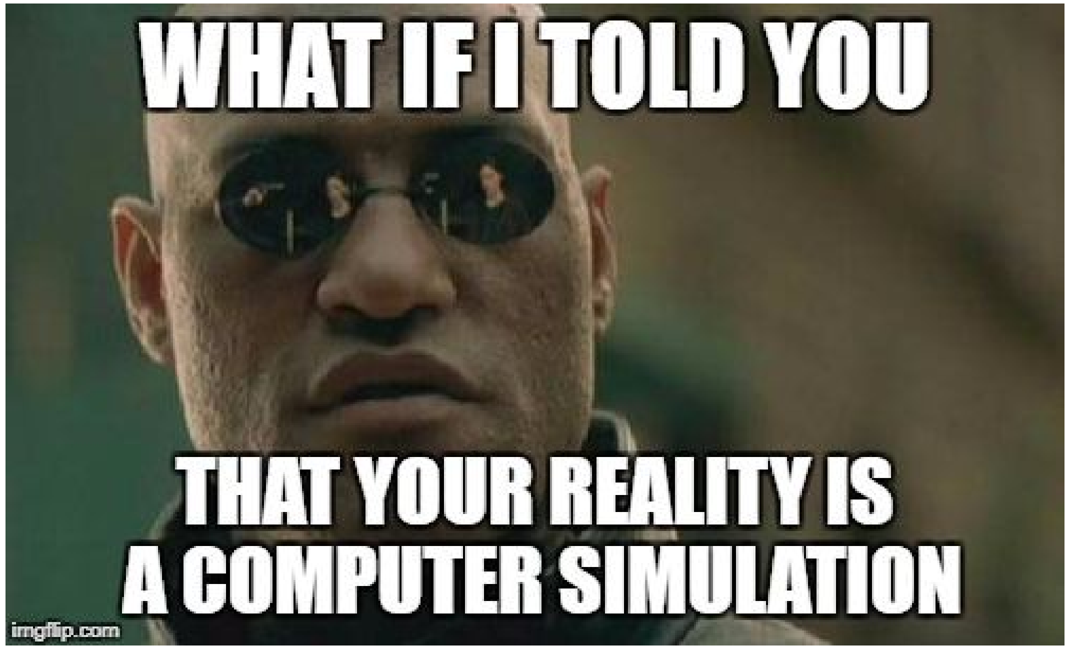
- For future reference, we note that, since V_{in} and I_0R are constants, we can write a general solution for V_c as:

$$V_C=V_1\,exp\left(-\frac{t}{RC}\right)+V_2 \tag{5}$$

where V_1 and V_2 are constants.

Simulating the response

- Let´s simulate the response of this circuit using *LTSpice*



Response to a sine wave

and taking the time derivative of this and rearranging yields

$$R\frac{dI}{dt} + \frac{I}{C} = \frac{dV_{in}}{dt} \tag{6}$$

Note that, unlike the switching problem, the derivative of the input voltage is not zero. To solve eq.(2) we asume that:

- $V_{in} = V_p \, sin\omega t$
- $I = I_p \, sin(\omega t + \phi)$, where I_p and ϕ are constants to be determined

Replacing in eq(2) results in:

$$R\omega I_p \, cos(\omega t + \phi) + \frac{I_p}{C} sin(\omega t + \phi) = \omega V_p \, cos\omega t \tag{7}$$

We now proceed to solve for I_p and ϕ.

Response to a sine wave

We obtain:

$$\left(\cos\phi + \frac{1}{\omega RC}\sin\phi - \frac{V_p}{I_p R}\right)\cos\omega t \;+\; \left(-\sin\phi + \frac{1}{\omega RC}\cos\phi\right)\sin\omega t = 0 \tag{8}$$

- At $t = 0$ equation (4) reduces to:

$$\cos\phi + \frac{1}{\omega RC}\sin\phi - \frac{V_p}{I_p R} = 0 \tag{9}$$

- Alternatively, if we choose t such that $\omega t = \frac{\pi}{2}$ we obtain:

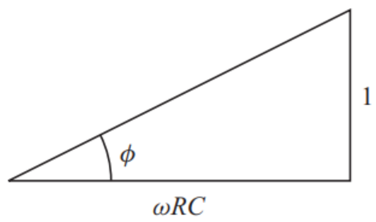
$$-\sin\phi + \frac{1}{\omega RC}\cos\phi = 0 \tag{10}$$

- This last equation can now be solved for ϕ:

$$\frac{\sin\phi}{\cos\phi} = \frac{1}{\omega RC} \;\rightarrow\; \phi = \tan^{-1}\left(\frac{1}{\omega RC}\right) \tag{11}$$

Response to a sine wave

- A right triangle satisfying Eq. (11) is shown in Fig. (2)



- It thus follows that

$$sin\phi = \frac{1}{\sqrt{1+(\omega RC)^2}} \qquad cos\phi = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}} \tag{12}$$

- Using these expressions in Eq. (9) and after some algebra

$$I_p = \frac{\omega C}{\sqrt{1+(\omega RC)^2}} V_p sin(\omega t + \phi) \tag{13}$$

Response to a sine wave

- Since we have already solved for the current, getting this output voltage is easy:

$$V_{out} = IR = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}} V_p sin(\omega t + \phi) \tag{14}$$

- we see that the output voltage has changed in two ways:
  - It has shifted in phase. Since $\omega RC > 0$ we have $0 < \phi < \pi/2$
  - The amplitude has changed. It is useful here to ignore the time variation and phase shift and simply compare the magnitude of the input signal |Vin| with the magnitude of the output |Vout|:

$$\frac{|V_{out}|}{|V_{in}|} = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}} \tag{15}$$

- Let´s simulate the circuit response!