3)

Modelo de Heisenberg

$$\underline{H(\vec{h})} = - \sum_{i,j} \underline{J_{i,j}} \, \vec{S}_i \cdot \vec{S}_j - \vec{h} \cdot \sum_i \vec{S}_i$$

$\forall i \quad \underline{\vec{S}_i \cdot \vec{S}_i} = 1 \qquad \vec{S}_i = \begin{pmatrix} S_i^x \\ S_i^y \\ S_i^z \end{pmatrix}$

Si $\vec{h} = \vec{0}$

H tiene la simetría $O(3)$

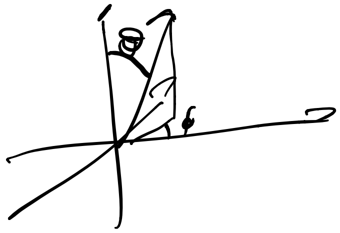
$$\boxed{\vec{S}_i \to \underset{3\times3}{T}\vec{S}_i} \;,\; \underline{T^t T} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\det T^2 = 1 \Rightarrow \det T = \pm 1$

$$\begin{cases} \det T = 1 \to SO(3) = \text{grupo de rotaciones en 3-D.} \\ \det T = -1 \to \underline{\vec{S} \to -\vec{S}} \end{cases}$$

$$\begin{pmatrix} S^x \\ S^y \\ S^z \end{pmatrix} \to \begin{pmatrix} -S^x \\ S^y \\ S^z \end{pmatrix}$$

$$\underline{Z = \int \prod_i d\vec{S}_i \; e^{-\beta H}}$$



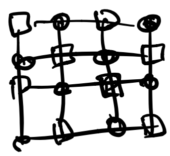
$$d\bar{S}_i = \sin\theta_i \, d\theta_i \, d\phi_i$$

$$\theta_i \in [0, \pi]$$

$$\phi_i \in [0, 2\pi[$$

$\forall i \quad \bar{S}_i \to -\bar{S}_i$

en una red bipartita



$$\forall i \in B \quad \bar{S}_i \to -\bar{S}_i \, , \quad J \to -J$$

$$H = -\sum_{\langle i,j \rangle} J \, \bar{S}_i \cdot \bar{S}_j$$

Ferro $\Longleftrightarrow$ Anti Ferro

en física cuántica

$\forall i \quad \bar{S}_i^2 = \hbar^2 s(s+1)$

$$[S^a, S^b] = i\hbar \epsilon^{abc} S^c$$

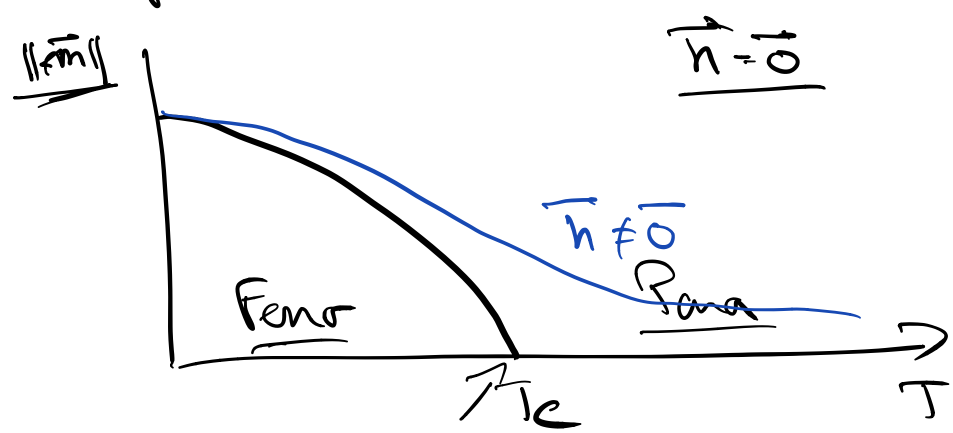
Ferro $\not\Longleftrightarrow$ AF.

magnetización $\bar{M} = \sum_i \langle \bar{S}_i \rangle \underset{\text{C.B.P.}}{=} N\bar{m}$

$$\boxed{\vec{m} = \langle \vec{S}_i \rangle_{\forall i}}$$

Susceptibilidad: $\chi^{\alpha\beta} = \dfrac{\partial m^\alpha}{\partial h^\beta} \sim \delta^{\alpha,\beta} \dfrac{\partial m^\alpha}{\partial h^\alpha}$

Diagrama de fases $D \geq 3$, Ferro.

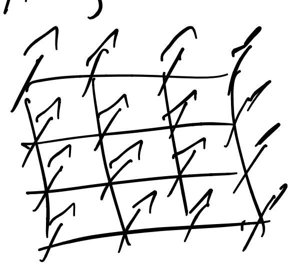


Ferro

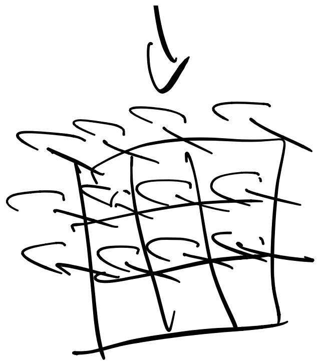
$$H = -J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

Se minimiza la energía

$$\vec{S}_i \parallel \vec{S}_j \quad \forall i,j$$



→ ruptura espontánea de $O(3)$ o $SO(3)$



6) Modelo "XY"

$$\forall i \quad \vec{S}_\ell = \begin{pmatrix} S^x_\ell \\ S^y_\ell \end{pmatrix} \leftarrow \qquad \underline{\vec{m}} = \langle \vec{S} \rangle$$

$$H = -\sum_{\langle i,j \rangle} J_{ij}\, \vec{S}_i \cdot \vec{S}_j - \vec{h} \cdot \sum_i \vec{S}_i$$

Si $\vec{h} = \vec{0}$,

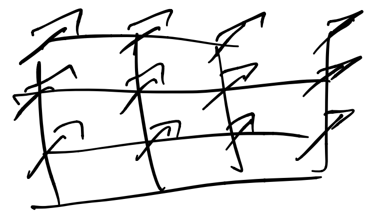
$\rightarrow$ simetría $O(2)$ $\quad \underline{(SO(2))} \cong U(1)$

$$\psi_\ell = S^x_\ell + \underset{\sqrt{-1}}{i}\, S^y_\ell \qquad \underline{\langle \psi \rangle}$$

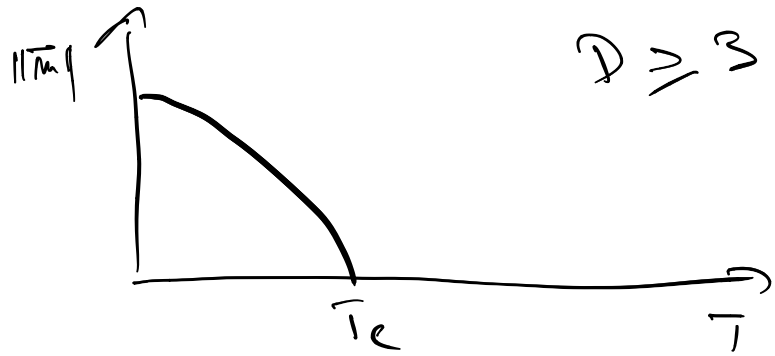
$$\begin{pmatrix} S^x \\ S^y \end{pmatrix} \rightarrow \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} S^x \\ S^y \end{pmatrix}$$

$$\psi \rightarrow \underbrace{e^{i\theta}}_{\in U(1)} \psi$$

estado de más baja energía (ferro)



$\bar{m} = \langle \vec{S} \rangle$



$D \geq 3$

- ejemplos:

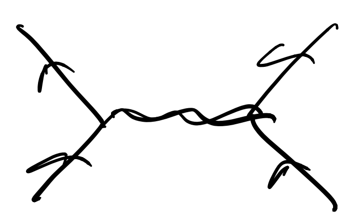
* $H = -\sum_{\langle i,j \rangle} J_{ij} \left(\cancel{\Delta S_i^z S_j^z} + S_i^x S_j^x + S_i^y S_j^y \right)$

$\Delta \ll 1$

* Superconductividad. orbitales "s" o "d"

1957 BCS

$$H_{BCS} = \sum_{\vec{k}, \sigma = \pm} \varepsilon(\vec{k}) \, c^{\dagger}_{\sigma,\vec{k}} \, c_{\sigma,\vec{k}}$$



$$+ \sum_{\vec{k},\vec{k}'}' V(\vec{k},\vec{k}')\, C^{+}_{\vec{k}\uparrow} C^{+}_{-\vec{k},\downarrow} C_{\vec{k}'\uparrow} C_{-\vec{k}'\downarrow}$$

Simetría $U(1)$ $\forall \sigma, \vec{k}$ $C_{\sigma,\vec{k}} \to e^{i\theta} C_{\sigma,\vec{k}}$

$C^{+}_{\sigma,\vec{k}} \to e^{-i\theta} c^{+} \ldots$

$$\to \underbrace{\langle C_{\vec{k}\uparrow} C_{-\vec{k}\downarrow} \rangle}_{} \neq 0$$

↳ ruptura espontánea de $U(1)$

* Superfluidez.

He^4 bosón

He^3 fermión. → mecanismo BCS

orbitales "l"

→ condensación de Bose-Einstein. ~2K

$$H = \sum_{k} \varepsilon(\vec{k})\, b^{+}_{\vec{k}} b_{\vec{k}} + \sum_{\substack{\vec{k}_1,\vec{k}_2,\\ -\vec{k}_1,\vec{k}_2}}' V(\vec{k})\, b^{+}_{\vec{k}_1} b^{+}_{\vec{k}_2} b_{\vec{k}_1} b_{\vec{k}_2}$$

$b_{\vec{k}} \to e^{i\theta} b_{\vec{k}}$

$b^{+} \to e^{-i\theta} b^{+}$

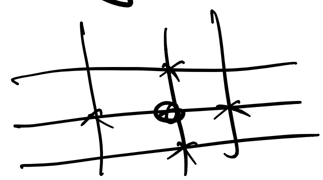
Condensación B.E. $\langle b(\vec{r}=0)\rangle \neq 0$

$m = \langle \sigma \rangle$ m e, E, B

7) Solución del modelo de Ising con la aproximación de campo medio

(Bragg & Williams, 1934)

Modelo de Ising ferro, con interacción a 1ros vecinos



$$H(h) = -J\sum_{\langle i,j\rangle} \sigma_i\sigma_j - h\sum_i \sigma_i$$

$J > 0$

si $J = 0$

$$\to H = -h\sum_i \sigma_i$$

$$Z(h) = \sum_{\{\sigma\}} e^{-\beta H}$$

$$= \sum_{\{\sigma_i\}} e^{\beta h \sum_i \sigma_i} = \sum_{\substack{\{\sigma_1=\pm1,\, \sigma_2=\pm1, \\ \dots\, \sigma_N=\pm1\}}} e^{\beta h \sigma_1} e^{\beta h \sigma_2} \cdots e^{\beta h \sigma_N}$$

$$= \sum_{\sigma_1=\pm1} e^{\beta h \sigma_1} \times \sum_{\sigma_2=\pm1} e^{\beta h \sigma_2} \times \cdots \sum_{\sigma_N=\pm1} e^{\beta h \sigma_N}$$

$$= \left(\sum_{\sigma=\pm1} e^{\beta h \sigma}\right)^N = \left(e^{\beta h} + e^{-\beta h}\right)^N$$

$$= 2^N (\cosh \beta h)^N = Z_1^N$$

$$Z = e^{-\beta F} \qquad F = -\frac{1}{\beta} \ln Z$$

$$F = -k_B T N \ln 2 - k_B T N \ln(\mathrm{ch}(\beta h))$$

$$\underline{f} = \frac{F}{N} = -k_B T \ln 2 - k_B T \ln(\mathrm{ch}(\beta h))$$

$$\langle \sigma_i \rangle = \frac{1}{Z} \sum_{\{\sigma\}} \sigma_i e^{-\beta H}$$

$$= \frac{1}{Z_1^N} Z_1^{N-1} \sum_{\sigma_i=\pm1} \sigma_i e^{\beta h \sigma_i}$$

$$= \frac{1}{Z_1} \underbrace{\sum_{\sigma_i=\pm 1} e^{\beta h \sigma_i} \sigma_i}$$

$$= \frac{1}{Z_1} \left(e^{\beta h} - e^{-\beta h} \right) = 2 Sh(\beta h)$$

$$Z_1 = 2\, ch(\beta H)$$

$$\langle \sigma_i \rangle = m = tgh(\beta h) \leftarrow$$

<u>obs</u> $m = -\frac{\partial}{\partial h} f$ ej.

<u>obs</u> $m = \frac{1}{\beta} \frac{\partial}{\partial h} Z_1$

<u>ahora</u> $J \neq 0$

→ ~~ahora~~ tomar en cuenta productos de fluctuaciones (C.B.P)

$$\langle \sigma_i \rangle = m$$

$$\sigma_i = m + \delta\sigma_i \quad \text{con} \quad \delta\sigma_i = \sigma_i - m$$

$$\sigma_i \sigma_j = (m + \delta\sigma_i)(m + \delta\sigma_j)$$

$$= m^2 + \underline{m}\,\delta\sigma_i + \underline{m}\,\delta\sigma_j + \cancel{\delta\sigma_i \delta\sigma_j}$$

$$\delta\sigma_i = \sigma_i - m$$
$$\delta\sigma_j = \sigma_j - m$$

$$\Rightarrow \sigma_i\sigma_j \simeq m^2 + m(\sigma_i - m) + m(\sigma_j - m) + \dots$$
$$\hookrightarrow = \underline{m(\sigma_i + \sigma_j) - m^2}$$

$$H = -\sum_{\langle i,j\rangle}' J\sigma_i\sigma_j - h\underline{\sum_i \sigma_i}$$

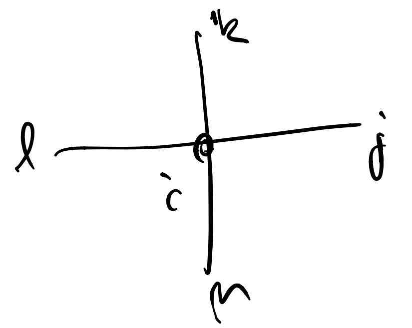
$$\downarrow$$

$$H \simeq -\sum_{\langle i,j\rangle}' J\left(m(\underline{\sigma_i} + \underline{\sigma_j}) - m^2\right) - h\sum_i \sigma_i$$

$$= -\frac{1}{2}\sum_{i,j}' J(m\sigma_i + m\sigma_j - m^2) - h\sum_i \sigma_i$$

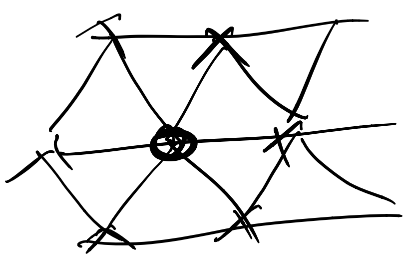
$$= -\frac{1}{2}\sum_i \underbrace{\sum}_{\text{vecinos de } i}\left(J\,2m\sigma_i - Jm^2\right) - h\sum_i \sigma_i$$

$$\sum_{\text{vecinos } i} = z \quad \text{coordinación de la red}$$



$z = 4$ red cuadrada

$z = 6$ '' cúbica



$z = 6$ '' triangular

$$\Rightarrow H \simeq -\sum_i{}' \left(J m z \sigma_i - \frac{J z m^2}{2} \right) - h \sum_i{}' \sigma_i$$

$$H \simeq \frac{J z m^2}{2} N - h_{eff} \sum_i \sigma_i$$

$$h_{eff} = h + J m z$$

$$Z = \sum_{\{\sigma\}}{}' e^{-\beta H} = e^{-\beta \frac{J z m^2 N}{2}} \sum_{\{\sigma\}}{}' e^{\beta h_{eff} \sum_i \sigma_i}$$

$$Z = \left(e^{-\beta \frac{J z m^2}{2}} \right)^N \left(2\, \mathrm{ch}(\beta h_{eff}) \right)^N$$

F tq $Z = e^{-\beta F}$

$$F = N\frac{Jzm^2}{2} - k_B T \ln\left[2\,\mathrm{ch}(\beta h_{ef})\right]^N$$

$$f = \frac{F}{N}$$

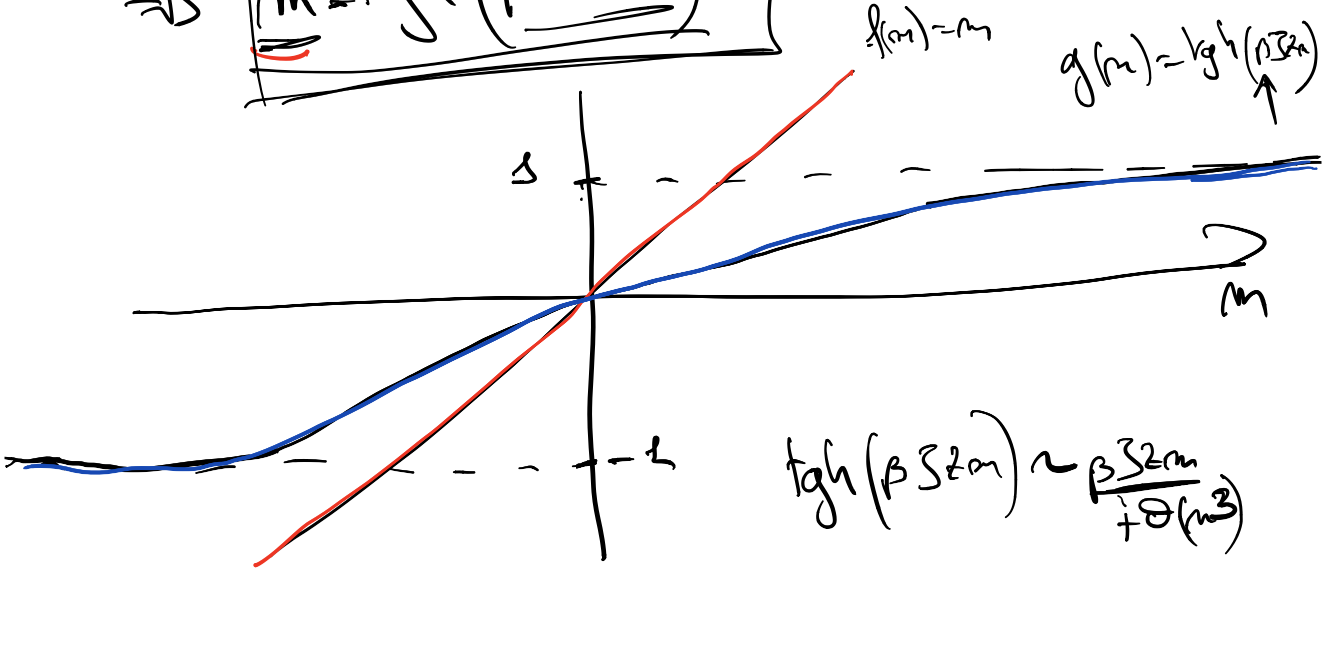
$$\boxed{m = \langle\sigma\rangle = \mathrm{tgh}(\beta h_{ef})}$$

$$\Rightarrow m = \mathrm{tgh}\left(\beta\left(Jzm + h\right)\right)$$

ecuación de autoconsistencia

ahora $h = 0$

$$\Rightarrow \boxed{m = \mathrm{tgh}(\beta J z m)}$$ (1934 B & W)

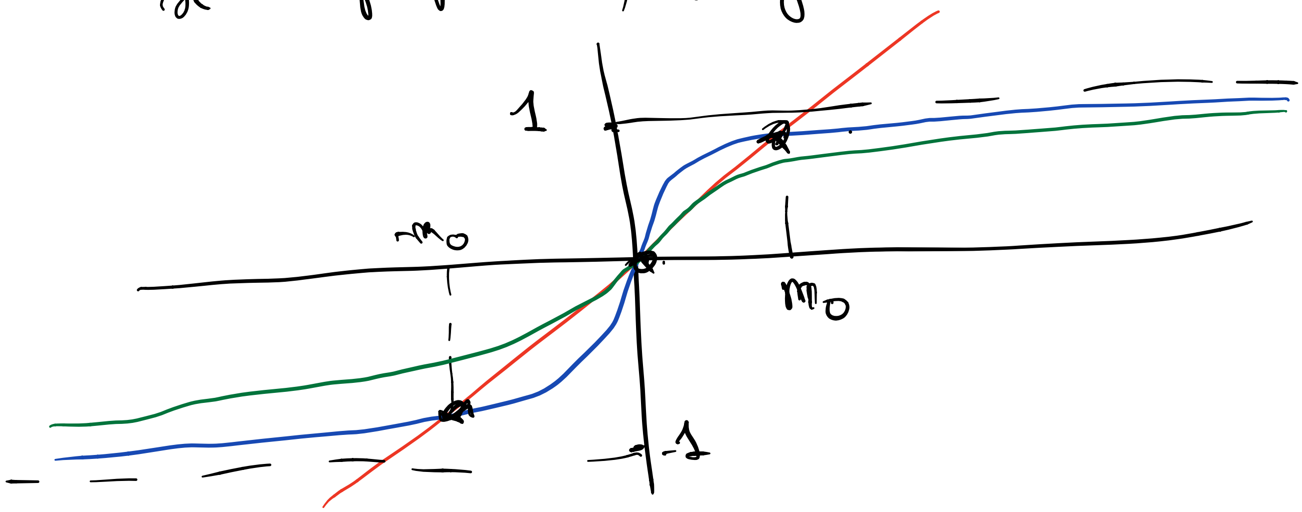


$$\mathrm{tgh}(\beta J z m) \sim \beta J z m + O(m^3)$$

si β pequeño, T grande

Solución es $m = 0$ (fase para)

si T pequeña, β grande



hay 3 soluciones posibles

$m = 0, \pm m_0$

→ la mejor es la que minimiza F

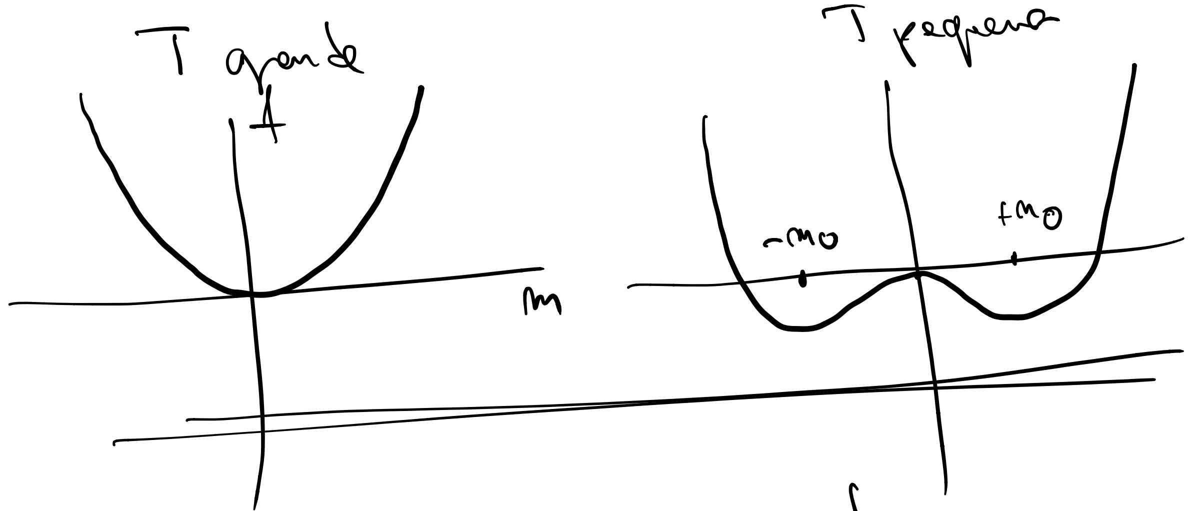
o f.

$$f = \frac{J z m^2}{2} - k_B T \ln\left[2 \operatorname{ch}(\beta h_{eff})\right]$$

ej: $\frac{df}{dm} = 0 \rightarrow m = \operatorname{tgh}(\beta J z m)$

$$f \simeq \text{cte} + \frac{Jz}{2} m^2 - k_B T \ln\left[1 + \frac{(\beta z J m)^2}{2} + \dots\right]$$

$$\simeq \underline{\text{cte}} + \frac{Jz}{2}\left(1 - \frac{Jz}{k_B T}\right) m^2 + \mathcal{O}(m^4)$$



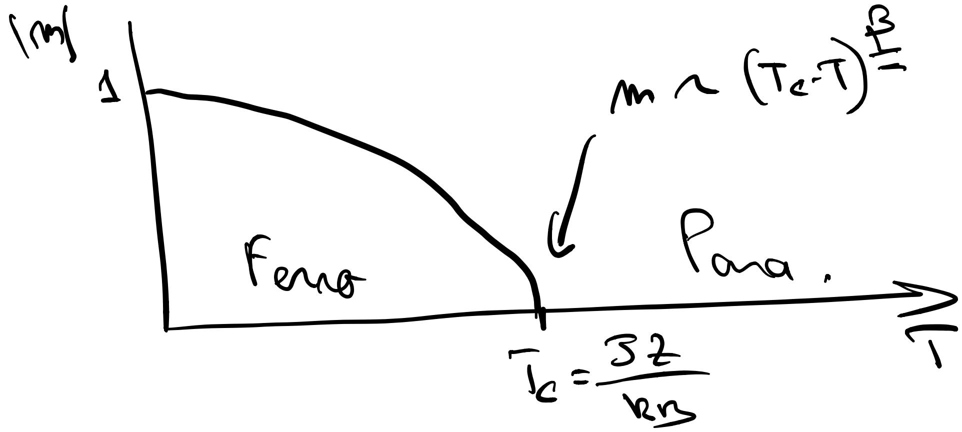
$\Rightarrow$ la solución es $m = \pm m_0$!

o sea $\langle \sigma \rangle = \pm m_0$

$\tanh(\beta J z m) \simeq \beta J z m$ tiene pendiente 1

cuando $\beta J z = 1$ o sea $\frac{Jz}{k_B T_c} = 1$

$$\Rightarrow \boxed{T_c = \frac{Jz}{k_B}}$$



$$m = \mathrm{tgh}\left[\frac{JZ}{k_B T} m\right]$$

$$\cancel{m} \simeq \frac{JZ}{k_B T}\cancel{m} - \frac{\left(\frac{JZ}{k_B T}\right)^3 m^{\cancel{3}2}}{3} + \ldots O(m^5)$$

$$\left(1 - \frac{JZ}{k_B T}\right) = - \frac{\left(\frac{JZ}{k_B T}\right)^3 m^2}{3}$$

$$m^2 \sim \frac{3}{\left(\frac{JZ}{k_B T}\right)^3}\left(\frac{JZ}{k_B T} - 1\right)$$

$$\sim \frac{3}{T\left(\frac{JZ}{k_B T}\right)^3}\left(\underbrace{\frac{JZ}{k_B}}_{= T_c} - T\right)$$

$$\Rightarrow m^2 \sim f(T)\,(T_c - T) \quad \text{si } T \sim T_c$$

$$\sim \underline{f(T_c)}\,(T_c - T)$$

$$m \sim \sqrt{T_c - T} \quad \text{ó} \quad \sim (T_c - T)^{1/2}$$

$$\Rightarrow \beta = \frac{1}{2} \qquad \boxed{\beta = \frac{1}{2}}$$

$z = 2$ No hay T. F.