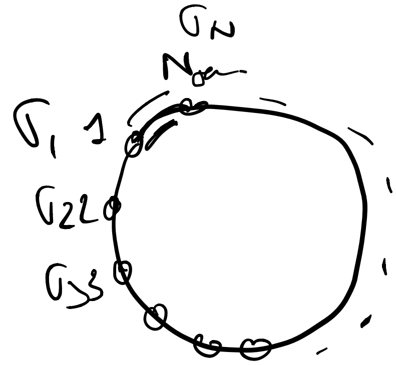
8) Modelo de Ising en 1-D

Cond. de Borde periódicas.



$$\sigma_{N+1} \equiv \sigma_1 \quad ; \quad \sigma_{N+2} \equiv \sigma_2$$

$$H = -J \sum_{i=1}^{N} \sigma_i \sigma_{i+1} - h \sum_{i=1}^{N} \sigma_i$$

$$Z = \sum_{\{\sigma\}} e^{-\beta H} \quad ; \quad K = \frac{J}{k_B T} = \beta J$$

$$\hat{h} = \frac{h}{k_B T}$$

$$Z = \sum_{\{\sigma\}}' e^{K\sum_{i=1}^{N} \sigma_i\sigma_{i+1} + \frac{\hat{h}}{2}\sum_{i=1}^{N}\sigma_i + \frac{\hat{h}}{2}\sum_{i=1}^{N}\sigma_{i+1}}$$

$$= \sum_{\{\sigma\}}' e^{\sum_i' \left(K\sigma_i\sigma_{i+1} + \frac{\hat{h}}{2}\sigma_i + \frac{\hat{h}}{2}\sigma_{i+1}\right)}$$

$\sigma_1 = \pm 1,\ \sigma_2 = \pm 1$
$\ldots\ \sigma_N = \pm 1$

$$= \left(\sum_{\substack{i_1 = 1,2 \\ i_2 = 1,2 \\ \vdots \\ i_N = 1,2}}' T_{i_1, i_2} T_{i_2, i_3} T_{i_3, i_4} \cdots T_{i_N, i_1} \right.$$

matriz de transferencia

$$T_{11} = e^{K\sigma_i\sigma_{i+1} + \frac{\hat{h}}{2}\sigma_i + \frac{\hat{h}}{2}\sigma_{i+1}}\Big|_{\substack{\sigma_i = 1 \\ \sigma_{i+1} = 1}}$$

$$T_{12} = \text{------}\Big|_{\substack{\sigma_i = 1 \\ \sigma_{i+1} = -1}}$$

$$T_{21} = \text{------}\Big|_{\substack{\sigma_i = -1 \\ \sigma_{i+1} = 1}}$$

$$T_{22} = \text{------}\Big|_{\sigma_i = \sigma_{i+1} = -1}$$

$$T = \begin{pmatrix} e^{k+h} & e^{-k} \\ e^{-k} & e^{k-h} \end{pmatrix}$$

$$\boxed{Z = \mathrm{tr}\{T^N\}}$$

$$\sum_i M_{ii} = \mathrm{tr}\, M$$

$$\exists\, O \ \text{tq}\ O^t O = \underline{I_d = \mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$$

$$O^t T O = \begin{pmatrix} \epsilon_1 & 0 \\ 0 & \epsilon_2 \end{pmatrix} = D$$

$$O^t T^N O = O^t T \underset{O^t O}{\cdot} T \underset{O^t O}{\cdot} T \underset{O^t O}{\cdot} T O = D^N$$

$$O^t T^N O = D^N \qquad \mathrm{tr}\{O^t T^N O\}$$

$$\mathrm{tr}\{D^N\} = \mathrm{tr}\{T^N \underbrace{O O^t}_{I_d}\} = \mathrm{tr}\{T^N\}$$

$$D^N = \begin{pmatrix} \epsilon_1^N & 0 \\ 0 & \epsilon_2^N \end{pmatrix}$$

$\Rightarrow \mathrm{tr}\{T^N\} = \epsilon_1^N + \epsilon_2^N$

$$\epsilon_{1/2} = e^{K} ch(\tilde{h}) \pm \left(e^{2K} sh^2(\tilde{h}) + e^{-2K}\right)^{1/2} \quad \text{ej.}$$

$\epsilon_1 > \epsilon_2 > 0$ ej.

$Z = \epsilon_1^N + \epsilon_2^N$ si $N \gg 1$

$$= \epsilon_1^N \left(1 + \left(\frac{\epsilon_2}{\epsilon_1}\right)^N\right) \simeq \epsilon_1^N$$

$\frac{\epsilon_2}{\epsilon_1} < 1$

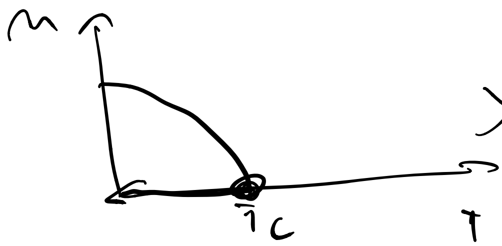
$F = -k_B T \ln Z \simeq -k_B T N \ln \epsilon_1$

$m = \frac{1}{N} \frac{\partial}{\partial h} \ln Z = \frac{\partial}{\partial h} \ln \epsilon_1$

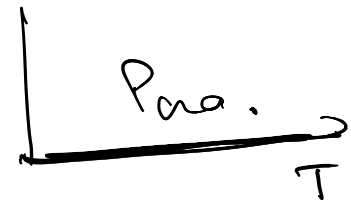
$$\Rightarrow \boxed{m = \frac{\sinh(\tilde{h})}{k_B T \sqrt{\sinh^2(\tilde{h}) + e^{-4K}}}}$$

obs si $\tilde{h} \to 0$, $m \to 0$ $\forall T$.

$\Rightarrow$ no hay fase ferro!



$\chi \rightarrow$



Funciones de correlación: $\underline{h=0}$

$$\langle \sigma_i \sigma_j \rangle = \frac{1}{Z}\sum_{\{\sigma\}} e^{-\beta H}\sigma_i\sigma_j$$

$j>i$

$$\sum_{\substack{\sigma_1=\pm1\\ \sigma_2=\pm1\\ \vdots\\ \sigma_N=\pm1}} e^{K\sigma_1\sigma_2+\frac{h}{2}\sigma_1+\frac{h}{2}\sigma_2}\, e^{\sigma_2\sigma_3\cdots}\cdots e^{K\sigma_{i-1}\sigma_i+\frac{h}{2}\sigma_{i-1}+\frac{h}{2}\sigma_i}\,\sigma_i\; e^{K\sigma_i\sigma_{i+1}+\frac{h}{2}\sigma_i+\frac{h}{2}\sigma_{i+1}}\cdots e^{\sigma_{j-1}\sigma_j\cdots}\,\sigma_j\, e^{\sigma_j\sigma_{j+1}}\cdots e^{\sigma_N\sigma_1}$$

$$\Sigma = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$= \mathrm{tr}\left\{\underbrace{T\cdot T\cdot T\cdots T}_{i}\,\Sigma\,\underbrace{T\cdot T\cdots T}_{j-i}\,\Sigma\,\underbrace{T\cdots T}_{N-j}\right\}$$

$$\Rightarrow \langle \sigma_i \sigma_j \rangle = \frac{1}{Z}\,\mathrm{tr}\left\{T^{i}\,\Sigma\, T^{j-i}\,\Sigma\, T^{N-j}\right\}$$

$$T = \begin{pmatrix} e^{K} & e^{-K} \\ e^{-K} & e^{K} \end{pmatrix} \; : \; \overline{\theta = \theta^t} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\theta^t \theta = \theta\theta = Id$$

$$\underline{\theta T \theta} = \underline{\begin{pmatrix} \epsilon_1 & 0 \\ 0 & \epsilon_2 \end{pmatrix}}$$

$$\theta T^n \theta = \begin{pmatrix} \epsilon_1^n & 0 \\ 0 & \epsilon_2^n \end{pmatrix}$$

$$\downarrow$$

$$\underset{\hat{\theta}^2}{T \cdot T} \cdot \underset{\hat{\theta}^2}{T} \cdot T$$

$$\Rightarrow \langle \sigma_i \sigma_j \rangle = \frac{\operatorname{tr}\left\{ D^i \theta \Sigma \theta D^{j-i} \theta \Sigma \theta D^{N-j} \right\}}{Z}$$

$$\theta \Sigma \theta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\Rightarrow \langle \sigma_i \sigma_j \rangle = \frac{1}{Z} \operatorname{tr}\left\{ \begin{pmatrix} \epsilon_1^i & 0 \\ 0 & \epsilon_2^i \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \epsilon_1^{j-i} & 0 \\ 0 & \epsilon_2^{j-i} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \cdots \begin{pmatrix} \epsilon_1^{N-j} & 0 \\ 0 & \epsilon_2^{N-j} \end{pmatrix} \right.$$

$$\Rightarrow \langle \sigma_i \sigma_j \rangle = \frac{\underbrace{\epsilon_1^{N-j+i}}\,\epsilon_2^{j-i} + \epsilon_2^{N-j+i}\,\epsilon_1^{j-i}}{\underbrace{\epsilon_1^{N}} + \epsilon_2^{N}}$$

$j-i$ fijo, $N \to \infty$

$$\langle \ \rangle \sim \frac{\cancel{\epsilon_1^{N}}\left(\epsilon_2^{j-i}\,\epsilon_1^{i-j} + \ldots\right)}{\cancel{\epsilon_1^{N}}\left(1 + \ldots\right)}$$

$$\Rightarrow \langle \sigma_i \sigma_j \rangle = \left(\frac{\epsilon_2}{\epsilon_1}\right)^{j-i} = \left[\operatorname{tgh}(K)\right]^{j-i}$$

$$= e^{-\frac{j-i}{\xi}}$$

$$\operatorname{tgh}(K) \equiv e^{-\frac{1}{\xi}}$$

longitud de correlación:

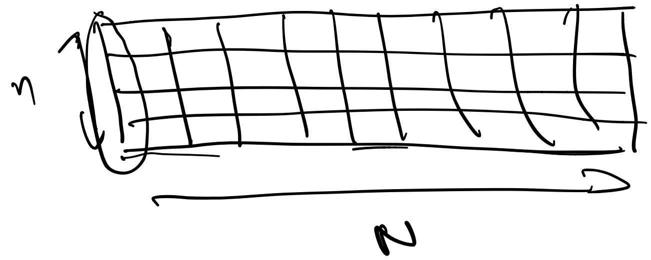
$$\xi = \frac{1}{\ln\left(\operatorname{ctgh}(K)\right)} = \frac{1}{\ln\left(\operatorname{ctgh}\left(\frac{J}{k_B T}\right)\right)}$$

<u>obs</u> si $T \to \infty$, $\xi \to 0$

si $\underline{T \to 0}$, $\underline{\xi} \to \infty$

1944, Onsager.

$\to$ Sol. para 2-D



$Z = \mathrm{tr}(\underline{T}^N)$

n espines. $\to$ 2^n posibilidades

$\underline{T}_{2^n \times 2^n} \to \epsilon_1, \epsilon_2, \ldots \epsilon_{2^n}$

$n \sim 20$

$n \sim 6, 8, 10$ Kerson Huang.

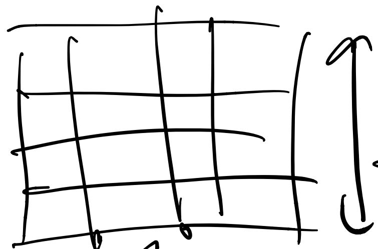
II Del modelo de Ising a la "acción" de Ginzburg-Landau.

$\int dk\, f(k)$ problema si $k \to \infty$

cut-off $k_{max} = \Lambda$ cut-off ultra-violeta.

$\int \frac{dk}{k^n}$ Problema si $k \to 0$

cut-off $k_{min} = \lambda$ cut-off Infra-rojo



$L \sim \frac{2\pi}{k_{min}}$.

$a \sim \frac{2\pi}{k_{max}}$

$a \to 0$

$L \to \infty$

1) transformación de Hubbard-Stratonovich

$$H = -\frac{1}{2}\sum_{\forall\, i,j} J_{ij}\,\sigma_i\sigma_j \qquad (J)_{N\times N}$$

J_{ij} es una matriz simétrica.

$$M = M_S + M_A$$

$$M = \frac{M+M^t}{2} + \frac{M-M^t}{2}$$

Y J_{ij} definida positiva (todos sus autovalores son >0)

$M \to \lambda_1, \lambda_2, \ldots, \lambda_N$ autovalores.

$\tilde{M} = M + \underline{\alpha}\, I_d$

$\tilde{M} \to \lambda_1 + \alpha,\ \lambda_2 + \alpha, \ldots, \lambda_N + \alpha$

α tq $\lambda_N + \alpha > 0$

obs $\tilde{J} = J + \alpha I_d$

$$\sum_{i,j} \alpha (I_d)_{ij}\, \sigma_i \sigma_j = \underline{N\alpha}$$

$\sigma_i^2 = 1$

$(J)_{N \times N}$ matriz de conectividad.

si $|i-j| \to \infty$ $\quad J_{ij} \to 0$

distancia entre i y j

$$H = -\frac{1}{2}\, \sigma^t J \sigma$$

$$\sigma^t J \sigma = (\sigma_1, \sigma_2, \ldots, \sigma_N)\, J \begin{pmatrix} \sigma_1 \\ | \\ | \\ \sigma_N \end{pmatrix}$$

$$Z = \sum_{\{\sigma\}} e^{\frac{\beta}{2}\sigma^t J \sigma}$$

ej: Se $A_{N\times N} > 0$ sim., y real.

$$I = \int_{-\infty}^{\infty} \underbrace{\prod_{i=1}^{N} dx_i}_{dx_1\, dx_2 \cdots dx_N} \; e^{-\frac{1}{2}\sum_{i,j}^{} x_i A_{ij} x_j + \sum_{i=1}^{N} x_i Y_i}$$

$$= \int_{-\infty}^{\infty} \underbrace{d\vec{x}}_{dx_1 \cdots dx_N} \; e^{-\frac{1}{2}\vec{x}^t A \vec{x} + x^t \cdot Y}$$

$$= \sqrt{\frac{(2\pi)^N}{\operatorname{Det}(A)}} \; e^{\frac{1}{2}\sum_{i,j} Y_i (A^{-1})_{ij} Y_j}$$

Dem $\vec{x} = \vec{\tilde{x}} + A^{-1}\vec{Y} \to x_i = \tilde{x}_i + \sum_j (A^{-1})_{ij} Y_j$

$$d\vec{x} = d\vec{\tilde{x}}$$

$$\to I = \left(\int d\vec{\tilde{x}} \; e^{-\frac{1}{2}\vec{\tilde{x}}^t A \vec{\tilde{x}}} \right) \times e^{\frac{1}{2} \vec{Y}^t A^{-1} \vec{Y}}$$

$$\exists\, O \text{ tq } O^t O = I_d$$

$$\text{tq } O^t A O = D = \underbrace{\begin{pmatrix} d_1 & & & 0 \\ & d_2 & & \\ & & \ddots & \\ 0 & & & d_N \end{pmatrix}}$$

$$\bar{x} = O\,\bar{\tilde{x}} \qquad \det O = 1$$

$$d\bar{x} = d\bar{\tilde{x}}$$

at last!

$$\underbrace{\int_{-\infty}^{\infty} dx\, e^{-\frac{\alpha x^2}{2}} = \sqrt{\frac{2\pi}{\alpha}} \qquad \alpha > 0}$$

d_i

$$\mathrm{Det}(A) = d_1 d_2 \ldots d_N$$

$$\beta J \longleftrightarrow A^{-1}$$

$$y_i \longleftrightarrow \sigma_i$$

$$x_i \longleftrightarrow \phi_i$$

$$e^{\frac{\beta}{2}\vec{\sigma}^t J \vec{\sigma}} = (2\pi)^{-N/2}\left[\mathrm{Det}(\beta J)\right]^{-1/2}$$

$$\times \int \prod_{i=1}^{N} d\phi_i\; e^{-\frac{1}{2\beta}\vec{\phi}^t J^{-1}\vec{\phi} + \sum_{i=1}^{N}\sigma_i\phi_i}$$

$$= \boxed{(2\pi)^{-N/2}\left[\mathrm{Det}(\beta J)\right]^{-1/2} \times \int \prod_i d\phi_i \, e^{-\frac{1}{2\beta}\sum_{i,j}' \phi_i (J^{-1})_{ij}\phi_j + \sum_i \sigma_i \phi_i}}$$

Pero $Z = \sum_{\{\sigma\}}' e^{\frac{\beta}{2}\vec{\sigma}^t J \vec{\sigma}}$

$$\Rightarrow Z = (2\pi)^{-N/2}\left[\mathrm{Det}(\beta J)\right]^{-1/2}\int \prod_i d\phi_i \, e^{-\frac{1}{2\beta}\vec{\phi}^t J^{-1}\vec{\phi}} \underline{\sum_{\{\sigma\}}' e^{\sum_i \sigma_i \phi_i}}$$

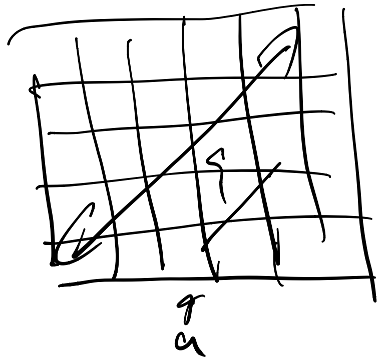
$$\sum_{\{\sigma\}}' e^{\sum_i \sigma_i \phi_i} = \underbrace{\left(\sum_{\sigma=\pm 1} e^{\sigma \phi_1}\right)}_{2\,\mathrm{ch}(\phi_1)} \underbrace{\left(\sum_{\sigma=\pm 1} e^{\sigma\phi_2}\right)}_{2\,\mathrm{ch}(\phi_2)} \cdots$$

$$= 2^N \prod_i \mathrm{ch}(\phi_i) = 2^N e^{\sum_i \ln[\cosh(\phi_i)]}$$

$$\Rightarrow \boxed{Z = (2\pi)^{-N/2}\, 2^N \left[\mathrm{Det}(\beta J)\right]^{-1/2} \times \int_{-\infty}^{\infty} \prod_i d\phi_i \, e^{-\frac{1}{2\beta}\vec{\phi}^t J^{-1}\vec{\phi} - V(\vec{\phi})}}$$

$$\text{con } U(\Phi) = -\sum_{i=1}^{N} \ln[\mathrm{ch}(\phi_i)]$$

2) El límite continuo



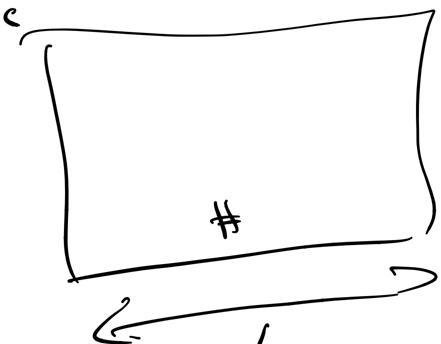
$$\langle \sigma_i \sigma_j \rangle \sim \frac{e^{-\frac{|r_i - r_j|}{\xi}}}{|i-j|^{\cdot}}$$

si $T \to T_c$

$\xi \to \infty$

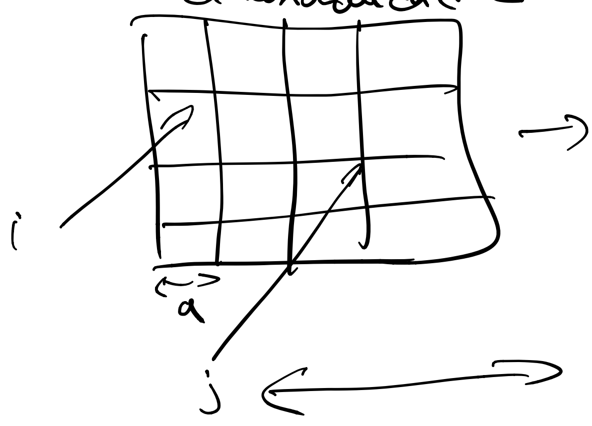
$\xi > a$

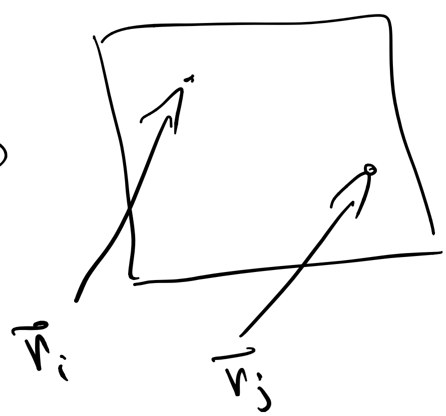
si $T \sim T_c$



$L \gg a$

dimensionalidad d





$$\sum_i \to \frac{1}{a^d}\int d^d \vec{r}_i$$

$$\delta_{i,j} \longrightarrow a^d \delta^d(\vec{r}_i - \vec{r}_j)$$

$$\sum_j \delta_{i,j} = 1 \longrightarrow \frac{1}{a^d}\int d\vec{r}_j \, a^d \delta(\vec{r}_i - \vec{r}_j) = 1$$

J_{ij} depende de $\underline{|i-j|}$

$$\hookrightarrow J(|\vec{r}_i - \vec{r}_j|)$$

$$J^{-1}_{i,j} \longrightarrow J^{-1}(|\vec{r}_i - \vec{r}_j|)$$

$$\sum_j \underline{J_{ij} J^{-1}_{jk}} = \delta_{ik}$$

$$\boxed{\frac{1}{a^d}\int d\vec{r}_j \; J(|\vec{r}_i - \vec{r}_j|) \, J^{-1}(|\vec{r}_j - \vec{r}_k|) = a^d \delta(\vec{r}_i - \vec{r}_k)} \quad (*)$$

$$\phi_i \longrightarrow \phi(\vec{r}_i)$$

definimos las transformadas de Fourier (T.F.)

$$\hat{\phi}(\vec{k}) = \int d^d\vec{r} \, e^{i\vec{k}\cdot\vec{r}} \phi(\vec{r})$$

$$\phi(\vec{r}) = \frac{1}{(2\pi)^d}\int d^d\vec{k}\, e^{-i\vec{k}.\vec{r}}\,\hat{\phi}(\vec{k})$$

$\underline{\text{obs}}$ $\phi(\vec{r})$ es real $\Rightarrow$ $\hat{\phi}^*(\vec{r}) = \hat{\phi}(-\vec{r})$

$$\hat{J}(\vec{k}) = \int d^d\vec{r}\, e^{i\vec{k}.\vec{r}}\, J(\vec{r})$$

$$J(\vec{r}) = \frac{1}{(2\pi)^d}\int d^d\vec{k}\, e^{-i\vec{k}.\vec{r}}\,\hat{J}(\vec{k})$$

$$\hat{J}^{-1}(\vec{k}) = \int d^d\vec{r}\, e^{i\vec{k}.\vec{r}}\, J^{-1}(\vec{r})$$

$$\underline{\int d^d\vec{r}\, e^{i(\vec{k}+\vec{k}').\vec{r}} = (2\pi)^d\,\delta(\vec{k}+\vec{k}')}$$

$$\underline{\int d^d\vec{k}\, e^{i\vec{k}.(\vec{r}-\vec{r}')} = (2\pi)^d\,\delta(\vec{r}-\vec{r}')}$$

ej:

$$\boxed{\hat{J}^{-1}(\vec{k}) = \frac{a^{2d}}{\hat{J}(\vec{k})}}$$

$$\sum_{i,j}\phi_i\, J^{-1}_{ij}\,\phi_j \rightarrow \frac{1}{a^{2d}}\int d^d\vec{r}\, d^d\vec{r}'\,\phi(\vec{r})\, J^{-1}(\vec{r}-\vec{r}')\,\phi(\vec{r}')$$

$$\xrightarrow{T.F.} \frac{1}{a^{2d}} \frac{1}{(2\pi)^d} \int d\vec{k} \, |\hat{\phi}(\vec{k})|^2 \, \hat{J}^{-1}(\vec{k})$$

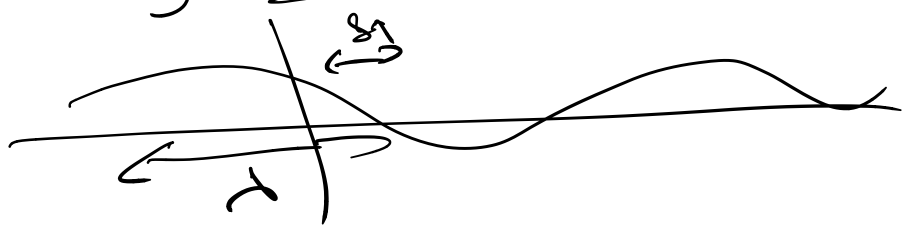
$$\left(\hat{\phi}(-\vec{k}) = \hat{\phi}^*(\vec{k}) \right)$$

$$= \frac{1}{(2\pi)^d} \int d\vec{k} \, \frac{|\hat{\phi}(\vec{k})|^2}{\hat{J}(\vec{k})}$$

→ distancias grandes

→ $|\vec{k}|$ pequeños.

$$\int e^{ikx} f(x) \to \hat{f}(k) \xrightarrow[k \gg \frac{1}{\lambda}]{} 0$$



$x \to x + \delta x \qquad \delta x \ll \lambda$

$f(x+\delta x) \simeq f(x)$

$e^{ikx + ik\delta x} \qquad e^{ik\delta x} \neq 1$

$$\int_{k_{min}}^{k_{max}} dk\, e^{ikx} \simeq 0 \qquad \delta k \gg \frac{1}{x}$$

$$\hat{J}(\vec{k}) = \int d^d\vec{r}\; e^{i\vec{k}\cdot\vec{r}}\; \underline{J(|\vec{r}|)}$$

$$\hat{J}(\vec{k}) = \hat{J}(\vec{0}) + \sum_{\alpha=1,..,d} k^\alpha \underbrace{\frac{\partial \hat{J}}{\partial k^\alpha}\Big|_{\vec{k}=0}}_{=0}$$

$$+ \frac{1}{2} \sum_{\alpha,\beta} k^\alpha k^\beta \frac{\partial^2 \hat{J}}{\partial k^\alpha \partial k^\beta}\Big|_{\vec{k}=0} + \underbrace{O(k^3)}_{=0} + O(k^4)$$

$$\int d^d\vec{r}\; r^\alpha\; J(|\vec{r}|)$$

sim $\underline{r^\alpha \to -r^\alpha}$

$$\int d^d\vec{r}\; r^\alpha r^\beta\; J(|\vec{r}|) \quad \text{si } \alpha \neq \beta$$

$$= 0$$

$$\hat{J}(\vec{k}) = \hat{J}(0) + \frac{1}{2} \sum_{\alpha} k^{\alpha 2} \frac{\partial^2 \hat{J}}{(\partial k^\alpha)^2}\Big|_{\vec{k}=0} + \dots$$

$$= -\frac{1}{d} \int d^d\vec{r}\; |\vec{r}|^2\; J(|\vec{r}|)$$

$$\parallel$$

11

ℓ: radio de alcance de la matriz de conectividad.

def $\quad \ell^2 \equiv \dfrac{\frac{1}{2d}\int d^d\vec{r}\;|\vec{r}|^2\, J(|\vec{r}|)}{\int d^d\vec{r}\; J(|\vec{r}|)}$

$$\Rightarrow \hat{J}(\vec{k}) = \hat{J}(0)\left[1 - \ell^2 \vec{k}^2 + \cdots\right]$$

$$\frac{1}{\hat{J}(\vec{k})} \sim \frac{1}{\hat{J}(0)}\left[1 + \ell^2 \vec{k}^2 + \cdots\right]$$

$$\Rightarrow \frac{1}{2\beta}\,\vec{\phi}^{\,t}\,J^{-1}\vec{\phi} \to \frac{1}{2\beta\hat{J}(0)}\frac{1}{(2\pi)^d}\int d^d\vec{k} \cdots$$

$$\cdots |\hat{\phi}(\vec{k})|^2\left[1 + \ell^2 \vec{k}^2 + \cdots\right]$$

$$= \frac{1}{2\beta\hat{J}(0)}\int d\vec{r}\left[\phi^2(\vec{r}) + \ell^2 |\vec{\nabla}\phi|^2 + \cdots\right]$$

$$\sum_i \ln \mathrm{ch}\,\phi_i \to \frac{1}{a^d}\int d^d\vec{r}\, \ln[\mathrm{ch}(\phi)]$$

$$\ln[\mathrm{ch}(\phi)] \simeq \ln\left[1 + \frac{\phi^2}{2} + \frac{\phi^4}{4!} + \cdots\right] = \frac{\phi^2}{2} - \frac{\phi^4}{12}$$

$\cdots$

$$e^{-\frac{1}{2\beta}\vec{\phi}^{t} J^{-1}\vec{\phi} - U(\vec{\phi})}$$

$$\partial_\mu\phi\,\partial_\mu\phi$$

$$= e^{-\frac{1}{2\beta\hat{J}(0)}\int d^d\vec{r}\,\left[\phi(\vec{r})^2 + \ell^2|\vec{\nabla}\phi|^2 - \frac{1}{a^d}\left(\frac{\phi^2}{2} - \frac{\phi^4}{12}\right)\right]}$$

$$C = (2\pi)^{-N/2}\left(\mathrm{Det}(\beta J)\right)^{-1/2} 2^N$$

$$\int \mathcal{D}(\phi) = \lim_{N\to\infty}\int \prod_i d\phi_i$$

acción de Ginzburg-Landau.

$$S(\phi) = \int d^d\vec{r}\left[\frac{c}{2}(\vec{\nabla}\phi)^2 + \frac{a_2}{2}\phi^2 + a_4\phi^4 + \dots\right]$$

$$c = \frac{\ell^2}{\beta\hat{J}(0)}\ ;\ a_2 = \frac{k_B T}{\hat{J}(0)} - \frac{1}{a^d}\ ,\ a_4 = \frac{1}{12a^d}$$

$$\boxed{Z = C\int \mathcal{D}\phi\, e^{-S(\phi)}}$$

$$\begin{pmatrix} -1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \to \begin{pmatrix} \phi & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

$$(\vec{\nabla}\phi)^2 \longleftrightarrow \partial_\mu\phi\,\partial_\mu\phi$$

$$a_2 \longleftrightarrow m^2\ ,\ a_4 \longleftrightarrow \lambda$$

$$e^{\frac{i}{\hbar} S} \qquad e^{\frac{i}{\hbar}\int dt\, \mathcal{L}}$$

$i d\tau$

rotación de Wick $\quad t = i\tau$

$$\partial_t^2 \to -\partial_\tau^2$$

$$\partial_t^2 - \Delta \to -\partial_t^2 - \Delta$$

$$\begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \to \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$U \to e^{\frac{i}{\hbar} H(t-t')}$$

$$e^{\frac{i}{\hbar} H \Delta t}\, e^{\frac{i}{\hbar} H \Delta t}$$

$$\int dq\, |q\rangle\langle q|$$

$$e^{-\beta H} \to e^{-\Delta\beta H}\, e^{-\Delta\beta H}$$

$$\hat{J}$$

$$(\bar{S}_i \bar{S}_j) \qquad \bar{J}_i^2,\ \bar{S}_j^2 = \hbar\, S(S+1)$$

$$\left(\bar{S}_i - \bar{S}_j \right)^2$$