Ejemplo el Teorema de Mermin-Wagner para el modelo XY.

Acción de G.L. para un campo vectorial

$$\vec{\phi}(\vec{r}) = \begin{pmatrix} \phi_1(\vec{r}) \\ \phi_2(\vec{r}) \end{pmatrix}$$ de 2 componentes ϕ_1 y ϕ_2 reales.

$$S = \int d^d\vec{r} \left[\frac{c}{2}\left((\vec{\nabla}\phi_1)^2 + (\vec{\nabla}\phi_2)^2 \right) + \frac{a_2}{2}\underbrace{\vec{\phi}\cdot\vec{\phi}}_{\vec{\phi}^2} + \frac{a_4}{4}\left(\vec{\phi}^2\right)^2 \right]$$

→ simetría de rotaciones. $SO(2)$

$$\vec{\phi} \rightarrow \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \vec{\phi} \qquad \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \rightarrow \begin{pmatrix} \phi_1 \\ -\phi_2 \end{pmatrix}$$

α indep de $\vec{r}$.

también superfluidez y superconduct.

$$\psi(\vec{r}) \in \mathbb{C} \quad ; \quad \psi(\vec{r}) = \phi_1(\vec{r}) + i\,\phi_2(\vec{r})$$

$$S = \int d\vec{r} \left[\frac{c}{2} \left|\vec{\nabla}\psi\right|^2 + \frac{a_2}{2}|\psi|^2 + \frac{a_4}{4}|\psi|^4 \right]$$

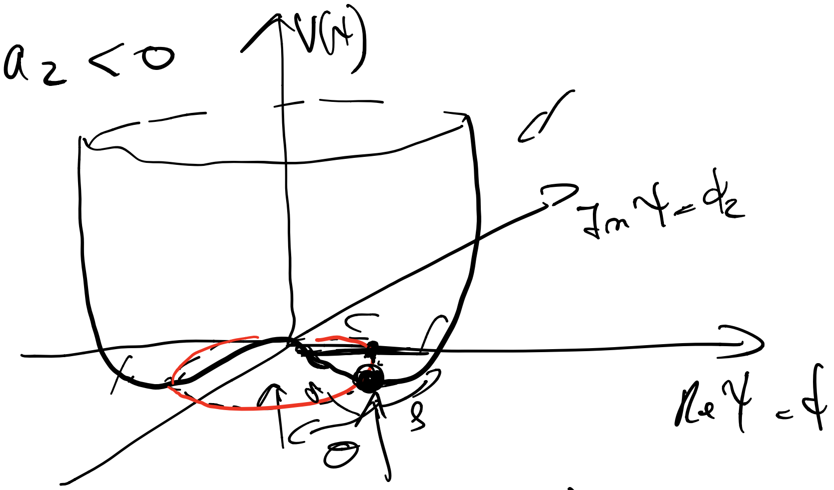
||

$$\vec{\nabla}\Psi^* \cdot \vec{\nabla}\Psi \qquad V(\Psi)$$

Simetría $\Psi(\vec{r}) \to e^{i\alpha}\Psi(\vec{r})$

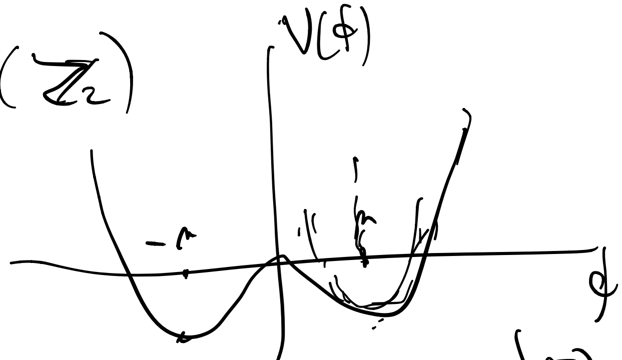
$a_2 = 0 \leftarrow$ punto crítico:

si $a_2 < 0$



· ruptura espontánea de simetría:

Para Ising ($\mathbb{Z}_2$)



$\phi(\vec{r}) = m + \delta\phi(\vec{r})$

el mínimo que tomamos es

$$\Psi_0 = \sqrt{\frac{-a_2}{a_4}}\, e^{i\theta_0}$$ con θ_0 arbitrario.

→ estudiar las fluctuaciones a nivel gaussiano al rededor de ψ_0 ($\theta_0 = 0$)

$$\psi_{(\vec{r})} = (\psi_0 + \rho(\vec{r}))\, e^{i\theta(\vec{r})}$$

$\rho \in \mathbb{R}$

$\theta \in \mathbb{R}$.

$$\Rightarrow S = \int d\vec{r} \left[\frac{c}{2}\left[(\vec{\nabla}\rho)^2 + m^2 \rho^2\right] + \mathcal{O}(\rho^3) + \frac{c}{2}\psi_0^2 (\vec{\nabla}\theta(\vec{r}))^2 + \dots \right]$$

$\rho(\vec{r})(\vec{\nabla}\theta)^2 + \dots$

$$m^2 = \frac{8}{c}\left(\frac{a_2}{2} + \frac{3a_4}{2}\psi_0^2\right) = \frac{-2a_2}{c} \Leftarrow (a_2 < 0)$$

$m^2 > 0$.

→ 2 campos reales desacoplados.

$$S_\rho = \int d\vec{r}\, \frac{c}{2}\left[(\vec{\nabla}\rho)^2 + m^2\rho\right]$$

$$\rightarrow \langle \rho(0)\rho(\vec{r})\rangle \sim \frac{e^{-\frac{|\vec{r}|}{\xi}}}{|\vec{r}|^\beta}$$

$\xi = \frac{1}{m}$

ρ campo de corto alcance.

obs $\rho = \rho_0 \neq 0$

$\rightarrow$ cuesta una energía $\sim V m^2 \rho$

$$S = \int d^d\vec{r}\ \kappa (\vec{\nabla}\theta)^2$$

modelo gaussiano "crítico"

o de largo alcance.

$\theta = \theta_0 \Rightarrow \nabla\theta_0 \Rightarrow$ no cuesta energía...

deformación de $\theta(\vec{r}) \sim \theta_0 e^{i\vec{k}\vec{r}}$

$\rightarrow$ modulación con vector de onda $\vec{k}$

$$\varepsilon \sim \kappa \vec{k}^2 \Rightarrow \varepsilon \xrightarrow[|\vec{k}| \to 0]{} 0$$

$\rightarrow$ Modo de Goldstone.

$\rightarrow$ a grandes escalas, nos podemos olvidar de ρ y sólo tomar en cuenta θ.

$$\langle \psi^*(\vec{0}) \psi(\vec{r}) \rangle \sim \langle \vec{\phi}(\vec{0}) \cdot \vec{\phi}(\vec{r}) \rangle$$

$\langle \rho \rangle = 0$

$$\xrightarrow[\vec{r} \to \infty]{} \underbrace{\langle \rho(0)\rho(\vec{r}) \rangle \langle e^{i\theta(0)} e^{i\theta(\vec{r})} \rangle}_{0} + \rho_0^2 \langle e^{i\theta(0)} e^{i\theta(\vec{r})} \rangle$$

→ vamos a integrar sobre los grados de libertad de "ρ"

$$Z = \int \mathcal{D}\psi \, e^{-S} \longrightarrow \int_{\Gamma} \mathcal{D}\rho \, \mathcal{D}\theta \, J(\cdot) \, e^{-S(\rho,\theta)}$$

$$\int \mathcal{D}\theta \, e^{-S_{eff}(\theta)}$$

$$e^{-S_{eff}(\theta)} = \int \mathcal{D}(\rho) J(\cdot) \, e^{-S(\rho,\theta)}$$

para $S_{eff}(\theta) = \int d^d r \left[\tilde{\kappa} (\nabla\theta)^2 + \tilde{\kappa}_2 \left((\nabla\theta)^2 \right)^2 + \dots \right]$

por argumentos de simetría

$$\varepsilon \sim \tilde{\kappa} |\vec{k}|^2 + \tilde{\kappa}_2 |\vec{k}|^4 + \cancel{\tilde{m}^2}$$

→ no va a aparecer un término como

$$\tilde{m}^2 \theta^2$$

→ a causa de la simetría original $U(1)$ (o $SO(2)$)

$$\psi \to e^{i\alpha}\psi \to \theta(\vec{r}) \to \theta(\vec{r}) + \alpha$$

$\vec{\nabla}\theta$ → son invariantes bajo $\theta(\vec{r}) \to \theta(\vec{r}) + \alpha$

pero $\tilde{m}^2 \theta^2$ → no es invariante bajo $\theta(\vec{r}) \to \theta(\vec{r}) + \alpha$

prohibido por la simetría $U(1)$

¿y porqué no $\partial_x \theta$?

→ prohibido por la simetría $x \to -x$

$y \to -y$

$z \to z$

¿y porqué $\vec{\nabla}\theta \cdot \vec{\nabla}\theta$

$\partial_x \cancel{\partial_y \theta}$

→ simetría $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \to R \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$$S_{eff} = \int d^d\vec{r}\, \tilde{K} (\vec{\nabla}\theta)^2$$

campo de Goldstone.

$$\langle \psi^\dagger(\vec{0})\, \psi(\vec{r}) \rangle = \psi_0^2 \langle e^{-i\theta(0)}\, e^{i\theta(\vec{r})} \rangle$$

$$\langle \vec{\Phi}(\vec{0}) \cdot \vec{\Phi}(\vec{r}) \rangle \sim \langle \vec{S}_0 \cdot \vec{S}_r \rangle$$

$$G(\vec{r}) = \psi_0^2 \left\langle e^{-i\theta(0) + i\theta(\vec{r})} \right\rangle$$

ej.: $$\int_{-\infty}^{\infty} dx\, e^{\alpha x} e^{-\sigma x^2} = \langle e^{\alpha x} \rangle$$

$$= e^{\frac{\alpha^2}{2}\langle x^2 \rangle}$$

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} dx\, x^2 e^{-\sigma x}$$

$$\langle \exp[\alpha q] \rangle = \exp\left(\frac{\alpha^2}{2}\langle q^2 \rangle\right)$$

con q = cualquier combinación lineal de $\theta(\vec{r})$, $\theta(\vec{0})$ ...

o sea $$\boxed{\left\langle e^{-i\theta(\vec{0}) + i\theta(\vec{r})} \right\rangle = e^{-\frac{1}{2}\langle [\theta(\vec{0}) - \theta(\vec{r})]^2 \rangle}}$$

$$\langle [\theta(0) - \theta(\vec{r})]^2 \rangle = \langle \theta(0)^2 \rangle + \langle \theta(\vec{r})^2 \rangle - 2\langle \theta(0)\,\theta(\vec{r}) \rangle$$

$$= 2\langle \theta(0)^2 \rangle - 2\langle \theta(0)\,\theta(\vec{r}) \rangle$$

$\parallel$ cte $= f_0^{2-\epsilon}$ $\Leftarrow$

$$\langle \theta(\vec{r})\,\theta(\vec{r}\,')\rangle = \begin{cases} -\dfrac{|x-x'|}{2K} & d=1 \\[2ex] -\dfrac{1}{2K\pi}\ln|\vec{r}-\vec{r}\,'| & d=2 \\[2ex] \dfrac{cte}{|\vec{r}-\vec{r}\,'|^{d-2}} & d\geq 3 \end{cases}$$

obs $\langle \theta(0)^2\rangle = \langle \theta(0)\,\theta(0)\rangle$

$= \lim\limits_{\vec{r}\to\vec{0}} \langle \theta(0)\,\theta(\vec{r})\rangle \sim \rho_0^{2-d}$ para $d>1$

$$\to \langle \psi^*(\vec{0})\,\psi(\vec{r})\rangle \sim \underbrace{\psi_0^2\, e^{-cte\,\rho_0^{2-d}}}_{\psi_{0\,eff}^2}\; e^{\langle \theta(\vec{0})\,\theta(\vec{r})\rangle}$$

si $\vec{r}\to\infty$?

1. d: $\langle \psi^*(0)\,\psi(\vec{r})\rangle \sim cte\; e^{-\frac{|x-x'|}{2K}} \longrightarrow 0$

Short range order.

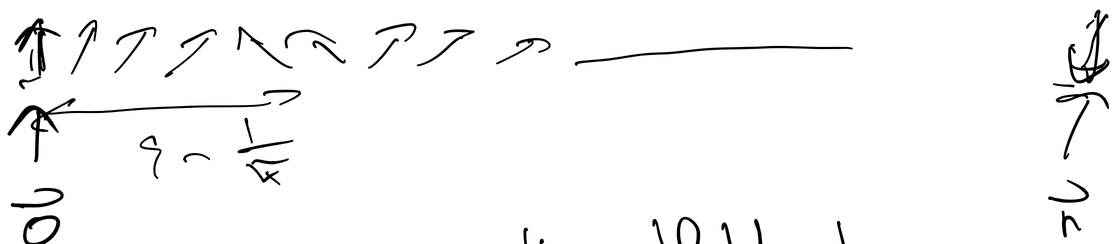
2. d: $\langle \psi^*(0)\,\psi(\vec{r})\rangle \sim cte\,\dfrac{1}{|\vec{r}|^{\frac{1}{2K\pi}}}$

Quasi Long range order.

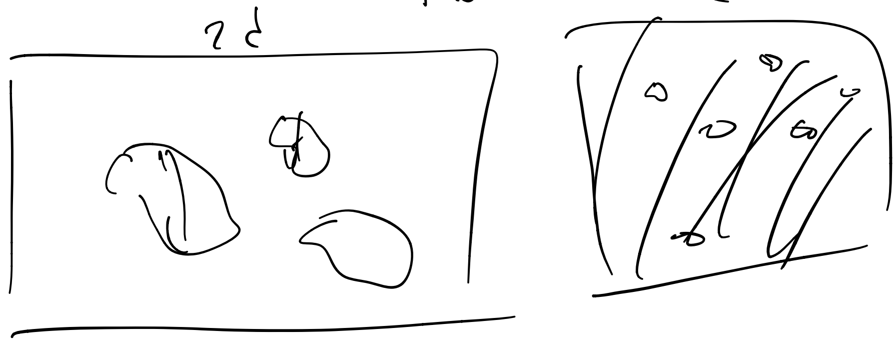
$d \geq 3$ $\quad \langle \psi^*(0)\, \psi(\vec{x}) \rangle \sim \psi_{0,eq}^2 \, e^{-\frac{cre}{|x|^{d-2}}} \to$ long range order.

$\psi_{0,eq}^2 \neq 0$

→ Sólo para $d \geq 3$ el sistema está ordenado.



Versión más sencilla del th. de Mermin-Wagner.



→ Restauración de simetría para Ising en 1-D.

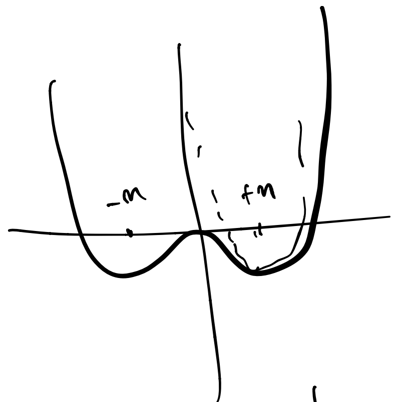
(obs esto no es Mermin-Wagner)

G.L.

$$S = \int dx \left\{ \frac{1}{2}\left(\frac{d}{dx}\phi\right)^2 - \frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 \right\}$$

$a_2 = -\mu^2 < 0$

$a_4 \to \lambda$



$m = \frac{\mu}{\sqrt{\lambda}}$

si uno escribe $\phi(x) = m + \psi(x)$

$$\Rightarrow S = S_0 + \int dx \left\{ \left(\frac{d\psi}{dx}\right)^2 + \mu^2\psi^2 + \dots \right\}$$

$$\langle \phi(x_1)\,\phi(x_2) \rangle = \langle (m + \psi(x_1))\,(m + \psi(x_2)) \rangle$$

$$\sim m^2 + \underbrace{\langle m\,\cancel{\psi(x_1)} \rangle}_{0} + \underbrace{\langle m\,\cancel{\psi(x_2)} \rangle}_{0} + \langle \psi(x_1)\,\psi(x_2) \rangle$$

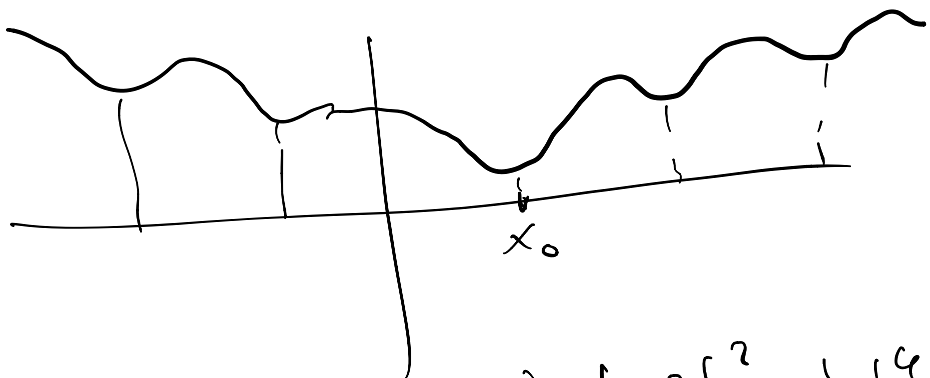
$$\langle \phi(x_1)\,\phi(x_2) \rangle \sim m^2 + e^{-\frac{|x_1 - x_2|}{\xi}} \xrightarrow[|x_1 - x_2| \to \infty]{} m^2 \neq 0$$

$\xi = \frac{1}{\mu}$

$$I = \int_{-\infty}^{\infty} dx\, e^{-f(x)}$$

x_0 cerca del mínimo

$$f(x) = f(x_0) + \frac{1}{2} f''(x_0)(x-x_0)^2 + \dots$$

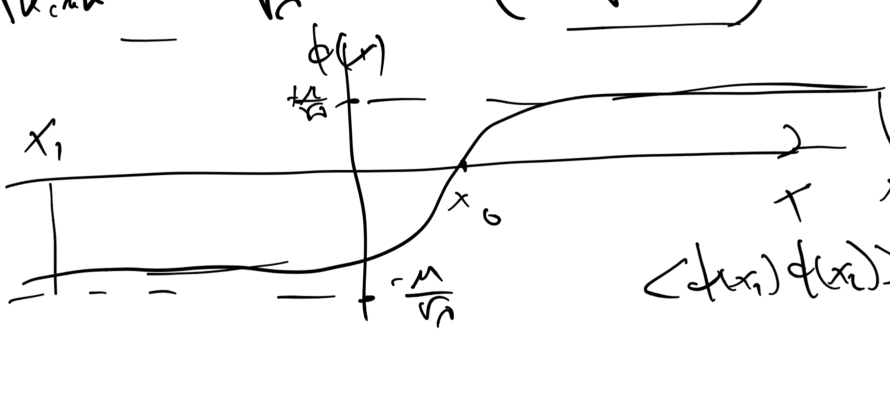


Para $S = \int dx \left\{ \frac{1}{2}\left(\frac{d\phi}{dx}\right)^2 - \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4 \right\}$

hay otros mínimos a parte de $\phi = \pm m$

$$\frac{\delta S}{\delta \phi} = 0 \Rightarrow \phi'' + \mu^2\phi - \lambda\phi^3 = 0$$

$$\phi_{kink}(x) = \frac{\mu}{\sqrt{\lambda}}\, tgh\left(\frac{\mu(x-x_0)}{\sqrt{2}}\right) \quad \forall x_0$$

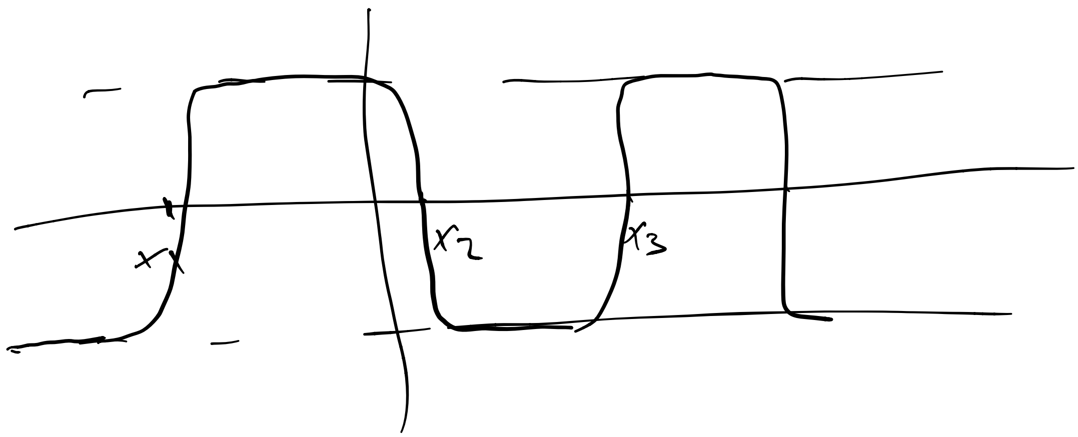


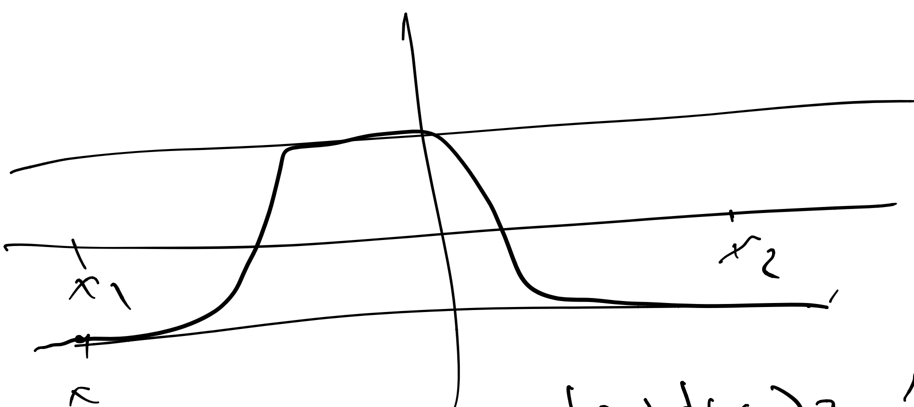
$$\langle \phi(x_1)\phi(x_2)\rangle = -\frac{\mu^2}{\lambda}$$

$\downarrow$

$\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow$

$$S_{\text{kink}} = S\{\phi_{\text{kin}}(x)\} = \boxed{\frac{2\sqrt{2}\,\mu^3}{3\lambda}}$$





$$\langle \phi(x_1)\,\phi(x_2)\rangle \sim \frac{\mu^2}{\lambda}$$

$$\langle \phi(x_1)\,\phi(x_2)\rangle = \frac{\mu^2}{\lambda}\sum_{N=0}^{\infty}(-1)^N e^{-N S_{\text{kink}}}\int_{x_1}^{x_2} dx_1^k\,dx_2^k\cdots dx_N^k$$

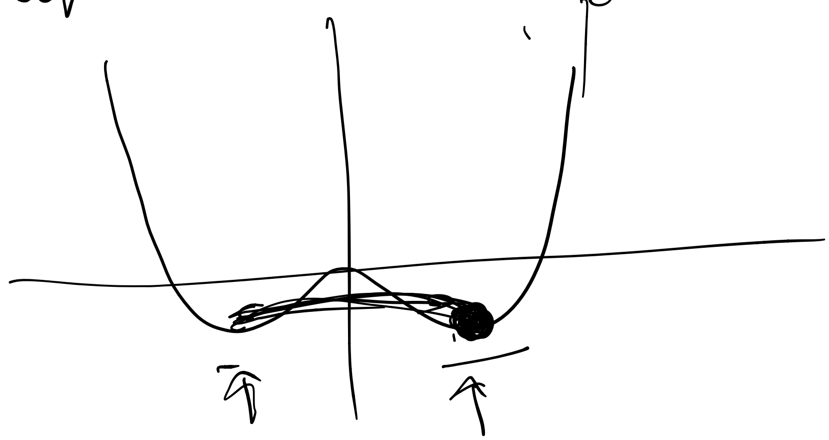
$$x_1 < x_1^k < x_2^k \cdots < x_N^k < x_2$$

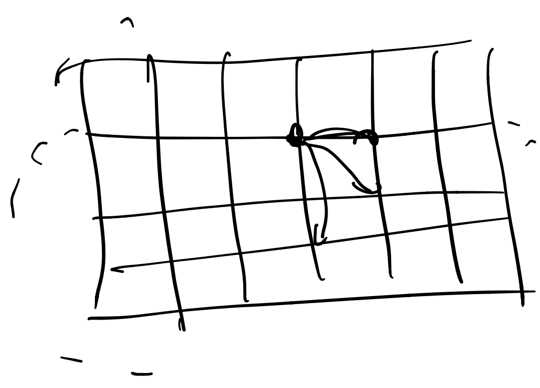
$$\to \langle \phi(x_1)\phi(x_2)\rangle \simeq \frac{\mu^2}{\lambda}\sum_{N=0}^{\infty}(-1)^N e^{-NS_{\min}}\frac{\left(\int_{x_1}^{x_2}dx_i^k\right)^N}{N!}$$

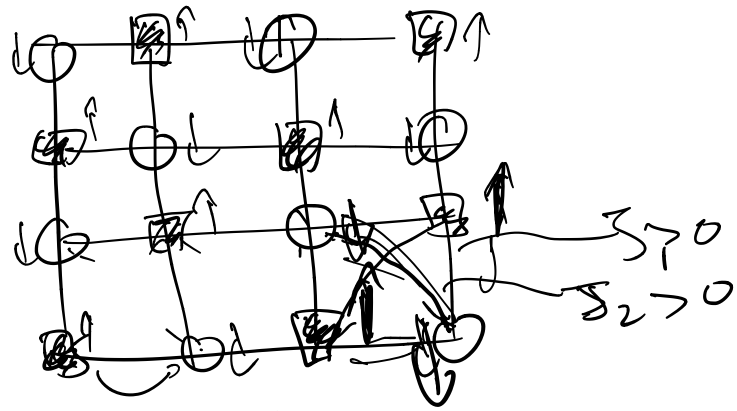
$$\Rightarrow \langle \phi(x_1)\phi(x_2)\rangle \simeq \frac{\mu^2}{\lambda}e^{-\frac{|x_1-x_2|}{\xi}}$$

$$\text{donde } \frac{1}{\xi} = e^{-S_{\text{inst.}}}$$

integral de camino de Feynman



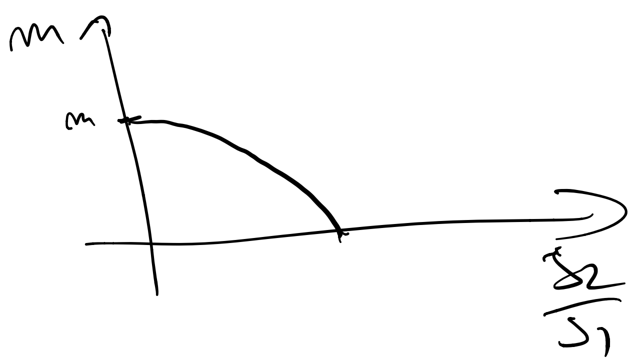




Ferro $\leftrightarrow$ AF.

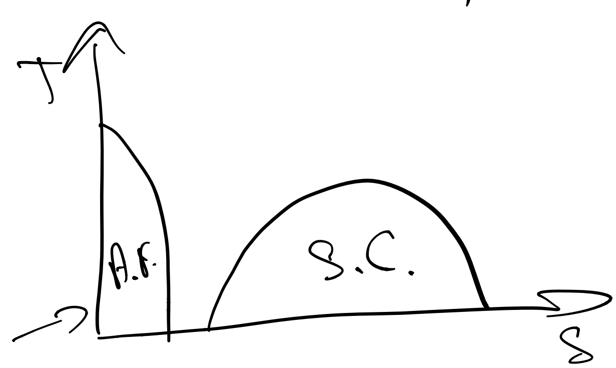
$$\left\{ \begin{array}{l} \sigma_{\square} \to -\sigma_{\square} \\ \sigma_{O} \to \sigma_{O} \end{array} \right| \quad J \to -J$$

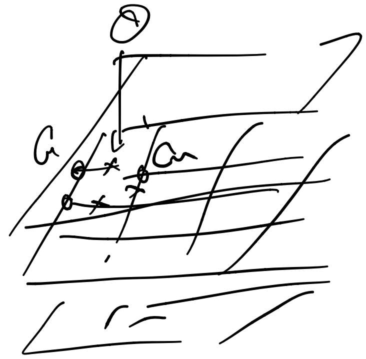
$T < T_c$



Cu-O

High T_c.

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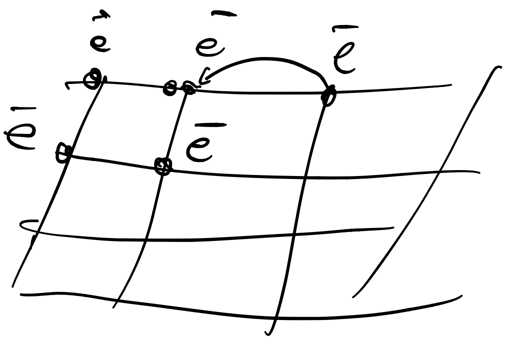
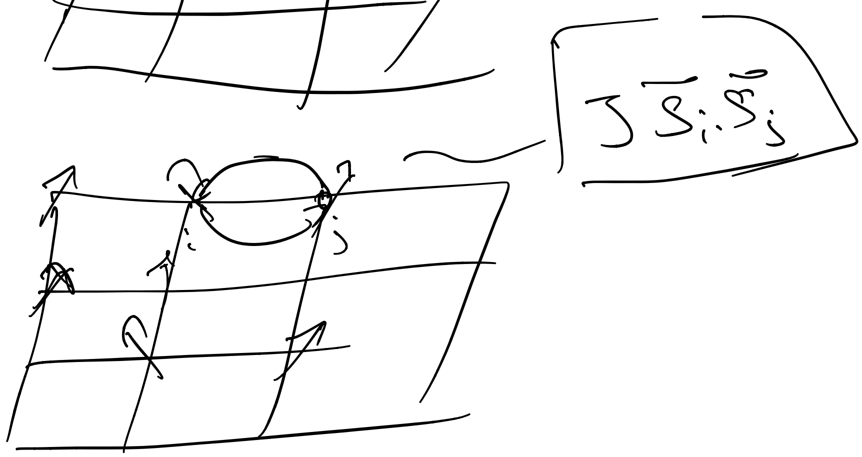
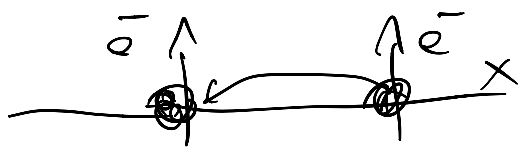
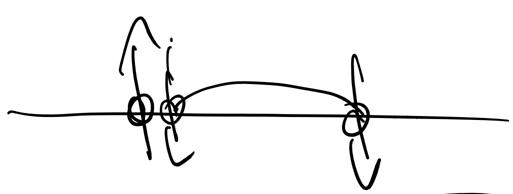
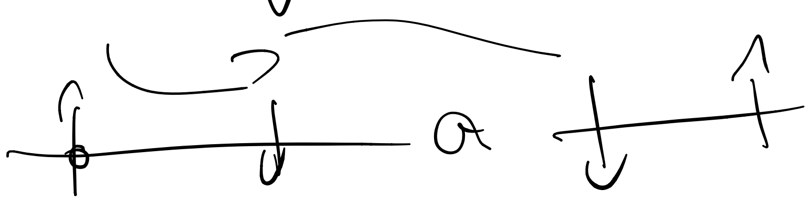

e⁻
e⁻
e⁻
e⁻
e⁻


i
j
J S_i · S_j


e⁻
e⁻
x

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