IV El grupo de Renormalización

1| invariancia de escala.

* transición de fase de 2do órden o más

$\xi \to \infty$, $\to$ límite al continuo, $a_0 \to 0$.

$\to$ Estado fractal.

$\to$ las funciones de correlación

Ya no tienen $e^{-\frac{|\vec{r}|}{\xi}}$

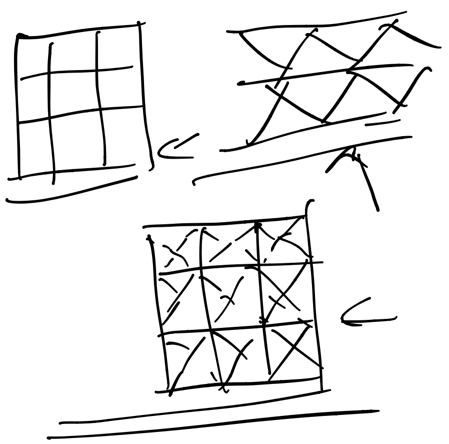
$\rightsquigarrow \ln |\vec{r}|$ en 2-D

$\rightsquigarrow \frac{1}{|\vec{r}|^{2-d+\eta}}$ en general.

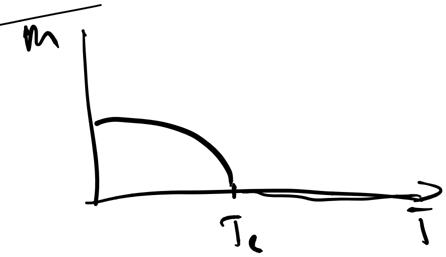
$\to$ los detalles micro. pierden su importancia

$\to$ el sistema está descrito para una "teoría de campos universal".

Ising en 2-D
Ferro.



diferentes T_c



→ en el punto crítico
→ misma "T.C. universal"
→ los exp. críticos, ν, β, γ, etc.

son los mismos.

→ clase de universalidad.

depende de: * la dimensión del sistema.

* de la simetría que se rompe: $\mathbb{Z}_2$ (Ising), $O(3)$ Heisenberg etc...

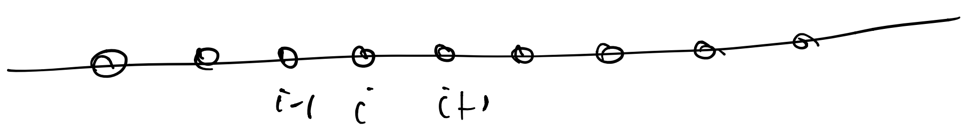
* casos particulares, puntos multi-críticos.

La idea del grupo de renormalización R.G., es "ver el sistema de más en más lejos" para borrar los detalles micro "superfluos" y quedarnos con la "teoría universal".
(Kadanoff, Migdal, K. Wilson, M. Fischer..)

2) R.G. en espacio real

→ el modelo de Ising en 1-D.

$\sigma_{N+1} \equiv \sigma_1$

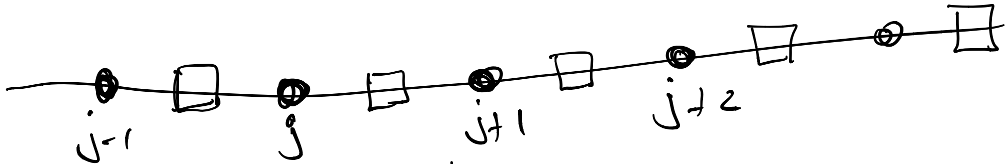


$$Z = \sum_{\{\sigma_i\}} e^{K\sum_i \sigma_i \sigma_{i+1}}$$

$$Z = \mathrm{Tr}\left\{T^N\right\}, \quad T = \begin{bmatrix} e^{K} & e^{-K} \\ e^{-K} & e^{K} \end{bmatrix}$$

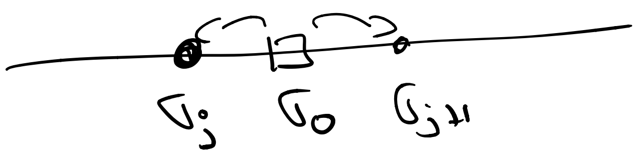
en la base $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ y $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ $\quad O = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$

$$O^T T O \to T_d = \begin{pmatrix} 2\cosh\kappa & 0 \\ 0 & 2\sinh\kappa \end{pmatrix}$$



de $N \Rightarrow \frac{N}{2}$ sitios.

$$Z = \sum_{\{\sigma_\bullet\}\{\sigma_\square\}} e^{\kappa \sum_i \sigma_i \sigma_{i+1}} \to \tilde{Z} = \sum_{\{\sigma_\bullet\}} e^{\tilde{\kappa} \sum_j \sigma_j \sigma_{j+1}}$$



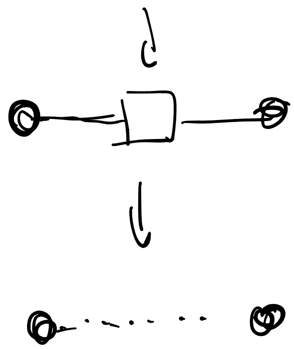
$$\sum_{\{\sigma_j\}\{\sigma_{j+1}\}\{\sigma_0\}} e^{\kappa(\sigma_j \sigma_0) + \kappa(\sigma_0 \sigma_{j+1})}$$

$$= \sum_{\substack{\sigma_j = \pm 1,\, \sigma_{j+1} = \pm 1 \\ \sigma_0 = \pm 1}} e^{\kappa \sigma_0 (\sigma_j + \sigma_{j+1})}$$

$$= \sum_{\sigma_j=\pm1,\, \sigma_{j+1}=\pm1} 2\,\mathrm{ch}\left[K(\sigma_j+\sigma_{j+1})\right]$$

$$\| $$

$$e^{\tilde{K}\sigma_j\sigma_{j+1} + \tilde{c}}$$



des de el comienzo

$$\boxed{Z = \sum_{\{\sigma\}} e^{K\sum_i(\sigma_i\sigma_{i+1} + c)}}$$

$\downarrow$ decimación

$$\tilde{Z} = \sum_{\{\sigma\}} e^{\tilde{K}\sum_j \sigma_j\sigma_{j+1} + \tilde{c}}$$

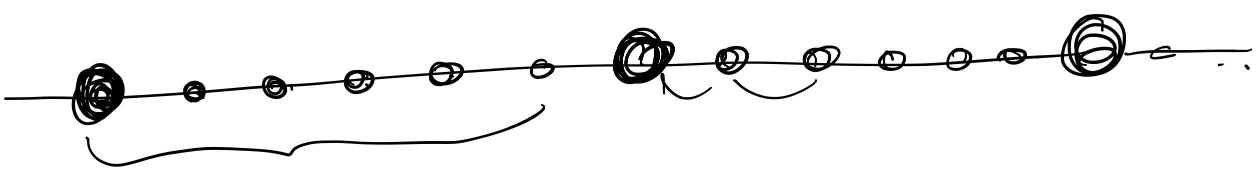
$\downarrow$ decimación

$$\tilde{\tilde{Z}} = \dots e^{\tilde{\tilde{K}}\dots + \tilde{\tilde{c}}}$$

$$Z(K,C) = \mathrm{tr}\left\{T^N\right\}$$

$$T = e^{C}\begin{pmatrix} e^{K} & e^{-K} \\ e^{-K} & e^{K} \end{pmatrix}$$

$$T_d = e^{C}\begin{pmatrix} 2\operatorname{ch}K & 0 \\ 0 & 2\operatorname{sh}K \end{pmatrix}$$



b espines.

Sumar sobre los b-1 espines intermedios

$N \to \frac{N}{b}$ espines.

$$Z(K,C) \to \tilde{Z}(\tilde{K},\tilde{C})$$

$$Z = \mathrm{tr}\left\{\Big[\underbrace{e^{C}\begin{pmatrix} 2\operatorname{ch}K & 0 \\ 0 & 2\operatorname{sh}K \end{pmatrix}}_{T}\Big]^{N}\right\}$$

$$\tilde{Z} = \mathrm{tr}\left\{\Big[\underbrace{e^{\tilde{C}}\begin{pmatrix} 2\operatorname{ch}\tilde{K} & 0 \\ 0 & 2\operatorname{sh}\tilde{K} \end{pmatrix}}_{\tilde{T}}\Big]^{\frac{N}{b}}\right\}$$

$\tilde{T} = T^b$

$$e^{bc}\begin{pmatrix} 2^b(\mathrm{ch}\,k)^b & 0 \\ 0 & 2^b(\mathrm{sh}\,k)^b \end{pmatrix} = e^{\tilde{c}}\begin{pmatrix} 2\,\mathrm{ch}(\tilde{k}) & 0 \\ 0 & 2\,\mathrm{sh}(\tilde{k}) \end{pmatrix}$$

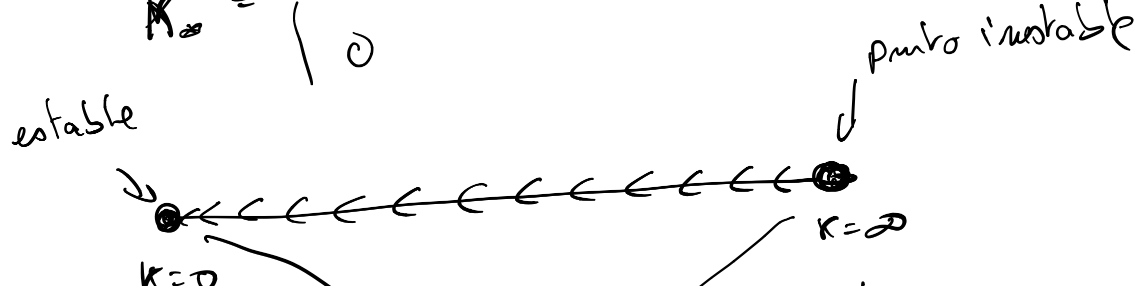
$$\Rightarrow \boxed{\mathrm{th}\,\tilde{k} = (\mathrm{th}(k))^b} \quad \longleftarrow \quad b \simeq (1+\delta\ell)$$

puntos fijos k_*

$$\mathrm{th}(k_*) = [\mathrm{th}(k_*)]^b$$

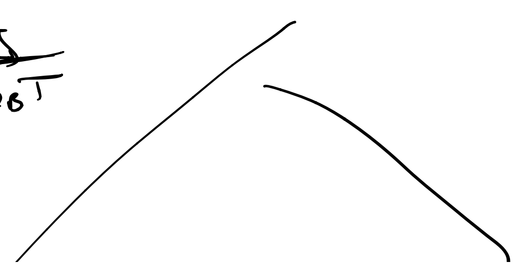
$$x = \mathrm{th}(k_*) \Rightarrow x = x^b \rightarrow \begin{cases} x = 0 \\ x = 1 \end{cases}$$

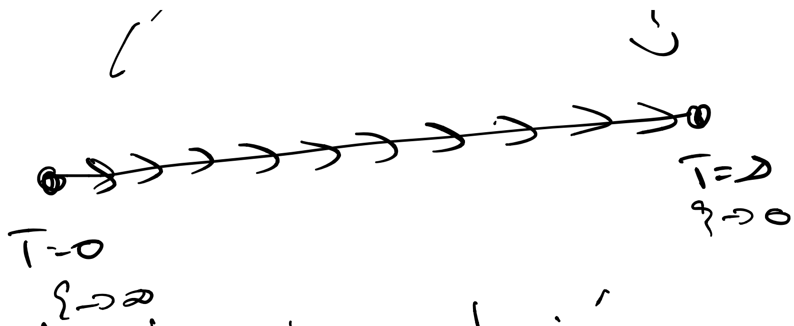
$$k_* = \begin{cases} \infty \\ 0 \end{cases}$$



$x = (1-\epsilon)$ $\quad x^b < x \quad , (x^b)^b < x^b \dots$

<u>obs</u> $k = \frac{J}{k_B T}$

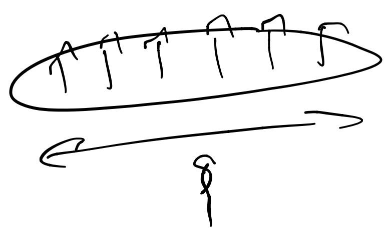


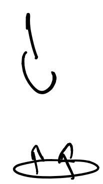


ev : longitud de correlación

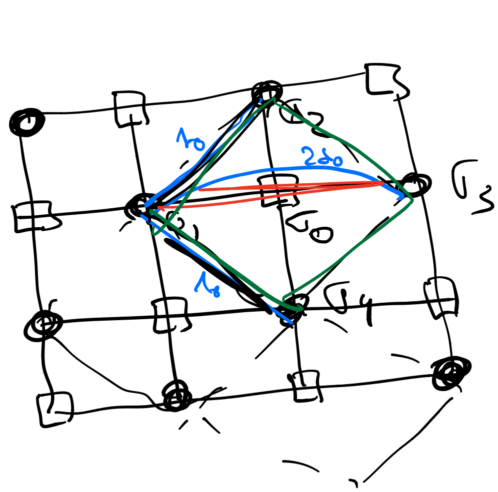
$$\xi(K) \to \hat{\xi}(\tilde{K})$$

$$\hat{\xi} = \frac{\xi}{b}$$





2-D ?



$$Z = \sum_{\{\sigma_0\}\{\sigma_a\}} e^{K\sum_{\langle ij\rangle}\sigma_i\sigma_j}$$

$$\sum_{\sigma_0=\pm 1} e^{K[\sigma_0\sigma_1+\sigma_0\sigma_2+\sigma_0\sigma_3+\sigma_0\sigma_4]}$$

$$= \sum_{\sigma_0=\pm 1} e^{K\sigma_0[\sigma_1+\sigma_2+\sigma_3+\sigma_4]}$$

$$= \underline{2\,\mathrm{ch}\left[K[\sigma_1+\sigma_2+\sigma_3+\sigma_4]\right]}$$

$$\underline{\sigma_i \to -\sigma_i}$$

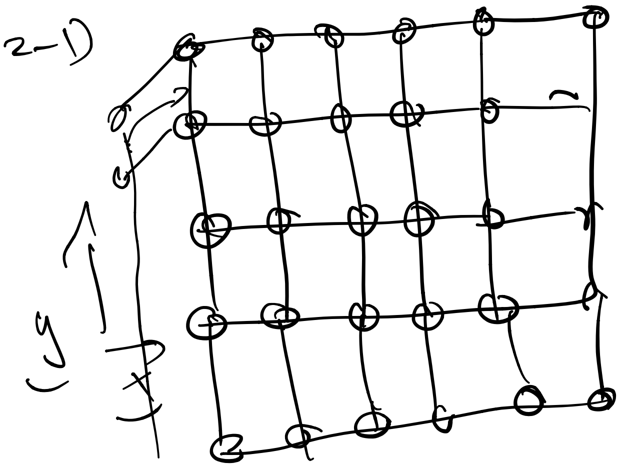
$$\sim e^{\tilde{c}+\tilde{K}[\sigma_1\sigma_2+\sigma_1\sigma_3+\sigma_1\sigma_4+\dots] + \cancel{\tilde{\lambda}[\sigma_1\sigma_2\sigma_3\sigma_4]}}$$

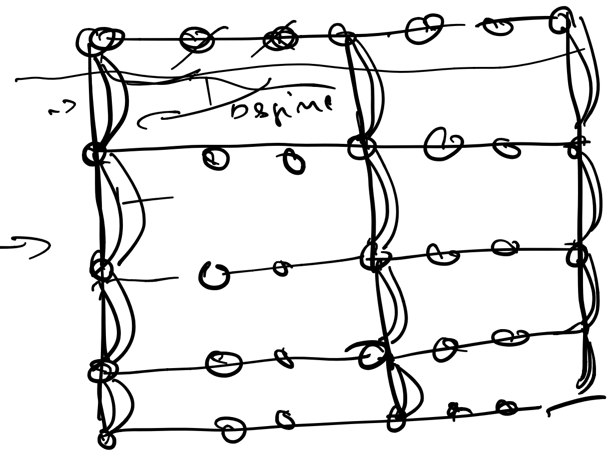
$$H(K,c) \to \hat{H}(\hat{K}_1,\hat{K}_2,\hat{\lambda}) \to \hat{\hat{H}}(\hat{\hat{K}}_1,\hat{\hat{K}}_2,\hat{\hat{\lambda}}_1,\hat{\hat{\lambda}}_2\dots)$$

→ <u>truncación</u>.

→ la decimación de <u>Migdal</u>-Kadanoff.

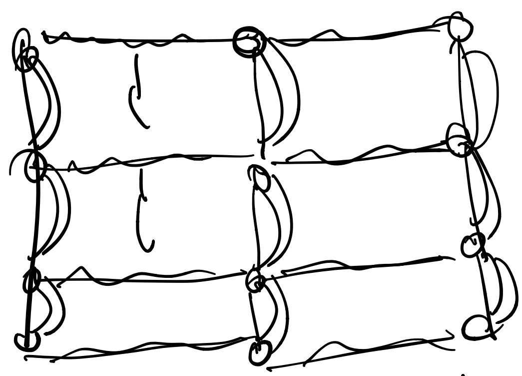
2-1)





$\tilde{K}_y = b K_y$ y luego $th(\tilde{K}_x) = th(K_x)^b$

$$\tilde{K}_x = th^{-1}\left[(th(K_x))^b\right]$$



→ luego se cambia $x \Leftrightarrow y$.

$$\boxed{\tilde{K} = b\, th^{-1}\left[(th(bK))^b\right]}$$

→ prolongación a $b \in \mathbb{R}$, $b \sim e^{\delta \ell} \sim (1+\delta\ell)$

$$th^{-1}\left[\left(th\left[(1+\delta\ell)K\right]\right)^{(1+\delta\ell)}\right] =$$

$$\simeq (1+\delta\ell)\,K + \delta\ell\left[\frac{1}{2}\,\mathrm{sh}\,2K \cdot \ln(\mathrm{th}(K))\right] + O(\delta\ell^2)$$

$$\tilde{K} - K = K + \delta\ell\left[1 + \frac{1}{2}\left[\mathrm{sh}\,2K\,\ln(\mathrm{th}K)\right]\right]$$

$\tilde{K} - K \to dK$

$\delta\ell \to d\lambda$

$$\Rightarrow \frac{dK}{d\ell} = K + \frac{1}{2}\left[\mathrm{sh}(2K)\ln\left[\mathrm{th}\,K\right]\right]$$

en D dimensiones

$$\frac{dK}{d\ell} = (D-1)K + \frac{1}{2}\left[\mathrm{sh}(2K)\ln\left[\mathrm{th}(K)\right]\right] \quad (*)$$

obs si D=1 la misma recurrencia de antes

tratamos D como una variable real

$$\epsilon = D-1$$

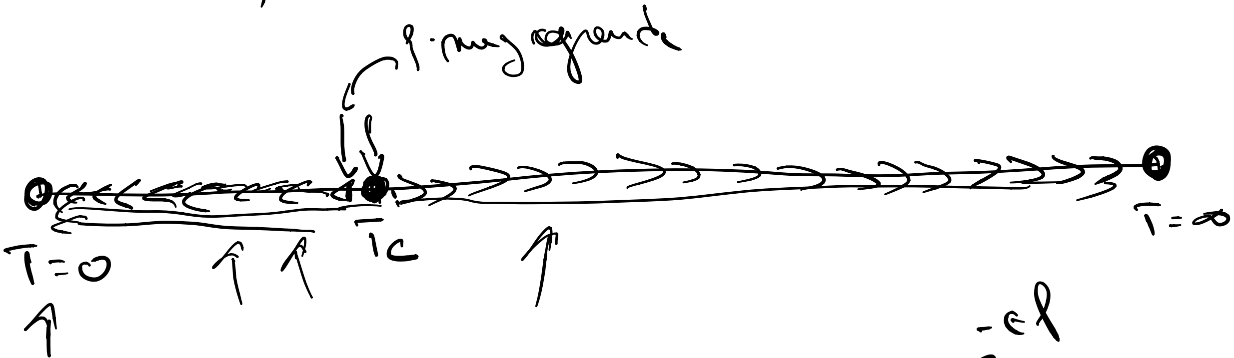
$$K = \frac{J}{k_B T}$$

(*) para T pequeño.

$$\to \text{es} \quad \frac{dT}{d\ell} = -\epsilon T + \frac{k_B}{2J}T^2 + O(T^3) \ldots$$

→ dos puntos fijos:

$$T = 0 \quad , \quad T = T_c = \frac{2\epsilon J}{k_B}$$



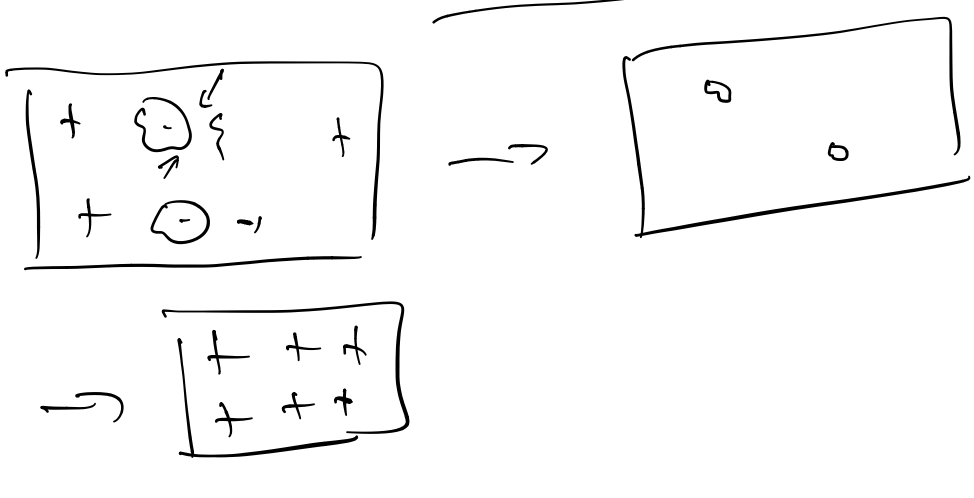
$T \sim 0 \qquad \frac{dT}{dl} = -\epsilon T \quad \rightarrow \quad T \sim T_0 e^{-\epsilon l}$

$\epsilon = D - 1 > 0$

Si ponemos $T = T_c + \delta T$

ev: $\frac{d\,\delta T}{dl} = \epsilon\,\delta T + \mathcal{O}(\delta T^2)$

$\rightarrow \underline{\delta T = \delta T_0 e^{\epsilon l}}$



$$\delta T \sim \delta T_0 e^{\epsilon \ell}$$

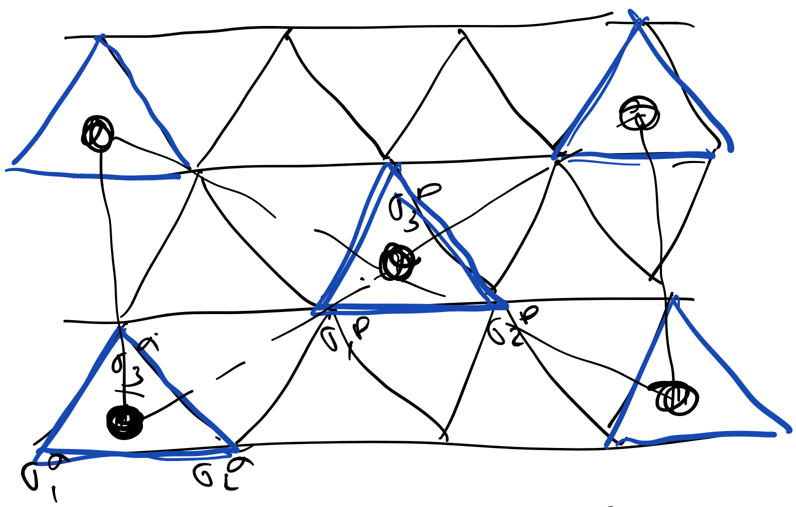
$$a_0 \to e^{\ell} a_0 , \quad \xi(\ell) \sim e^{-\ell} \xi(0)$$

$$\frac{\xi(\ell)}{\xi_0} \sim e^{-\ell} = \left(\frac{\delta T}{\delta T_0}\right)^{-\nu} = e^{-\ell \epsilon \nu}$$

[illegible] ν

$$\Rightarrow \nu = \frac{1}{\epsilon} = \frac{1}{D-1}$$

[illegible]



$$Z = \sum_{\{\sigma\}} e^{K_1 \sum_{\Delta} \sigma_i^\alpha \sigma_j^\alpha + K_2 \sum \sigma_i^\alpha \sigma_j^\beta}$$

$$= \sum_{\{\sigma\}} e^{H_0 + U}$$

$$Z = \sum_{\{\sigma\}} e^{H_0}\left(1 + U + \frac{1}{2}U^2 + \dots\right)$$

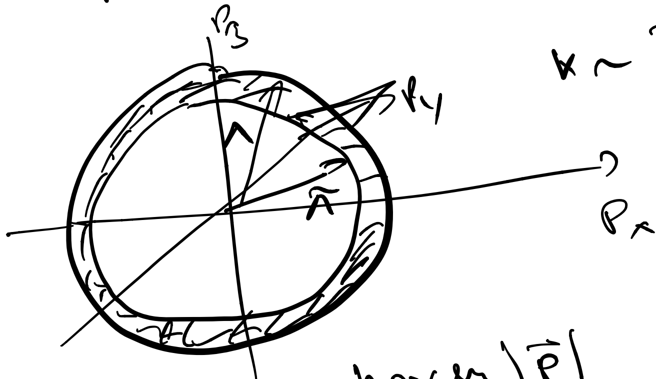
$$= \left(1 + \langle U\rangle + \frac{1}{2}\langle U^2\rangle + \dots\right)$$

$$\Rightarrow e^{\langle U\rangle + \frac{1}{2}\left[\langle U^2\rangle - \langle U\rangle^2\right] + \dots}$$

$$\hookrightarrow Z_{eff} = \sum_{\{\sigma_\bullet\}} e^{\tilde{K}_{ef} \sum \sigma_j \sigma_j}$$

3) El R.G. para teorías de campos

→ la decimación no se hace en espacio real pero en el espacio de Fourier.



$$k \sim \frac{2\pi}{a}$$

hay un $|\vec{p}|_{max} = \Lambda$

$$Z = \int \prod_x D\phi(x) \rightarrow \int \prod_{\vec{p}} d\tilde{\phi}(\vec{p})$$

$\hat{\Lambda} < \Lambda$

$$\int \prod_{\vec{p}} d\hat{\phi}_{\vec{p}}\, e^{-S} = \int \prod_{|\vec{p}|<\hat{\Lambda}} d\hat{\phi}_{\vec{p}} \underbrace{\prod_{\hat{\Lambda}\leq|\vec{p}|<\Lambda} d\hat{\phi}_{\vec{p}}\; e^{-S}}$$

G.L

$$= \int \prod_{|\vec{p}|<\hat{\Lambda}} d\hat{\phi}_{\vec{p}}\; e^{-\tilde{S}}$$

ejemplo invariante de escala:

→ modelo gaussiano sin "masa"

acción del M.G.

$$S = \int d\vec{n} \left[\frac{c}{2} |\nabla\phi|^2 + \frac{m^2}{2}\phi^2 \right]$$

$m \to 0$

$$S_{i.escala} = \int d\vec{n} \left[\frac{c}{2} |\nabla\phi|^2 \right]$$

$\vec{n} \to d\vec{n} \qquad \frac{\partial}{\partial n^\mu} \to \frac{1}{d}\frac{\partial}{\partial n^\mu}$

$\phi \to d^{2-D}\phi \;;\; c \to c$

$$S(a_2, e, a_4, a_6 \ldots) \to \widetilde{S}(\underset{4}{\widehat{a_2}}, \tilde{e}, \underset{4}{\widetilde{a_4}}\, \underset{7}{\widehat{a_6}}, \ldots)$$