# Física de Partículas

## Interacción Débil

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Latin American alliance for Capacity buildiNG in Advanced physics
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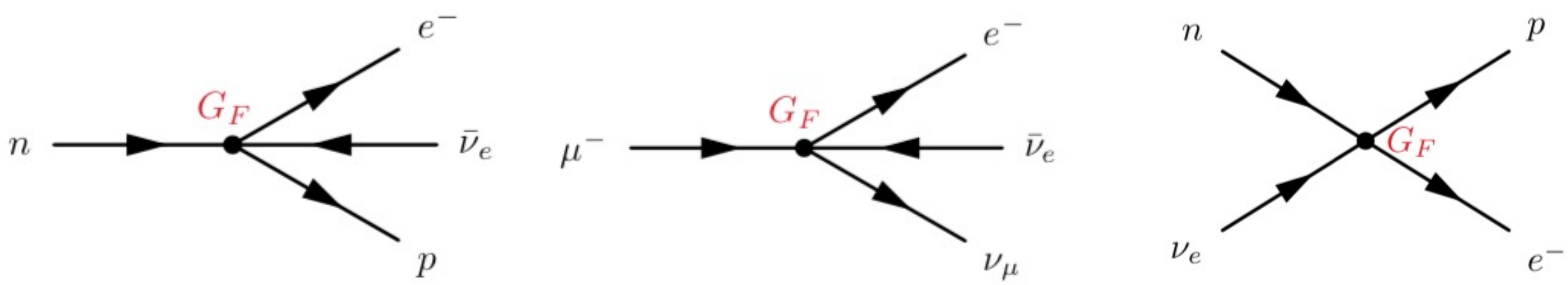
Cofinanciado por el programa Erasmus+ de la Unión Europea

# The Weak Interaction

- By the 1930's it was clear there was a "weak" interaction responsible for beta decay and other decays involving Pauli's neutrino:

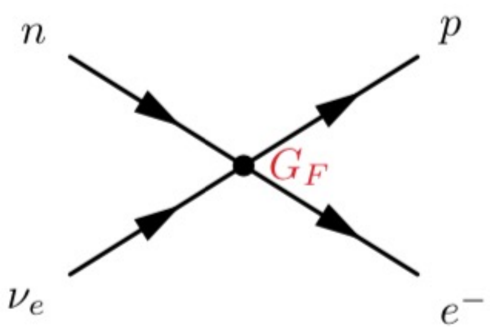
$$n \rightarrow p^+ + e^- + \bar{\upsilon}$$
$$\pi \rightarrow \mu + \upsilon$$
$$\mu \rightarrow e + 2\upsilon$$

- Fermi's explanation was a 4-fermion contact interaction




- Where the coupling strength is the Fermi constant $G_F$

- Neutrino-neutron scattering in the Fermi theory
- The differential cross section (remember the Born approximation):




$$\frac{d\sigma}{d\Omega} = \frac{E^2}{(2\pi)^2} |\mathcal{M}|^2$$

- And the matrix element:

$$|M_{fi}|^2 \approx 4G_F^2$$

- So the differential cross section reads:

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 E_e^2}{\pi^2}$$

- Fermi theory breaks down at high energies

# Parity in QED and QCD

- Remember that the parity operator for the Dirac spinors is the $\gamma^0$ matrix:

$$u \xrightarrow{\hat{P}} \hat{P}u = \gamma^0 u$$

$$\overline{u} \xrightarrow{\hat{P}} \overline{u}\gamma^0$$

- So the electron current in the $eq \to eq$ process, transforms as:

$$j_e^\mu = \overline{u}(p_3)\gamma^\mu u(p_1) \xrightarrow{\hat{P}} \overline{u}(p_3)\gamma^0\gamma^\mu\gamma^0 u(p_1)$$

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- The time component: $j_{\mathrm{e}}^{0} \xrightarrow{\hat{P}} \overline{u}\gamma^0\gamma^0\gamma^0 u = \overline{u}\gamma^0 u = j_{\mathrm{e}}^{0}$

- The spatial component: $j_{\mathrm{e}}^{k} \xrightarrow{\hat{P}} \overline{u}\gamma^0\gamma^k\gamma^0 u = -\overline{u}\gamma^k\gamma^0\gamma^0 u = -\overline{u}\gamma^k u = -j_{\mathrm{e}}^{k}$

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- So the four-vector product of the QED matrix element:

$$j_e \cdot j_q = j_e^0 j_q^0 - j_e^k j_q^k \xrightarrow{\hat{P}} j_e^0 j_q^0 - (-j_e^k)(-j_q^k) = j_e \cdot j_q$$

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- The form of the QCD vertex is the same except for colour factors: Parity is conserved in QCD

# Wu's experiment

- In 1957 Wu and collaborators studied the beta decay of polarised cobalt-60:

$$^{60}\mathrm{Co} \rightarrow {}^{60}\mathrm{Ni}^{*} + e^{-} + \overline{\nu}_{e}$$

- For a fixed magnetic field, the nuclear magnetic moment of the cobalt was aligned
- Electrons were detected at different angles with respect to the direction of the field
- The rate at which electrons were emitted on either direction of the field, should be the same (if parity were conserved)

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- Electrons were detected at different angles with respect to the direction of the field
- The rate at which electrons were emitted on either direction of the field, should be the same (if parity were conserved)
- However, more electrons were observed in the direction opposite the magnetic field – Parity Violation of Weak Interactions

# Weak interaction structure

- From the observation of parity violation, it is clear the weak interaction vertex must have a different form to the QED and QCD vertices:

$$j^{\mu} = \overline{u}(p')\gamma^{\mu}u(p)$$

- We need a Lorentz Invariant Matrix element, and there are only 5 such bilinear covariants :

| Type | Form | Components | Boson spin |
|---|---|---|---|
| Scalar | $\overline{\psi}\phi$ | 1 | 0 |
| Pseudoscalar | $\overline{\psi}\gamma^5\phi$ | 1 | 0 |
| Vector | $\overline{\psi}\gamma^{\mu}\phi$ | 4 | 1 |
| Axial vector | $\overline{\psi}\gamma^{\mu}\gamma^5\phi$ | 4 | 1 |
| Tensor | $\overline{\psi}(\gamma^{\mu}\gamma^{\nu} - \gamma^{\nu}\gamma^{\mu})\phi$ | 6 | 2 |

# Weak interaction structure

- The most general Lorentz Invariant form for the interaction between a fermion and a boson is a linear combination of the bilinear covariants
- If the boson has spin 1

$$j^{\mu} \propto \overline{u}(p')(g_V \gamma^{\mu} + g_A \gamma^{\mu}\gamma^5)u(p) = g_V j_V^{\mu} + g_A j_A^{\mu}$$

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where the current has been split into vector and axial components

- Parity transformation for axial current:

$$j_A^\mu = \overline{u}\gamma^\mu\gamma^5 u \xrightarrow{\hat{P}} \overline{u}\gamma^0\gamma^\mu\gamma^5\gamma^0 u = -\overline{u}\gamma^0\gamma^\mu\gamma^0\gamma^5 u$$

- Time component: $j_A^0 = \xrightarrow{\hat{P}} -\overline{u}\gamma^0\gamma^0\gamma^0\gamma^5 u = -\overline{u}\gamma^0\gamma^5 u = -j_A^0$

- Spatial component: $j_A^k = \xrightarrow{\hat{P}} -\overline{u}\gamma^0\gamma^k\gamma^0\gamma^5 u = +\overline{u}\gamma^k\gamma^5 u = +j_A^k$

# Weak interaction structure

- The vector and axial currents under a parity transformation:

$$j_V^0 \xrightarrow{\hat{P}} +j_V^0, \quad j_V^k \xrightarrow{\hat{P}} -j_V^k, \quad \text{and} \quad j_A^0 \xrightarrow{\hat{P}} -j_A^0, \quad j_A^k \xrightarrow{\hat{P}} +j_A^k$$

- The product of two vector or two axial currents is invariant under parity, but not the product of vector and axial currents
- This structure could therefore explain the parity violation observed in the weak interaction

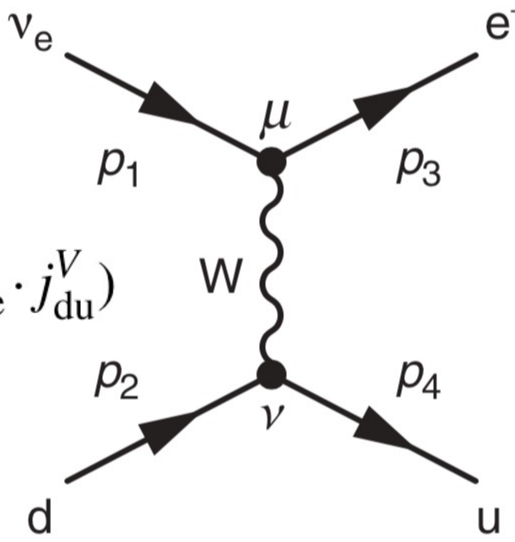
# Weak interaction structure

- Consider the inverse beta decay process:

$$j^{\mu}_{\nu e} = \overline{u}(p_3)(g_V\gamma^{\mu} + g_A\gamma^{\mu}\gamma^5)u(p_1) = g_V j^V_{\nu e} + g_A j^A_{\nu e}$$

$$j^{\nu}_{du} = \overline{u}(p_4)(g_V\gamma^{\nu} + g_A\gamma^{\nu}\gamma^5)u(p_2) = g_V j^V_{du} + g_A j^A_{du}$$

$$\mathcal{M}_{fi} \propto j_{\nu e}\cdot j_{du} = g_V^2 j^V_{\nu e}\cdot j^V_{du} + g_A^2 j^A_{\nu e}\cdot j^A_{du} + g_V g_A (j^V_{\nu e}\cdot j^A_{du} + j^A_{\nu e}\cdot j^V_{du})$$



# Weak interaction structure (V-A)

- Consider the inverse beta decay process:

$$j^{\mu}_{\nu e} = \overline{u}(p_3)(g_V\gamma^{\mu} + g_A\gamma^{\mu}\gamma^5)u(p_1) = g_V\, j^{V}_{\nu e} + g_A\, j^{A}_{\nu e}$$

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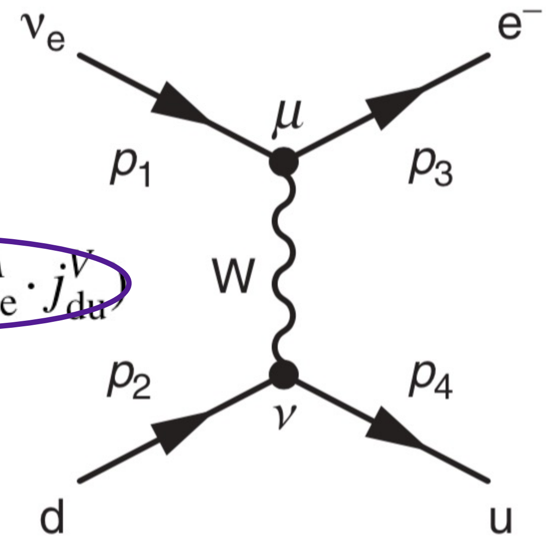


not invariant under Parity

- From experiments, we know the weak charged current due to the exchange of W bosons has the vertex factor:

$$\frac{-ig_W}{\sqrt{2}}\frac{1}{2}\gamma^{\mu}(1-\gamma^5)$$

- Remember the Chiral projection operators:
  - Any Dirac spinor can be decomposed into left and right handed chiral components through projection operators:

$$P_R = \tfrac{1}{2}(1+\gamma^5)$$
$$P_L = \tfrac{1}{2}(1-\gamma^5)$$

  - $P_R$ projects the right handed chiral particle states and the left handed chiral antiparticle states

$$P_R u_R = u_R$$
$$P_R u_L = 0$$
$$P_R v_R = 0$$
$$P_R v_L = v_L$$

  - $P_L$ projects the left handed chiral particle states and the right handed chiral antiparticle states

$$P_L u_R = 0$$
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$$P_L v_L = 0$$

# Chiral structure of the weak interaction

- Remember the Chiral projection operators
- We also saw that in QED, only two combinations of spinors give non-zero currents (chiral nature of QED)
- For the weak interaction vertex:

$$j_{RR}^{\mu} = \frac{g_W}{\sqrt{2}} \overline{u}_R(p')\gamma^{\mu}\frac{1}{2}(1-\gamma^5)u_R(p)$$
$$= \frac{g_W}{\sqrt{2}} \overline{u}_R(p')\gamma^{\mu}P_L u_R(p) = 0,$$

the only non-zero current for particle spinors involves only left-handed chiral states

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$$
\begin{aligned}
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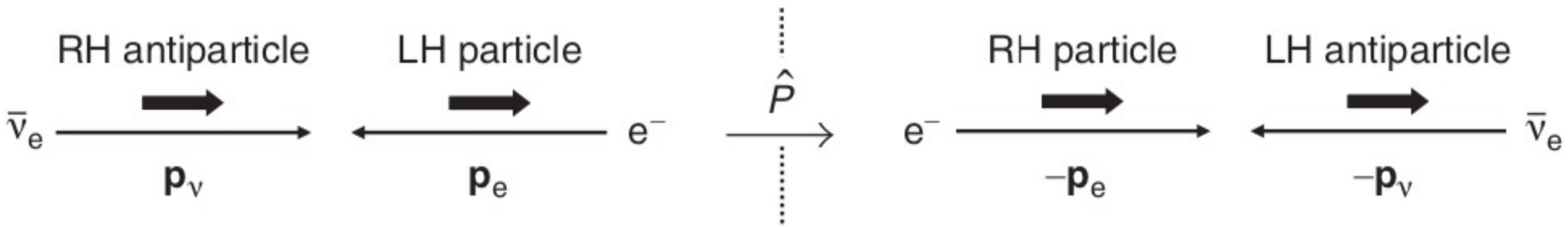
&= \frac{g_W}{\sqrt{2}}\,\overline{u}_R(p')\gamma^{\mu}P_L u_R(p) = 0,
\end{aligned}
$$

the only non-zero current for particle spinors involves only left-handed chiral states

- For anti-particle spinors, PL projects the right-handed chiral states, so only right handed chiral anti-particle states interact with the charged weak interaction

- Going back to the Wu experiment:

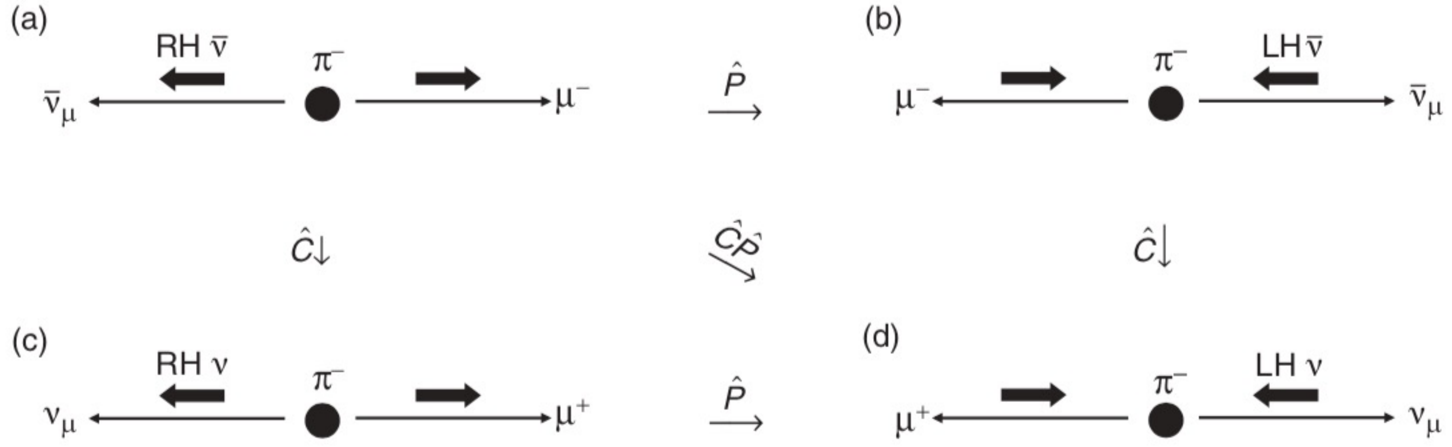



allowed　　　　　　not allowed

- So Parity violation implies we see more electrons in one direction than the opposite

# Chiral structure of the weak interaction

- Charge conjugation is also violated in the weak interactions
- However: Charge conjugation AND Parity could* be conserved
- This is called the combined CP symmetry




*CP violation is necessary to explain the difference in matter and anti-matter in the Universe. QED and QCD conserve CP, so the only place for CP violation in the Standard Model is the weak interaction. CP violation has been observed in meson systems, but this is not sufficient to account for the matter-antimatter asymmetry in the Universe

# The W boson

- For massive bosons have an additional degree of freedom of a longitudinal polarisation state:

$$\sum_{\lambda} \epsilon_{\mu}^{\lambda *} \epsilon_{\nu}^{\lambda} = -g_{\mu\nu} + \frac{q_{\mu} q_{\nu}}{m_{\mathrm{W}}^2}$$

so the Feynman rule for the propagator of the W boson:

$$\frac{-i}{q^2 - m_{\mathrm{W}}^2} \left( g_{\mu\nu} - \frac{q_{\mu} q_{\nu}}{m_{\mathrm{W}}^2} \right)$$

- In the limit $q^2 \ll m_W^2$:

$$\frac{-i g_{\mu\nu}}{q^2 - m_{\mathrm{W}}^2}$$

# Fermi theory revisited

- At low energies the propagator can be approximated by:

$$i\frac{g_{\mu\nu}}{m_{\mathrm{W}}^2}$$

- This would correspond to an interaction that occurs at a single point (Fermi theory):

$$\mathcal{M}_{fi} = G_{\mathrm{F}}\, g_{\mu\nu}[\overline{\psi}_3\gamma^{\mu}\psi_1][\overline{\psi}_4\gamma^{\nu}\psi_2]$$

- Including the V-A structure to account for parity violation:

$$\mathcal{M}_{fi} = \frac{1}{\sqrt{2}}G_{\mathrm{F}}\, g_{\mu\nu}[\overline{\psi}_3\gamma^{\mu}(1-\gamma^5)\psi_1][\overline{\psi}_4\gamma^{\nu}(1-\gamma^5)\psi_2]$$

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- And comparing to the full expression in the limit $q^2 \ll m_W^2$:

$$M_{fi} = \frac{g_W^2}{8m_W^2} g_{\mu\nu}[\overline{\psi}_3\gamma^\mu(1-\gamma^5)\psi_1][\overline{\psi}_4\gamma^\nu(1-\gamma^5)\psi_2]$$

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- At low energies the propagator can be approximated by:

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- This would correspond to an interaction that occurs at a single point (Fermi theory)

- We can relate both coupling constants:

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}$$

- The strength of the weak interaction is most precisely determined from low-energy measurements
- Muon lifetime ($m_\mu^2 \ll m_W^2$):

$$\Gamma(\mu^- \to e^- \nu_\mu \overline{\nu}_e) = \frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

with the measurements of the muon mass and lifetime:

$$m_\mu = 0.105\,658\,371\,5(35)\,\mathrm{GeV} \qquad \tau_\mu = 2.196\,981\,1(22) \times 10^{-6}\,\mathrm{s}$$

we can have a precise measurement of the Fermi constant:

$$G_F = 1.166\,38 \times 10^{-5}\,\mathrm{GeV}^{-2}$$

# Strength of the weak interaction

- The strength of the weak interaction is related to the Fermi constant by the mass of the W boson:

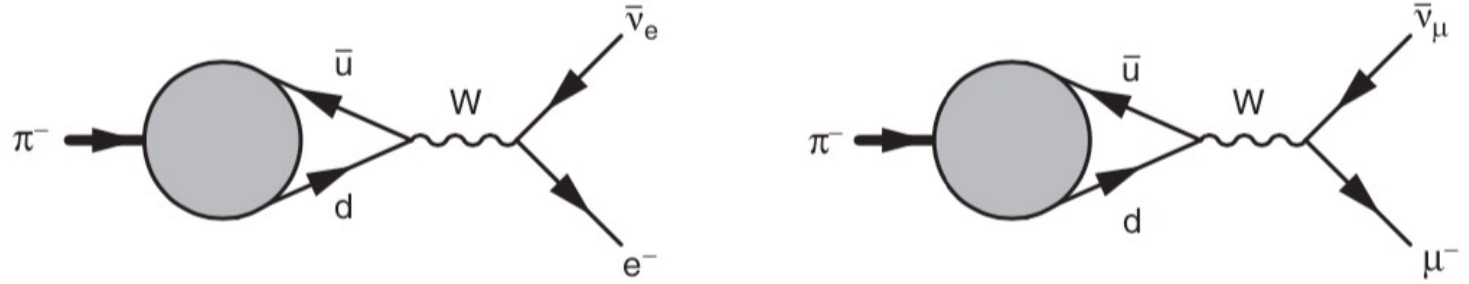
$$m_{\mathrm{W}} = 80.385 \pm 0.015\,\mathrm{GeV}$$

$$\alpha_W = \frac{g_{\mathrm{W}}^2}{4\pi} = \frac{8 m_{\mathrm{W}}^2 G_{\mathrm{F}}}{4\sqrt{2}\pi} \approx \frac{1}{30}$$

- The coupling constant itself is larger than the QED constant
- However: the presence of the mass of the W boson in the propagator, makes the weak interaction weaker than QED

# Pion decays

- Pions are the lightest mesons formed by the lightest quarks
- They can not decay via the strong interaction
- Pions decay via the weak interaction into leptons

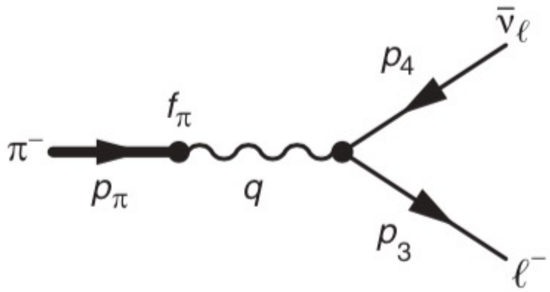


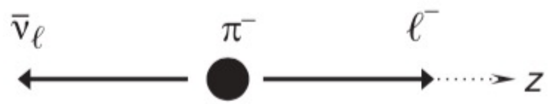

- Pions are found to decay much more frequently into muons than electrons:

$$\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} = 1.230(4) \times 10^{-4}$$

# Pion decays

- **Let us calculate the decay rate:**




$$p_\pi = (m_\pi, 0, 0, 0), \quad p_\ell = p_3 = (E_\ell, 0, 0, \mathrm{p}) \quad \text{and} \quad p_{\overline{\nu}} = p_4 = (\mathrm{p}, 0, 0, -\mathrm{p})$$

$$j_\ell^\nu = \frac{g_W}{\sqrt{2}} \overline{u}(p_3) \frac{1}{2} \gamma^\nu (1 - \gamma^5) v(p_4)$$

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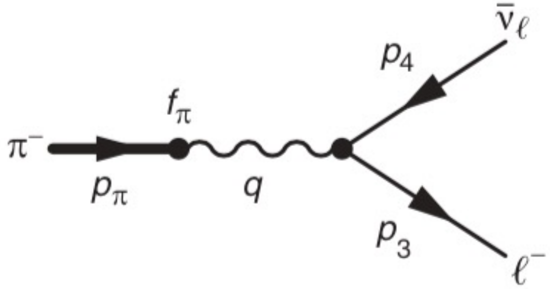


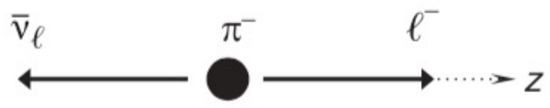


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- The pion current has to be a four-vector: pion four-momentum:

$$f_\pi p_\pi^\mu$$

where f is a constant associated with the decay

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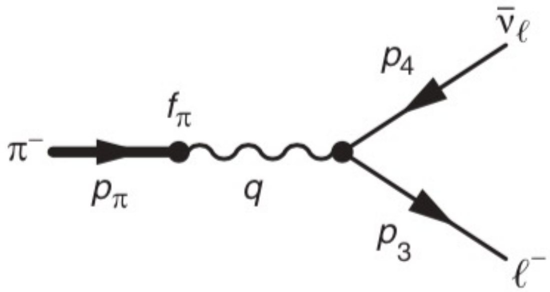


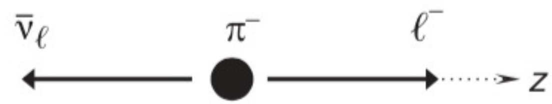


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- The matrix element:

$$\mathcal{M}_{fi} = \left[\frac{g_W}{\sqrt{2}} \frac{1}{2} f_\pi p_\pi^\mu\right] \times \left[\frac{g_{\mu\nu}}{m_W^2}\right] \times \left[\frac{g_W}{\sqrt{2}} \bar{u}(p_3)\gamma^\nu \frac{1}{2}(1-\gamma^5)v(p_4)\right]$$

pion current — Propagator (approx. Fermi) — lepton current

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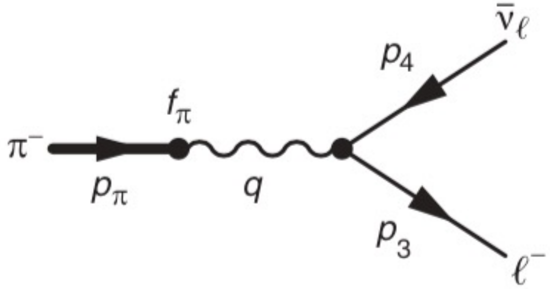


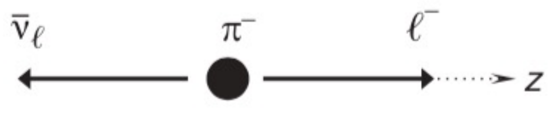


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$$= \frac{g_W^2}{4m_W^2} g_{\mu\nu} f_\pi p_\pi^\mu \bar{u}(p_3)\gamma^\nu \tfrac{1}{2}(1-\gamma^5)v(p_4),$$

where we will only have the time-like component

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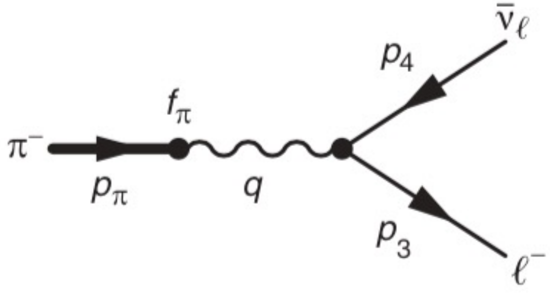


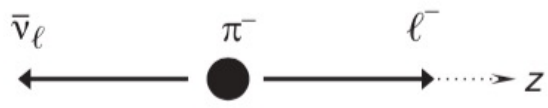


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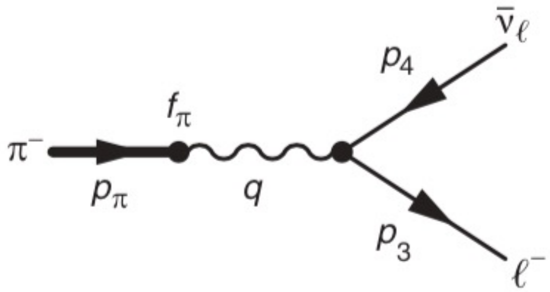
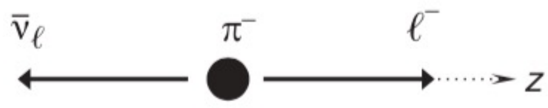


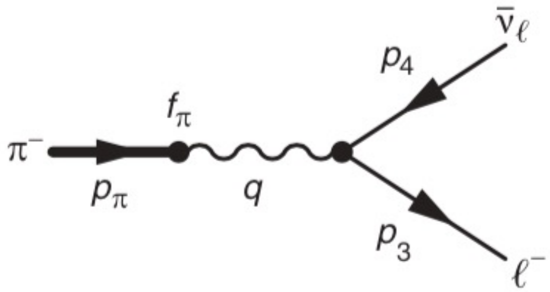

$$\mathcal{M}_{fi} = \frac{g_W^2}{4m_W^2} f_\pi m_\pi u^\dagger(p_3)\tfrac{1}{2}(1-\gamma^5)v(p_4)$$

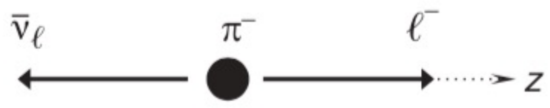
- For the neutrino the helicity eigenstates are the same as the chiral states ($m \ll E$):

$$\mathcal{M}_{fi} = \frac{g_W^2}{4m_W^2} f_\pi m_\pi u^\dagger(p_3)v_\uparrow(p_4)$$

# Pion decays

- Let us calculate the decay rate:




$$\mathcal{M}_{fi} = \frac{g_W^2}{4m_W^2} f_\pi m_\pi u^\dagger(p_3) v_\uparrow(p_4)$$

- The explicit form of the spinors for the lepton and neutrino:

$$u_\uparrow(p_3) = \sqrt{E_\ell + m_\ell}\begin{pmatrix} 1 \\ 0 \\ \frac{\mathrm{p}}{E_\ell + m_\ell} \\ 0 \end{pmatrix} \qquad u_\downarrow(p_3) = \sqrt{E_\ell + m_\ell}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -\frac{\mathrm{p}}{E_\ell + m_\ell} \end{pmatrix} \qquad v_\uparrow(p_4) = \sqrt{\mathrm{p}}\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

only one combination is not zero

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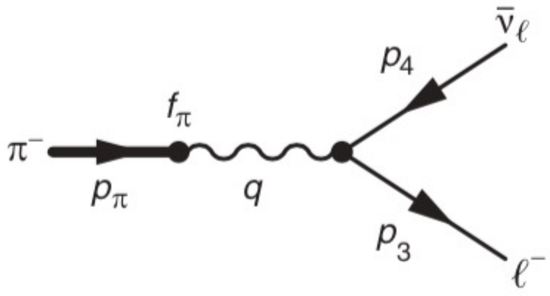


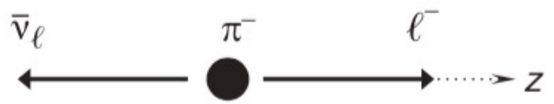


$$\mathcal{M}_{fi} = \frac{g_W^2}{4m_W^2} f_\pi m_\pi \sqrt{E_\ell + m_\ell}\,\sqrt{\mathrm{p}}\left(1 - \frac{\mathrm{p}}{E_\ell + m_\ell}\right)$$

- Which can be re-written using: $E_\ell = \dfrac{m_\pi^2 + m_\ell^2}{2m_\pi}$ $\quad \mathrm{p}_\ell = \dfrac{m_\pi^2 - m_\ell^2}{2m_\pi}$

to get:

$$\mathcal{M}_{fi} = \left(\frac{g_W}{2m_W}\right)^2 f_\pi m_\ell (m_\pi^2 - m_\ell^2)^{\frac{1}{2}}$$

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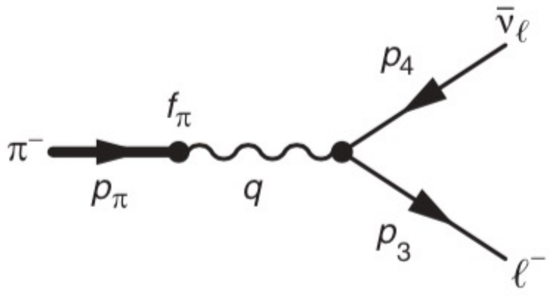


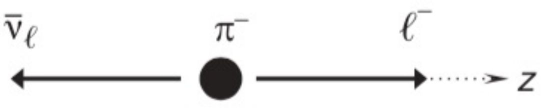


- The matrix-element squared and corresponding decay rate:

$$\langle|\mathcal{M}_{fi}|^2\rangle \equiv |\mathcal{M}_{fi}|^2 = 2G_F^2 f_\pi^2 m_\ell^2 (m_\pi^2 - m_\ell^2)$$

$$\Gamma = \frac{4\pi}{32\pi^2 m_\pi^2} \mathrm{p} \langle|\mathcal{M}_{fi}|^2\rangle = \frac{G_F^2}{8\pi m_\pi^3} f_\pi^2 \left[m_\ell (m_\pi^2 - m_\ell^2)\right]^2$$

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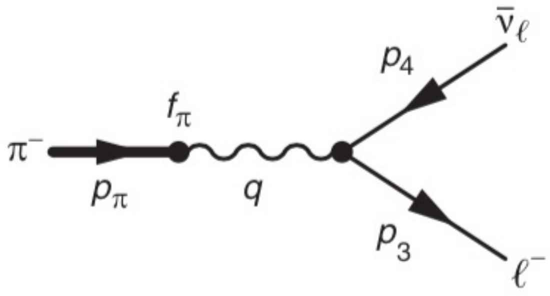



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- Finally, the ratio: $$\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} = \left[\frac{m_e(m_\pi^2 - m_e^2)}{m_\mu(m_\pi^2 - m_\mu^2)}\right]^2 = 1.26 \times 10^{-4}$$

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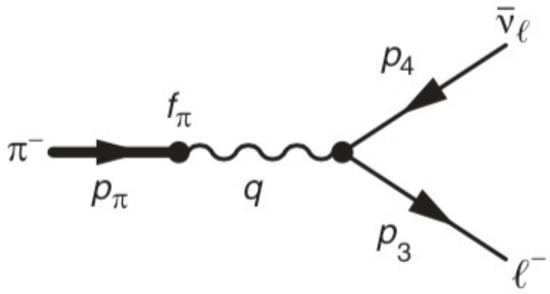


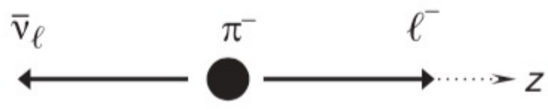


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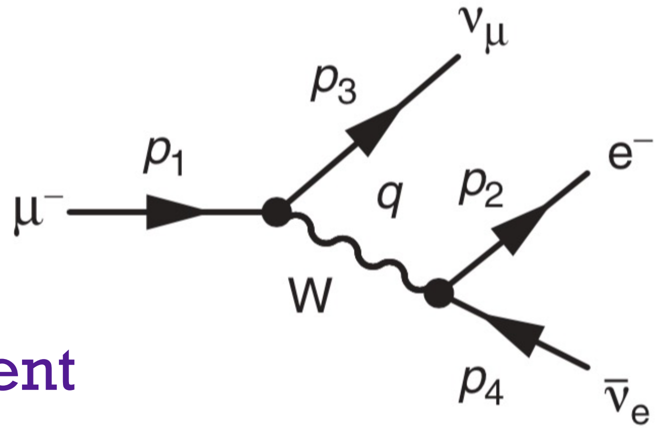
this is evidence of the V-A structure of the weak interaction

# Muon decay

- Muons do not decay via QED (the photon does not change flavour)
- Only the charged weak interaction changes lepton type

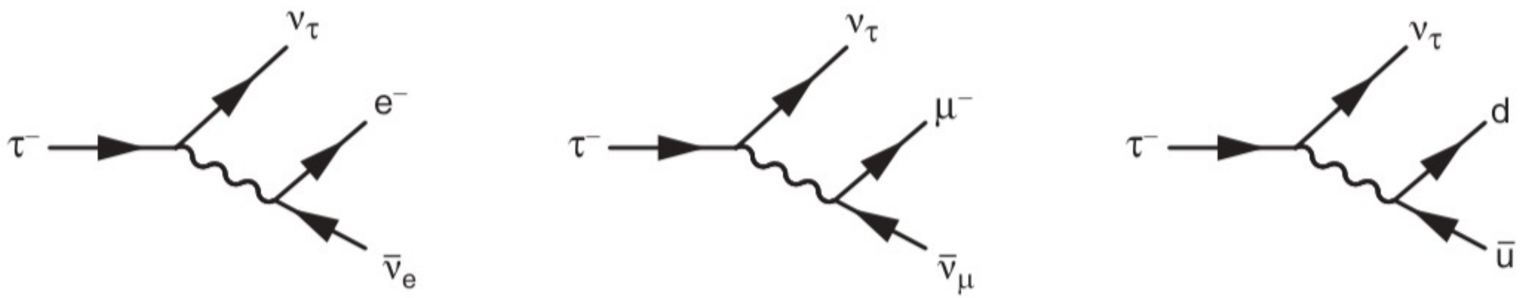
$$\Gamma(\mu^- \to e^- \overline{\nu}_e \nu_\mu) \equiv \frac{1}{\tau_\mu} = \frac{G_F^{(e)} G_F^{(\mu)} m_\mu^5}{192\pi^3}$$




in principle the coupling to different leptons could vary

# Tau decay

- Taus are sufficiently massive that they can decay into light quarks




$$Br(\tau^- \to e^-\overline{\nu}_e\nu_\tau) = \frac{\Gamma(\tau^- \to e^-\overline{\nu}_e\nu_\tau)}{\Gamma} = \Gamma(\tau^- \to e^-\overline{\nu}_e\nu_\tau) \times \tau_\tau$$

$$\Gamma(\tau^- \to e^-\overline{\nu}_e\nu_\tau) = \frac{G_F^{(e)} G_F^{(\tau)} m_\tau^5}{192\pi^3}$$

# Lepton Universality

- We can compare the lifetimes to obtain the ratio of the couplings

$$\frac{G_F^{(\tau)}}{G_F^{(\mu)}} = \frac{m_\mu^5 \tau_\mu}{m_\tau^5 \tau_\tau} Br(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$$

- Measuring masses, lifetimes, and decay fractions:

$$Br(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = 0.1783(5)$$
$$m_\mu = 0.1056583715(35)\,\text{GeV}$$
$$m_\tau = 1.77682(16)\,\text{GeV}$$

$$Br(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau) = 0.1741(4)$$
$$\tau_\mu = 2.1969811(22) \times 10^{-6}\,\text{s}$$
$$\tau_\tau = 0.2906(10) \times 10^{-12}\,\text{s}.$$

- We get a ratio for the couplings:

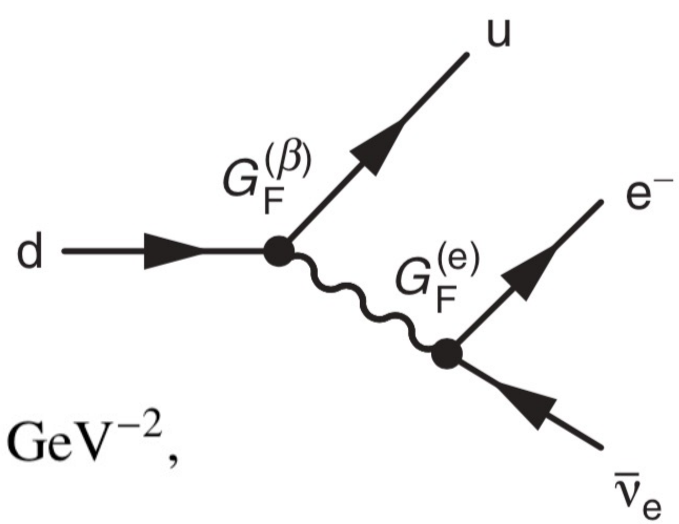
$$\frac{G_F^{(\tau)}}{G_F^{(\mu)}} = 1.0023 \pm 0.0033$$

$$\frac{G_F^{(e)}}{G_F^{(\mu)}} = 1.000 \pm 0.004$$

# Weak interactions of quarks

- Consider the underlying process for beta decay

$$|\mathcal{M}|^2 \propto G_F^{(e)} G_F^{(\beta)}$$




From observed decay rates, and comparing to the lepton coupling:

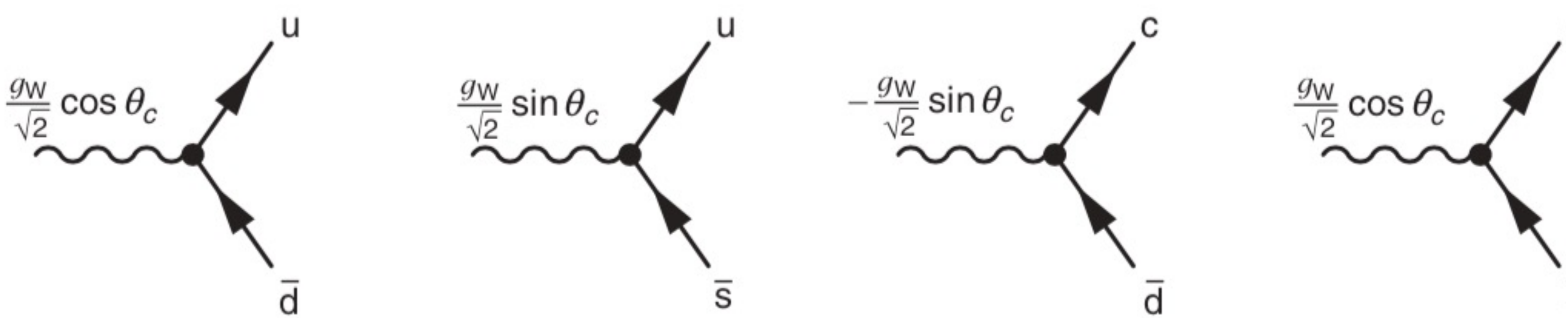
$$G_F^{(\mu)} = (1.166\,3787 \pm 0.000\,0006) \times 10^{-5}\,\mathrm{GeV}^{-2},$$
$$G_F^{(\beta)} = (1.1066 \pm 0.0011) \times 10^{-5}\,\mathrm{GeV}^{-2}.$$

- The coupling to $ud$ quarks is 5% smaller
- It is also found that the decay rate of $K^-(u\bar{s}) \to \mu^-\bar{\nu}_\mu$ is much smaller than for $\pi^-(u\bar{d}) \to \mu^-\bar{\nu}_\mu$
- The coupling to quarks does not seem to be universal

# Quark mixing

- Cabibbo hypothesis: weak interactions of quarks have the same strength as the leptons, but the weak eigenstates of quarks differ from the mass eigenstates

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$




- The differences in the decay rates can be explained by a Cabibbo angle of 13°

# CKM matrix

- The Cabibbo mechanism can be extended to three generations (the charm had not been discovered when Cabibbo proposed his hypothesis):

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

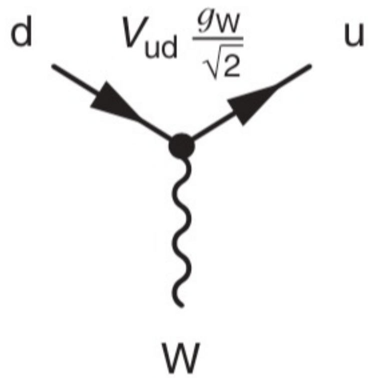
which is known as the Cabibbo-Kobayashi-Maskawa (CKM) matrix

- The weak charged current vertices are then:

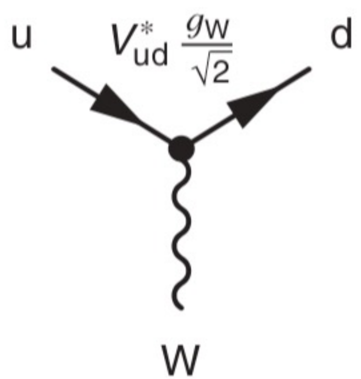
$$-i\frac{g_W}{\sqrt{2}}(\bar{u}, \bar{c}, \bar{t})\gamma^{\mu}\frac{1}{2}(1-\gamma^5)\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

# CKM matrix

- If the down-type quark is the adjoint spinor in the matrix element, the CKM element is the complex conjugate
- The CKM matrix is unitary and can be written in terms of 3 rotation angles and a complex phase




$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \times \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta'} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta'} & 0 & c_{13} \end{pmatrix} \times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$




$$s_{ij} = \sin\phi_{ij}$$
$$c_{ij} = \cos\phi_{ij}$$

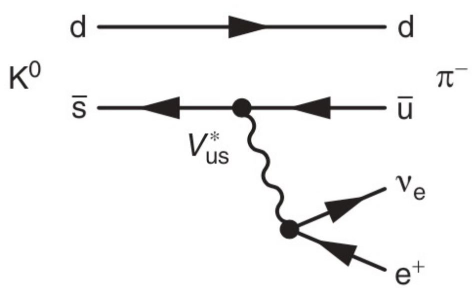
# CKM matrix

- From nuclear $\beta$ decay:

$$|V_{ud}| = \cos\theta_c = 0.974\,25(22)$$

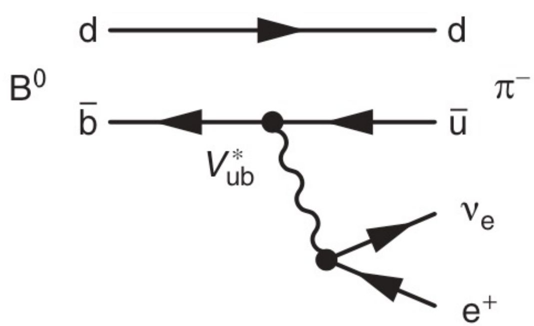
- From $K^0 \to \pi^- e^+ \nu_e$ ay:

$$|V_{us}| = 0.225\,2(9)$$




- Similarly, from B meson decays:

$$|V_{ub}| = (4.15 \pm 0.49) \times 10^{-3}$$



# CKM matrix

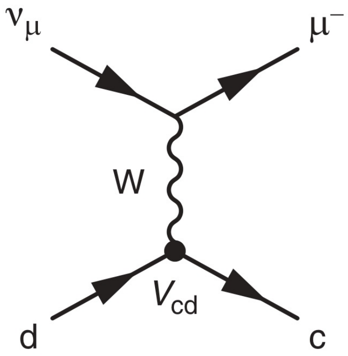
- From D meson decays :  $|V_{cs}| = 1.006 \pm 0.023$

  $D_s^+ \rightarrow \mu^+ \nu_\mu$

- From semi-leptonic B meson decays:

  $$|V_{cb}| = (40.9 \pm 1.1) \times 10^{-3}$$

- From neutrino-nucleon scattering:




$$|V_{cd}| = 0.230(11)$$

- For the top: $|V_{td}| = (8.4 \pm 0.6) \times 10^{-3}$ $|V_{ts}| = (42.9 \pm 2.6) \times 10^{-3}$

# CKM matrix

- The CKM matrix is unitary, so:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$
$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1$$
$$|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1$$

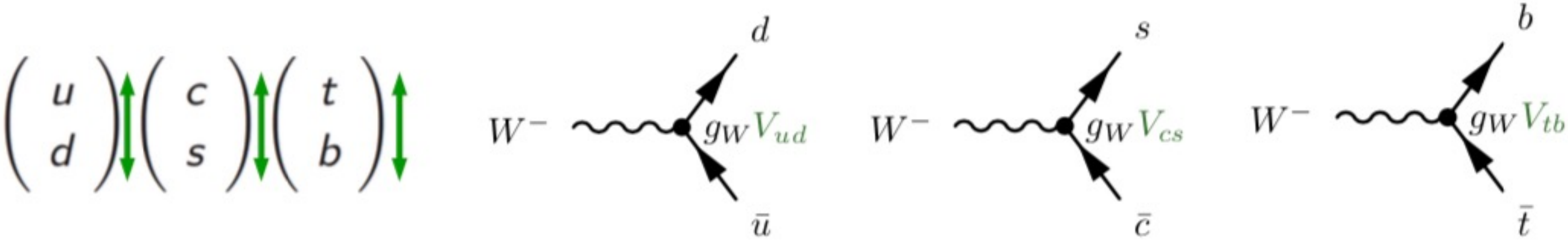
- Summarising:

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.225 & 0.004 \\ 0.225 & 0.973 & 0.041 \\ 0.009 & 0.040 & 0.999 \end{pmatrix}$$

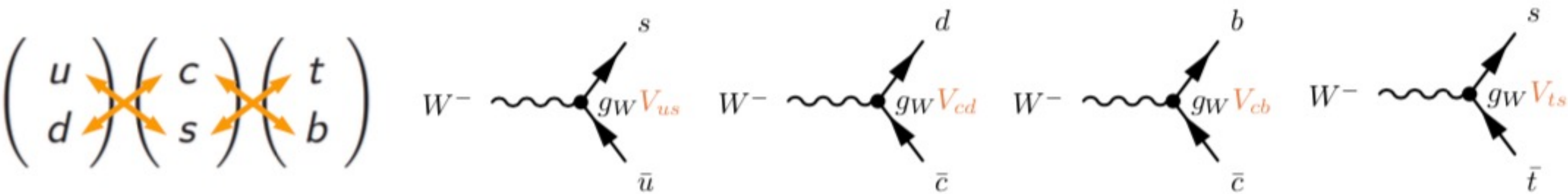
$$\phi_{12} = 13^\circ \qquad \phi_{23} = 2.3^\circ \qquad \phi_{13} = 0.2^\circ$$

# CKM matrix

- Between the same family: Cabibbo allowed




- Between quarks differing one family: Cabibbo suppressed




- Between quarks differing two families: Doubly Cabibbo suppressed